SAMARQAND VILOYATi PEDAGOGLARNI YANGI METODIKALARGA O‘RGATISH MILLIY MARKAZI

Z.OCHILOV, S. UMAROV

TRIGONOMETRIK TENGLAMALAR VA TENGSIZLIKLARNI O‘RGANISH USULLARI

(Umumta’lim maktablarning matematika fani o‘qituvchilari uchun uslubiy ko‘rsatma)

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Trigonometrik tenglamalar va tengsizliklarni o‘rganish usullari. (Umumta’lim maktablarning matematika fani o‘qituvchilari uchun uslubiy ko‘rsatma). Samarqand VPYMO‘MM. 2023-yil. 28 bet.

Tuzuvchilar: **Z.Ochilov -** Samarqand Davlat Universiteti Differensial tenglamalar kafedrasi dotsenti

S.Umarov– Samarqand viloyat pedagoglarni yangi metodikalarga o‘rgatish milliy markazi aniq va tabiiy fanlar metodikasi kafedrasi katta o‘qituvchisi

Taqrizchilar: **F.Gafarova**–Samarqand shahar 6-maktab o‘qituvchisi.

A.Samyayev–Samarqand VPYMO‘MM aniq va tabiiy fanlar metodikasi kafedrasi katta o‘qituvchisi

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Kirish

Ta’limni tubdan isloh etish jarayonida zamonaviy pedagogik texnologiyalarni qo‘llash orqali o‘quvchilar ta’lim-tayyorgarlik darajasini va bilim, ko‘nikma hamda malakalarini chuqurlashtirib, kengaytirish bugungi kunning asosiy vazifalaridan biridir. Bunga o‘z navbatida malaka oshirishdan olingan yangilik birinchi navbatda o‘quvchilarning o‘zlashtirish darajasini tahlili va natijasining monitoringiga, fan o‘qituvchilarining zamonaviy pedagogik texnologiyalar asosida noan’anaviy, interfaol hamda innovasion usullarni dars jarayonlariga qo‘llab, ta’lim-tarbiya samaradorligini oshirish orqali erishish mumkin.

O‘rta maktablarda matematika o‘qitishning maqsadi quyidagi uch omil bilan belgilanadi:

1. Matematika o‘qitishning umumta’limiy maqsadi.
2. Matematika o‘qitishning tarbiyaviy maqsadi.
3. Matematika o‘qitishning amaliy maqsadi.

Matematika o‘qitishning umumta’limiy maqsadi o‘z oldiga quyidagi vazifalarni qo‘yadi:

a) O‘quvchilarga ma’lum bir dastur asosida matematik bilimlar tizimini berish. Bu bilimlar tizimi matematika fani to‘g‘risida o‘quvchilarga yetarli darajada ma’lumot berishi, ularni matematika fanining yuqori bo‘limlarini o‘rganishga tayyorlashi kerak. Bundan tashqari, dastur asosida o‘quvchilar o‘qish jarayonida olgan bilimlarining ishonchli ekanligini tekshira bilishga o‘rganishlari, ya’ni isbotlash va nazorat qilishning asosiy metodlarini egallashlari kerak.

b) O‘quvchilarning og‘zaki va yozma matematik bilimlarini tarkib toptirish.

Matematikani o‘rganish o‘quvchilarning o‘z ona tillarida xatosiz so‘zlash, o‘z fikrini aniq, ravshan va lo‘nda qilib bayon eta bilish malakalarini o‘zlashtirishlariga yordam berishi kerak. Bu degan so‘z o‘quvchilarning har bir matematik qoidani o‘z ona tillarida to‘g‘ri gapira olishlariga erishish hamda ularni ana shu qoidaning matematik ifodasini formulalar yordamida to‘g‘ri yoza olish qobiliyatlarini atroflicha shakllantirish demakdir;

v) O‘quvchilarni matematik qonuniyatlar asosida real haqiqatlarni bilishga o‘rgatish. Bu yerda o‘quvchilarga real olamda yuz beradigan eng sodda hodisalardan tortib to murakkab hodisalargacha hammasining fazoviy formalari va ular orasidagi miqdoriy munosabatlarni tushunishga imkon beradigan hajmda bilimlar berish ko‘zda tutiladi.

Bunday bilimlar berish orqali esa o‘quvchilarning fazoviy tasavvur qilishlari shakllanadi hamda mantiqiy tafakkur qilishlari yanada rivojlanadi.

Matematika o‘qitishning tarbiyaviy maqsadi o‘z oldiga quyidagilarni qo‘yadi:

a) O‘quvchilarda ilmiy dunyoqarashni shakllantirish. Bu g‘oya bilish nazariyasi asosida amalga oshiriladi.

b) O‘quvchilarda matematikani o‘rganishga bo‘lgan qiziqishlarni tarbiyalash.

Bizga ma’lumki, matematika darslarida o‘quvchilar o‘qishning dastlabki kunlaridanoq mustaqil ravishda xulosa chiqarishga o‘rganadilar. Ular avvalo kuzatishlar natijasida, so‘ngra esa mantiqiy tafakkur qilish natijasida xulosa chiqaradilar. Ana shu chiqarilgan xulosalar matematik qonuniyatlar bilan tasdiqlanadi.

Matematika o‘qituvchisining vazifasi o‘quvchilarda mustaqil mantiqiy fikrlash qobiliyatlarini shakllantirish bilan birga ularda matematikaning qonuniyatlarini o‘rganishga bo‘lgan qiziqishlarini tarbiyalashdan iboratdir.

v) O‘quvchilarda matematik tafakkurni va matematik madaniyatni shakllantirish. Matematika darslarida o‘rganiladigan har bir matematik xulosa qat’iylikni talab qiladi, bu esa o‘z navbatida juda ko‘p matematik tushuncha va qonuniyatlar bilan ifodalanadi. O‘quvchilar ana shu qonuniyatlarni bosqichma-bosqich o‘rganishlari davomida ularning mantiqiy tafakkur qilishlari rivojlanadi, matematik xulosa chiqarish madaniyatlari shakllanadi. O‘quvchilarni biror matematik qonuniyatni ifoda qilmoqchi bo‘lgan fikrlarni simvolik tilda to‘g‘ri ifodalay olishlari va aksincha simvolik tilda ifoda qilingan matematik qonuniyatni o‘z ona tillarida ifoda qila olishlariga o‘rgatish orqali ularda matematik madaniyat shakllantiriladi.

3. Matematika o‘qitishning amaliy maqsadi o‘z oldiga quyidagi vazifalarni qo‘yadi:

a) Matematika kursida olingan nazariy bilimlarni kundalik hayotda uchraydigan elementar masalalarni yechishga tadbiq qila olishga o‘rgatish. Bunda asosan o‘quvchilarda nazariy bilimlarni amaliyotga bog‘lay olish imkoniyatlarini tarkib toptirish, ularda turli sonlar va matematik ifodalar ustida amallar bajarish malakalarini shakllantirish va ularni mustahkamlash uchun maxsus tuzilgan amaliy masalalarni hal qilishga o‘rgatiladi.

b) Matematikani o‘qitishda texnik vosita va ko‘rgazmali qurollardan foydalanish malakalarini shakllantirish. Bunda o‘quvchilarning matematika darslarida texnika vositalaridan, matematik ko‘rgazmali qurollar, jadvallar va hisoblash vositalaridan foydalana olish malakalari tarkib toptiriladi.

v) O‘quvchilarni mustaqil ravishda matematik bilimlarni egallashga o‘rgatish. Bunda asosan o‘quvchilarni o‘quv darsliklaridan va ilmiy-ommaviy matematik kitoblardan mustaqil o‘qib o‘rganish malakalarini shakllantirishdan iboratdir.

Trigonometrik funksiyalar qadimgi Gretsiyada astronomiya va geometriyadagi tekshirishlar bilan bog‘liq holda paydo bo‘ldi. Bizning eramizgacha bo‘lgan III asrda Arximed, Appolloniya Pergskogo, Yevklid va boshqalarning ishlarida uchragan bo‘lib, trigonometrik funksiyani aniqlanishga to‘g‘ri burchakli uchburchakning tomonlarini nisbatidan iboratdir. Trigonometrik funksiyalar nazariyasining hozirgi zamon shaklini va umuman trigonometriyani L.Eyler ta’riflagan. U trigonometrik funksiyalarni ta’rifini va hozirgi kundagi belgilashlarni kiritgan.Bu uslubiy ko‘rsatma matematikani chuqurlashtirish va murakkab tadbiqiy masalalarni yechishga katta yordam beradi.

Trigonometrik tenglamalar va tenglamalar sistemasi

1. $\sin x \cdot (1+\cos x) = 1 + \cos x + \cos^2 x$ tenglamani yeching.

Yechish. Agar $\cos x = 1$ bo‘lsa, u holda $\sin x = \frac{3}{2} > 1$ bo‘lib, bunday bo‘lishi mumkin emas.

Demak, $\cos x \neq 1$ yoki $1 - \cos x \neq 0$ U holda berilgan tenglama ikkala tomonini $1 - \cos x$ ga ko‘paytirib

$$\sin x \cdot (1-\cos^2 x) = 1 - \cos^3 x \text{ yoki } \sin^3 x + \cos^3 x = 1 \quad (*)$$

tenglikni hosil qilish mumkin.

$\sin^3 x \leq \sin^2 x$, $\cos^3 x \leq \cos^2 x$ va $\sin^2 x + \cos^2 x = 1$ munosabatlardan $\sin^3 x + \cos^2 x \leq 1$ bo‘lib, $(*)$ tenglama $\begin{cases} \sin^3 x = \sin^2 x, \\ \cos^3 x = \cos^2 x \end{cases}$ tenglamalar sistemasiga teng kuchli bo‘ladi, bundan:

$$\begin{cases} \sin x \cdot (\sin x - 1) = 0, \\ \cos x \cdot (\cos x - 1) = 0 \end{cases} \quad (**)$$

bo‘lib, $\cos x \neq 1$ shartga ko‘ra, $(**)$ tenglamalar sistemasining ikkinchi tenglamasidan $\cos x = 0$ bo‘lib, u holda $\sin x = \pm 1$ bo‘ladi. Ammo, birinchi tenglamadan $\sin x = 1$, ya’ni $x_1 = \frac{\pi}{2} + 2\pi n$ bu yerda n – butun son.

2. a,b,c,d-musbat sonlar biror arifmetik progressiyaning ketma-ket hadlari bo‘lsa, quyidagi tenglamani yeching

$$\sin ax \sin bx = \sin cx \sin dx.$$

Yechish. a,b,c,d sonlari arifmetik progressiyaning ketma-ket hadlari bo‘lganligidan:

$$b = a + r, c = a + 2r, d = a + 3r, \ (r - arifmetik\ progressiyaning\ ayirmasi)$$

$$\cos(2a + r)x - \cos(2a + 5r)x = 0,$$

$$\sin(2a + 3r)x \sin 2rx = 0.$$

Bu tenglamaning yechimlari quyidagicha bo‘ladi:

$$x_1 = \frac{\pi k}{2a+3r}, x_2 = \frac{\pi k}{2r}, k\epsilon Z.$$

Oxirgi tengliklar ma’noga ega, chunki

$$2a + 3r = b + c, r \neq 0.$$

Javob. $x_1 = \frac{\pi k}{b+c}, x_2 = \frac{\pi k}{2(b-a)}, k\epsilon Z.$

3. $1 + \cos 2x \cdot \cos 3x = \frac{1}{2}\sin^2 3x$ tenglamani yeching.

Yechish. Tenglamaning ikkala tomonini 2 ga ko‘paytirib va $\sin^2 3x = 1 - \cos^2 3x$ ayniyatdan foydalanib $\cos^2 3x + 2\cos 2x \cos 3x = -1$ tenglamani hosil qilish mumkin va bu tenglama ikkala tomoniga $\cos^2 2x$ qo‘shilsa, $(\cos 3x + \cos 2x)^2 = -\sin^2 2x$ tenglama hosil bo‘ladi va bu tenglama uning ikkala qismi ham

0 ga teng bo‘lgandagina o‘rinli bo‘ladi, ya’ni

$$\begin{cases} \cos 3x + \cos 2x = 0, \\ \sin 2x = 0 \end{cases} \qquad (*)$$

$cos 2x = 2\cos^2 x - 1$ va $\cos 3x = 4\cos^3 x - 3\cos x$ ayniyatlar yordamida sistemaning birinchi tenglamasi $\cos x$ ga nisbatan uchinchi darajali:

$$4\cos^3 x + 2\cos^2 x - 3\cos x - 1 = 0 \qquad (**)$$

tenglamaga teng kuchli bo‘ladi.

$(*)$ sistemaning ikkinchi tenglamasidan $\cos x = 0$ yoki $\sin x = 0$ bo‘ladi. Agar $\sin x = 0$ bo‘lsa, $\cos x = \pm 1$ bo‘lib, $\cos x = -1$ $(**)$ sistemani qanoatlantiradi. $\cos x = -1$ $(*)$ tenglamaning yoki berilgan tenglamaning ildizlari

$x_1 = \pi(2n + 1)$ bu yerda n – butun son.

4. $\cos 6x + \sin\frac{5}{2}x = 2$ tenglamani yeching.

Yechish. $\cos 6x \leq 1$ va $\sin\frac{5}{2}x \leq 1$. U holda:

$$\cos 6x + \sin\frac{5}{2}x \leq 2$$

bo‘lib, berilgan tenglamadan quyidagi tenglamalar sistemasi hosil bo‘ladi:

$$\begin{cases} \cos 6x = 1, \\ \sin\frac{5}{2}x = 1. \end{cases} \qquad (*)$$

Ushbu sistemaning yechimi undagi tenglamalar ildizlarining kesishmasidan iborat bo‘lib, bu kesishma bo‘sh to‘plamdan iborat bo‘lsa berilgan tenglama ildizlari mavjud emas. $(*)$ sistema tenglamalari ildizlari $\begin{cases} 6x = 2\pi n, \\ \frac{5}{2}x = \frac{\pi}{2} + 2\pi m \end{cases}$ sistemadan iborat, bu yerda m va n butun sonlar.

Bundan $\begin{cases} x = \frac{\pi n}{3} \\ x = \frac{\pi}{5}(4m + 1) \end{cases}$ bo‘lib $(**)$ ularning kesishmasini topish uchun $\frac{\pi n}{3} = \frac{\pi}{5}(4m + 1)$ qaraymiz. Demak, $5n = 12n + 3$ bo‘lib, n 3 ga karrali ya’ni ixtiyoriy butun k son uchun $n = 3k$. Natijada,

$15k = 12m + 3$ yoki $5k = 4m + 1$

bu son toq son bo‘lib, undan k ham toq son bo‘ladi ya’ni u butun son uchun: $k = 2u + 1$ u holda , $10u + 5 = 4m + 1$ yoki $5u = 2m - 2$. Ko‘rinib turibdiki, u juft son, ya’ni butun t lar uchun $u = 2t$ bo‘lib, $10t = 2m - 2$ yoki $m = 5t + 1$ u holda $(**)$ sistemaning ikkinchi tenglamasidan butun t sonlar uchun:

$$x_1 = \frac{\pi}{5}(4m + 1) = \frac{\pi}{5}(20t + 5) = \pi(4t + 1)$$

bo‘ladi.

5. $\frac{1}{\sin x} - \frac{1}{\sin 2x} = \frac{1}{\sin 4x}$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi: $\begin{cases} \sin x \neq 0, \\ \sin 2x \neq 0, \\ \sin 4x \neq 0 \end{cases}$ tengsizliklar sistemasi bilan aniqlanib, butun k- sonlari uchun $x \neq \frac{\pi}{4}k$ bo‘ladi.

Demak,

$$\frac{1}{\sin x} = \frac{1}{\sin 2x} + \frac{1}{\sin 4x}, \ \frac{1}{\sin x} = \frac{\sin 2x + \sin 4x}{\sin 2x \cdot \sin 4x},$$

$$\frac{1}{\sin x} = \frac{2\sin 3x \cdot \cos x}{2\sin x \cdot \cos x \cdot \sin 4x} \qquad (*)$$

bo‘lib, $x \neq \frac{\pi}{4}k$ shartdan $\sin x \neq 0$, $\cos x \neq 0$ va $(*)$ tenglamadan $\sin 4x - \sin 3x = 0$ yoki $2\sin\frac{x}{2}\cos\frac{7x}{2} = 0$, $\sin\frac{x}{2} \neq 0$ chunki, $x \neq \frac{\pi}{4}k$ (k-butun son), u holda $\cos\frac{7x}{2} = 0$ tenglama ildizlari $x_1 = \frac{\pi}{7}(2n+1)$ $(n - butun\ son)$ bo‘ladi. Ammo, x ning topilgan barcha qiymatlari berilgan tenglama ildizlari bo‘lmasligi mumkin. Argument x ning bu qiymatlaridan tenglamaning aniqlanish sohasiga kirmaydigan qiymatlarini chiqarib tashlashga to‘g‘ri keladi.

Shunday qilib, x ning yechim bo‘lmaydigan qiymatlarini $\frac{\pi}{7}(2n+1) = \frac{\pi}{4}k$ tenglik yordamida aniqlaymiz. Bundan $8n + 4 = 7k$, va $k = 4m$, m – butun son. Demak, $8n + 4 = 28m$ yoki $2n + 1 = 7m$ bo‘lib bunda m toq son ya’ni t- butun son uchun $m = 2t + 1$ bo‘lib, $2n + 1 = 14t + 7$ yoki $n = 7t + 3$.

Natijada, berilgan tenglama ildizlari $x_1 = \frac{\pi}{7}(2n+1)$ bo‘lib, bu yerda $n \neq 7t + 3$ va n, t – butun sonlar.

6 $sin^8x + cos^8x = \frac{17}{32}$ tenglamani yeching.

Yechish. Trigonometrik va qisqa ko‘paytirish ayniyatlaridan foydalanib berilgan tenglamada quyidagi shakl almashtirishlarni bajarish mumkin:

$$(sin^4x + cos^4x)^2 - 2sin^4xcos^4x = \frac{17}{32},$$

$$((sin^2x + cos^2x)^2 - \frac{1}{2}sin^2 2x)^2 - \frac{1}{8}sin^4 2x = \frac{17}{32},$$

$$1 - sin^2 2x + \frac{1}{8}sin^4 2x = \frac{17}{32},$$

$$sin^4 2x - 8sin^2 2x + \frac{15}{4} = 0.$$

Oxirgi tenglamadan

$$sin^2 2x = 4 \pm \frac{7}{2} \qquad yoki \quad sin^2 2x = \frac{1}{2} \quad bo‘lib,$$

$$2x = \frac{\pi}{4} + \frac{\pi k}{2} \quad yoki \ \ x = \frac{(2k+1)\pi}{8}. (k \epsilon Z)$$

7. $\cos^2 x + \frac{1}{\cos^2 x} = \cos 2x + \cos 4x$ tenglamani yeching.

Yechish. Koshi (3) tengsizligiga ko‘ra: $\cos^2 x + \frac{1}{\cos^2 x} \geq 2$ bo‘lib, ikkinchi tomondan $\cos 2x \leq 1$ va $cos4x \leq 1$, u holda: $\cos 2x + \cos 4x \leq 2$.

Demak, $\begin{cases} \cos^2 x + \frac{1}{\cos^2 x} = 2, \\ \cos 2x + \cos 4x = 2 \end{cases}$ bo‘lib, $\begin{cases} \cos^2 x = 1, \\ \cos 2x = 1, \\ \cos 4x = 1 \end{cases}$ tengliklar bajarilishi kerak bo‘ladi.

Bulardan $x = \pi k$, k- butun son .

8. $3\sin x + 4\cos 3x \cdot \cos x + 2\sin 5x = 7$ tenglamani yeching.

Yechish. Koshi-Bunyakovskiy (9) tengsizligidan foydalanib quyidagi tengsizlikni yozish mumkin:

$$(3\sin x + 4\cos 3x \cdot \cos x)^2 \leq (9 + 16\cos^2 3x) \cdot (\sin^2 x + \cos^2 x) \leq 25$$

Bundan $3\sin x + 4\cos 3x \cdot \cos x \leq 5$ bo‘ladi. $2\sin 5x \leq 2$ bo‘lganligidan,

$$3\sin x + 4\cos 3x \cdot \cos x + 2\sin 5x \leq 7$$

tengsizlikda tenglik bajarilishi uchun bir vaqtda $\frac{3}{4\cos 3x} = \frac{\sin x}{\cos x}$, $\cos^2 3x = 1$ va $\sin 5x = 1$ shartlar bajarilishi lozim.

Oxirgi ikkita tenglik bir vaqida bajarilishi uchun $\begin{cases} \cos 3x = \pm 1, \\ \sin 5x = 1 \end{cases}$ yoki $\begin{cases} x = \frac{\pi}{3} n, \\ x = \frac{\pi}{10}(4m + 1) \end{cases}$ bo‘lishi kerak bu yerda n, m – butur sonlar $\frac{\pi}{3} n = \frac{\pi}{10}(4m + 1)$ u holda $10m = 12m + 3$ bo‘lib, bu tenglikning chap tomoni juft o‘ng tomoni esa doim toq bo‘ladi. Demak, berilgan tenglama yechimga ega emas.

9. $sin^{10} x + cos^{10} x = \frac{29}{16} cos^4 2x$ tenglamani yeching.

Yechish. Darajani pasaytirish formulalaridan foydalarib berilgan tenglamada quyidagi almashtirishlarni bajarish mumkin:

$$(\frac{1 - \cos 2x}{2})^5 + (\frac{1 + \cos 2x}{2})^5 = \frac{29}{16} cos^4 2x,$$

$$(1 - \cos 2x)^5 + (1 + \cos 2x)^5 = 58 cos^4 2x,$$

$\cos 2x = y$ almashtirish yordamida haqiqiy ildizlari

$$y_{1,2} = \pm \frac{\sqrt{2}}{2}$$

bo‘lgan

$$24y^4 - 10y^2 - 1 = 0$$

bikvadrat tenglamani hosil qilish mumkin.

Demak, $\cos 2x = \pm \frac{\sqrt{2}}{2}$ bo‘lib, berilgan tenglamaning yechimi x=$\frac{2k+1}{8}\pi$, ($k \epsilon Z$) bo‘ladi.

10. $\sin x + 2\sin 2x = 3 + \sin 3x$ tenglamani yeching.

Yechish. Berilgan tenglamadan unga teng kuchli

$2\sin 2x + \sin x - \sin 3x = 3$, $2\sin 2x - 2\sin x \cdot \cos 2x = 3$

yoki

$$\sin 2x - \sin x \cdot \cos 2x = \frac{3}{2} \qquad (*)$$

tenglamani hosil qilish mumkin.

Koshi-Bunyakovskiy (9) tengsizligidan foydalanib $(*)$ tenglama chap tomoni uchun

$(\sin 2x - \sin x \cdot \cos 2x)^2 \leq (1 + \sin^2 x)(\sin^2 2x + \cos^2 2x) \leq 2$

tengsizlikni hosil qilamiz.

Bundan $2 \sin 2x - \sin x \cdot \cos 2x \leq \sqrt{2}$,ya'ni $\sin 2x - \sin x \cdot \cos 2x < \frac{3}{2}$.

Demak, $(*)$ tenglama va berilgan tenglama ildizlarga ega emas.

11. $\sqrt{2}(\sin x + \cos x) = tgx + ctg\, x$ tenglamani yeching.

Yechish. $\sin x + \cos x = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right)$ bo'lib, tenglama chap tomoni uchun $-2 \leq \sqrt{2}(\sin x + \cos x) \leq 2$ tengsizlik o'rinli bo'ladi.

$tgx + ctgx = tgx + \frac{1}{tgx}$ tenglikdan Koshi (3) va (4) tengsizligiga ko'ra: $|tgx + ctgx| \geq 2$ bo'lib, berilgan tenglama uning ikkala tomoni bir vaqtda -2 yoki 2 ga teng bo'lganda o'rinli bo'ladi. Natijada, $\begin{cases} \sqrt{2}(\sin x + \cos x) = -2, \\ tgx + ctgx = -2 \end{cases}$ va $\begin{cases} \sqrt{2}(\sin x + \cos x) = 2, \\ tgx + ctgx = 2 \end{cases}$ tenglamalar sistemasi hosil bo'ladi. Birinchi tenglamalar sistemasidan $\sin x + \cos x = -\sqrt{2}$ yoki $\sin x = \cos x = \frac{\sqrt{2}}{2}$ va $tgx = 1$ bo'ladi. Sistema umumiy yechimga ega emas,ikkinchi tenglamalar sistemasidan $\sin x = \cos x = -\frac{\sqrt{2}}{2}$ va $tgx = 1$ bo'lib, bu tenglamalarning ildizlari n- butun son uchun $x_1 = \frac{\pi}{4} + 2\pi n$ bo'ladi.

12. $\sin(\frac{\pi}{10} + \frac{3x}{2}) = 2 \sin(\frac{3\pi}{10} - \frac{x}{2})$ tenglamani yeching.

Yechish. $\frac{3\pi}{10} - \frac{x}{2} = y$ bo'lsin, u holda $\frac{\pi}{10} + \frac{3x}{2} = \pi - 3\left(\frac{3\pi}{10} - \frac{x}{2}\right) = \pi - 3y$ bo'lib, berilgan tenglama quyidagi ko'rinishga keladi:

$$\sin 3y = 2 \sin y,$$

$$\sin y\,(4sin^2 y - 1) = 0.$$

Oxirgi tenglamaning yechimlari:

$$y_1 = \pi k, y_2 = (-1)^k \frac{\pi}{6} + \pi k, y_3 = (-1)^{k+1} \frac{\pi}{6} + \pi k, k\epsilon Z$$

bo'ladi. Mos ravishda berilgan tenglamaning yechimlari:

$$x_1 = \frac{3\pi}{5} + \pi n, x_2 = \frac{3\pi}{5} + (-1)^n \frac{\pi}{3} + \pi n, x_3 = \frac{3\pi}{5} + (-1)^{n+1} \frac{\pi}{3} + \pi n, n\epsilon Z$$

13. $\cos^2 x - \cos^4 x = \sin^2 x \cdot \sin 3x - 1$ tenglamani yeching.

Yechish. Tenglamaning chap va o'ng tomonlarini quyidagicha baholaymiz: $|\cos x| \leq 1$ bo'lib, $\cos^4 x \leq \cos^2 x$ va $\cos^2 x - \cos^4 x \geq 0$ shuningdek, $\sin^2 x \leq 1$,

$\sin 3x \leq 1$ bo‘lib, $\sin^2 x \cdot \sin 3x - 1 \leq 0$. Demak, berilgan tenglama $\begin{cases} \cos^2 x - \cos^4 x = 0, \\ \sin^2 x \cdot \sin 3x = 1 \end{cases}$ $(*)$ tenglamalar sistemasiga teng kuchli bo‘lib,

$\cos^2 x - \cos^4 x = \cos^4 x \cdot (1 - \cos^2 x) = \sin^2 x \cdot \cos^2 x$, $0 \leq \sin^2 x \leq 1$ va $-1 \leq \sin 3x \leq 1$ bo‘ladi. U holda $(*)$ tenglamalar sistemasi $\begin{cases} \sin x \cdot \cos x = 0, \\ \sin^2 x = 1, \\ \sin 3x = 1 \end{cases}$ $(**)$ tenglamalar sistemasiga teng kuchli bo‘ladi.

Agar $\sin^2 x = 1$ bo‘lsa, u holda $\cos x = 0$ va $\sin x \cdot \cos x = 0$ bo‘lib, $\sin 3x = -4\sin^3 x + 3\sin x$ formulaga ko‘ra $(**)$ tenglamalar sistemasidan $\begin{cases} \sin x = \pm 1, \\ 4\sin^3 x - 3\sin x = -1 \end{cases}$ $(***)$ tenglamalar sistemasi hosil bo‘ladi va bu sistema $\sin x = -1$ tenglamaga teng kuchli bo‘lib, uning, ya’ni berilgan tenglamaning ildizlari $x_1 = -\frac{\pi}{2} + 2\pi k$ bu yerda k – butun son.

14. $\left(\sin x + \sqrt{3}\cos x\right) \cdot \sin 4x = 2$ tenglamani yeching.

Yechish. Berilgan tenglamadan osongina

$\left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right) \cdot \sin 4x = 1$ yoki $\sin\left(x + \frac{\pi}{3}\right) \cdot \sin 4x = 1(*)$ tenglamani hosil qilish mumkin.

Ma’lumki, $-1 \leq \sin\left(x + \frac{\pi}{3}\right) \leq 1$ va $-1 \leq \sin 4x \leq 1$ bo‘lib, u holda $(*)$ tenglamadan quyidagi ikkita

$$\begin{cases} \sin\left(x + \frac{\pi}{3}\right) = -1, \\ \sin 4x = -1 \end{cases} \quad \text{va} \quad \begin{cases} \sin\left(x + \frac{\pi}{3}\right) = 1, \\ \sin 4x = 1 \end{cases} \qquad (*)$$

tenglamalar sistemasi hosil bo‘lib, birinchi sistemaning yechimlari m, n – butun sonlar uchun:

$$\begin{cases} x + \frac{\pi}{3} = -\frac{\pi}{2} + 2\pi n, \\ 4x = -\frac{\pi}{2} + 2\pi m \end{cases} \text{yoki} \quad \begin{cases} x = -\frac{5\pi}{6} + 2\pi k = \frac{\pi}{6}(12n - 5), \\ x = -\frac{\pi}{8} + \frac{\pi}{2}m = \frac{\pi}{8}(4m - 1) \end{cases}$$ ularning kesishmasi (umumiysi)ni topish uchun

$\frac{\pi}{6}(12n - 5) = \frac{\pi}{8}(4m - 1)$ yoki $4(12n - 5) = 3(4m - 1)$ bo‘lib, oxirgi tenglikning chap tomoni doim juft, o‘ng tomoni esa toq bo‘ladi va $(*)$ sistemaning birinchisi yechimga ega emas.

Ikkinchi tenglamalar sistemasining yechimlari butun k va l lar uchun: $\begin{cases} x + \frac{\pi}{3} = \frac{\pi}{2} + 2\pi k, \\ 4x = \frac{\pi}{2} + 2\pi l \end{cases}$ va $\begin{cases} x = \frac{\pi}{6} + 2\pi k = \frac{\pi}{6}(12k + 1), \\ x = \frac{\pi}{8} + \frac{\pi}{2}l = \frac{\pi}{8}(4l + 1) \end{cases}$ bo‘lib,bundan $\frac{\pi}{6}(12k + 1) = \frac{\pi}{8}(4l + 1)$ va $4(12k + 1) = 3(4l + 1)$.

Yuqoridagi holga mos xulosa chiqarib berilgan tenglama haqiqiy ildizlarga ega emasligini ko‘rsatish mumkin.

15. $\sqrt{\frac{2\cos x}{3}}+\sqrt{\frac{2-2\cos x}{3}}=\frac{2}{3}\sin x+\frac{1}{2\sin x}$ tenglamani yeching.

Yechish. Shartga ko‘ra $\sin x>0$ va $\cos x\geq 0$ bo‘lib, butun n- soni uchun $2\pi n<x\leq\frac{\pi}{2}+2\pi n$ (x- birinchi chorakka tegishdi). Koshi-Bunyakovskiy (9) tengsizligiga ko‘ra berilgan tenglamaning chap tomoni:

$\left(\sqrt{\frac{2\cos x}{3}}+\sqrt{\frac{2-2\cos x}{3}}\right)^2\leq(1^2+1^2)\left(\frac{2\cos x}{3}+\frac{2-2\cos x}{3}\right)=\frac{4}{3}$,ya’ni $\sqrt{\frac{2\cos x}{3}}+\sqrt{\frac{2-2\cos x}{3}}\leq\frac{2}{\sqrt{3}}$ bo‘lib, $\sin x>0$ Koshi (1) tengsizligiga ko‘ra tenglamaning o‘ng tomoni: $\frac{2}{3}\sin x+\frac{1}{2\sin x}\geq 2\sqrt{\frac{2}{3}\sin x\cdot\frac{1}{2\sin x}}=\frac{2}{\sqrt{3}}$ bo‘ladi.

Demak, $\frac{2}{3}\sin x+\frac{1}{2\sin x}=\frac{2}{\sqrt{3}}$ bo‘lib, $\sin x=\pm\frac{\sqrt{3}}{2}$ yoki $\cos x=\pm\frac{1}{2}$ ammo, shartga ko‘ra $\sin x>0$ va $\cos x\geq 0$ bo‘lib, $\sin x=\frac{\sqrt{3}}{2}$ va $\cos x=\frac{1}{2}$ topilgan $\sin x$ va $\cos x$ qiymatlarini berilgan tenglamaga qo‘yib uning ildizlari butun n- soni uchun $x_1=\frac{\pi}{3}+2\pi k$ ekanligiga ishonch hosil qilish mumkin.

16. $sin^3x+sin^3 2x+sin^3 3x=(\sin x+\sin 2x+\sin 3x)^3$ tenglamani yeching.

Yechish. Avvaldan ma’lum:

$$(x+y+x)^3-x^3-y^3-z^3=3(x+y)(z+x)(z+y)$$

ayniyatdan foydalanib berilgan tenglamada quyidagicha almashtirishlar bajarish mumkin:

$$(\sin x+\sin 2x)(\sin 2x+\sin 3x)(\sin x+\sin 3x)=0,$$

$$\sin\frac{3x}{2}\sin 2x\sin\frac{5x}{2}\cos x\, cos^2\frac{x}{2}=0.$$

Oxirgi tenglikdagi har bir ko‘paytuvchini 0 ga tenglab quyidagi beshta yechimni yozish mumkin:

1) x=$\frac{2n_1}{3}\pi$; 2) x=$\frac{n_2}{2}\pi$; 3) $x=\frac{2n_3}{5}\pi$; 4) $x=\frac{2n_4+1}{2}\pi$; 5) $x=(2n_5+1)\pi$.

Bu yerda n_1,n_2,n_3,n_4,n_5-butun sonlar.

4 va 5 yechimlar 2-yechimda qatnashadi. Demak, berilgan tenglamaning yechimlari:

$x=\frac{2n_1}{3}\pi; x=\frac{n_2}{2}\pi; x=\frac{2n_3}{5}\pi.$ Bu yerda n_1,n_2,n_3-butun sonlar.

17. $\left(\sin^2 x+\frac{1}{\sin^2 x}\right)^2+\left(\cos^2 x+\frac{1}{\cos^2 x}\right)^2=12+\frac{\sin y}{2}$ tenglamani yeching.

Yechish. $n=2$ bo‘lganda (10) tengsizlikka ko‘ra: $a^2+b^2\geq\frac{(a+b)^2}{2}$ $(*)$. Bu tengsizlikdan foydalanib tenglamaning chap tomonini quyidagicha yozish mumkin:

$\left(\sin^2 x+\frac{1}{\sin^2 x}\right)^2+\left(\cos^2 x+\frac{1}{\cos^2 x}\right)^2\geq\frac{1}{2}\Big(\sin^2 x+\frac{1}{\sin^2 x}+\cos^2 x+$

$\left.\frac{1}{\cos^2 x}\right)^2 = \frac{1}{2}\left(1+\frac{1}{\sin^2 x\cdot\cos^2 x}\right)^2 == \frac{1}{2}\left(1+\frac{4}{\sin^2 2x}\right)^2 \geq \frac{1}{2}(1+4)^2 = 12\frac{1}{2}.$

Ikkinchi tomondan: $12+\frac{\sin u}{2} \leq 12\frac{1}{2}$. Natijada,

$$\begin{cases} \sin^2 x + \frac{1}{\sin^2 x} = \cos^2 x \frac{1}{\cos^2 x}, \\ \sin y = 1 \end{cases} \qquad (**)$$

tenglamalar sistemasini hosil qilish mumkin. $0 < x < 1$ bo'lganda $f(z) = z^2 + \frac{1}{z^2}$funksiyani qaraymiz u holda $(**)$ tenglamalar sistemasining birinchi tenglamasi $f(\sin x) = f(\cos x)$ ko'rinishidagi funksional tenglama bo'lib, $0 < z < 1$ uchun $f'(z) = \frac{2(z^4-1)}{z^3} < 0$ bo'lib, $f(x)$ funksiya shu oraliqda kamayuvchi bo'ladi. Shuningdek, $f(z)$ – juft funksiya ekanligidan $f(\sin x) = f(\cos x)$ tenglama $\sin x = \pm\cos x$ tenglamaga teng kuchli bo'lib, bu tenglamaning ildizi n- butun son uchun $x_1 = \frac{\pi}{4}(2n+1)$ bo'ladi. $(**)$ sistema ikkinchi tenglamasining ildizi k – butun son uchun $y_1 = \frac{\pi}{2}(4k+1)$ bo'ladi.

18. $\cos x + cosy - \cos(x+y) = \frac{3}{2}$ tenglamani yeching.

Yechish. Tenglamani quyidagicha shakl almashtirish mumkin:

$$2\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2} - 2\cos^2\frac{x+y}{2} + 1 = \frac{3}{2},$$

$$2\cos^2\frac{x+y}{2} - 2\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2} + \frac{1}{2} = 0,$$

$$4\cos^4\frac{x+y}{2} - 4\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2} +$$

$$+\cos^2\frac{x-y}{2} - \cos^2\frac{x-y}{2} + 1 = 0.$$

$$\left(2\cos\frac{x+y}{2} - 2\cos\frac{x-y}{2}\right)^2 + \sin^2\frac{x-y}{2} = 0 \ (*)$$

$(*)$ tenglamadan $\begin{cases} 2\cos\frac{x+y}{2} = \cos\frac{x-y}{2}, \\ \sin\frac{x-y}{2} = 0 \end{cases}$ $(**)$ tenglamalar sistemasini hosil qilish mumkin.

$(**)$ sistemaning ikkinchi tenglamasidan $\cos\frac{x-y}{2} = \pm 1$ bo'lib, u holda $(**)$ tenglamalar sistemasi

$$\begin{cases} \cos\frac{x+y}{2} = \pm\frac{1}{2}, \\ \sin\frac{x-y}{2} = 0 \end{cases} \qquad (***)$$

tenglamalar sistemasiga teng kuchli bo'ladi. $(***)$ tenglamalar sistemasining yechimlari n va k butun sonlar uchun: $\frac{x+y}{2} = \frac{\pi}{3}(3n \pm 1)$ va $\frac{x-y}{2} = \pi k$ bo'ladi.

Natijada, berilgan tenglama ildizlari butun n va k sonlar uchun $x_1 = \frac{\pi}{3}(3n + 3k + 1)$, $y_1 = \frac{\pi}{3}(3n - 3k + 1)$, $x_2 = \frac{\pi}{3}(3n + 3k - 1)$, $y_2 = \frac{\pi}{3}(3n - 3k - 1)$

bo‘ladi.

19. $\frac{\sin^{10}x+\cos^{10}x}{4}=\frac{\sin^6 x+\cos^6 x}{4\cos^2 2x+\sin^2 2x}$ tenglamani yeching.

Yechish. Tenglamaning chap tomonini quyidagicha shakl almashtirish mumkin: $\frac{\sin^6 x+\cos^6 x}{4\cos^2 2x+\sin^2 2x}=\frac{\sin^6 x+\cos^6 x}{4(\cos^2 x+\sin^2 x)+4\sin^2 x\cdot\cos^2 x}=\frac{(\sin^2 x+\cos^2 x)(\sin^4 x-\sin^2 x\cdot\cos^2 x+\cos^2 x)}{4(\sin^4 x-\sin^2 x\cdot\cos^2 x+\cos^4 x)}=\frac{1}{4}$. U holda berilgan tenglama

$\sin^{10}x+\cos^{10}x=1$ $(*)$ ko‘rinishga keladi.$sin^2x+cos^2x=1$ ayniyatdan foydalanib $(*)$ tenglamani quyidagi ko‘rinishda yozish mumkin: $\sin^{10}x+\cos^{10}x=\sin^2 x+\cos^2 x$, $\sin^2 x-\sin^{10}x+\cos^2 x-\cos^{10}x=0$ yoki $\sin^2 x\cdot(1-\sin^8 x)+\cos^2 x\cdot(1-\cos^8 x)=0$ $(**)$ Ma’lumki, $\sin^2 x\geq 0$, $\cos^2 x\geq 0$, $\sin^8 x\leq 1$, $\cos^8 x\leq 1$ va $1-\sin^8 x\geq 0$, $1-\cos^8 x\geq 0$ bo‘lib, ularga ko‘ra $(**)$ tenglamadan $\begin{cases}\sin x=\pm1,\\ \cos x=0\end{cases}$ yoki $\begin{cases}\sin x=0,\\ \cos x=\pm1\end{cases}$ bo‘lib, ildizlari n – butun son uchun $x_1=\frac{\pi}{2}n$ bo‘ladi.

20. $(\cos 4x-\cos 2x)^2=\sin 3x+5$ tenglamani yeching.

Yechish. Ma’lumki,

$$|\cos 4x-\cos 2x|\leq 2\ va\ \sin 3x+5\geq 4$$

munosabatlar o‘rinli bo‘lib,berilgan tenglama yechimga ega bo‘lishi uchun

$$|\cos 4x-\cos 2x|=2\ va\ \sin 3x=-1$$

tengliklar bajarilishi lozim.Bunda ikki holni qarashga to‘g‘ri keladi:

$$a)\ \cos 2x=1, x=\pi k;$$

$$\cos 4x=-1, x=\left(\frac{n}{2}+\frac{1}{4}\right)\pi;$$

$$\sin 3x=-1, x=-\frac{\pi}{6}+\frac{2\pi}{3}l.$$

n,k,l-butun sonlar. Ushbu holda umumiy yechim mavjud emas.

$$b)\ \cos 4x=1, x=\frac{n}{2}\pi;$$

$$\cos 2x=-1, x=\left(k+\frac{1}{2}\right)\pi;$$

$$\sin 3x=-1, x=-\frac{\pi}{6}+\frac{2}{3}\pi l=\frac{4l-1}{6}\pi.$$

n,k,l-butun sonlar. Bu holda umumiy yechim:

$$x=\left(2m+\frac{1}{2}\right)\pi, m\epsilon Z.$$

21. $1+\cos^6 x=2\cdot\sqrt[3]{\cos 2x}$ tenglamani yeching.

Yechish. Ma’lumki, $\cos 2x=2\cos^2 x-1$ bo‘lib, u holda berilgan tenglamani $1+\cos^6 x=2\cdot\sqrt[3]{2\cos^2 x-1}$ ko‘rinishda yozib $\cos^2 x=y$ almashtirish yordamida $0\leq y\leq 1$ bo‘lganda $1+y^3=2\cdot\sqrt[3]{2y-1}$ $(*)$ tenglamani yozish mumkin.U holda

$1+y^3 \geq 1$ bo‘lib, $(*)$ tenglamadan $2\cdot\sqrt[3]{2y-1} \geq 1$ yoki $y \geq \frac{9}{16}$ shunday qilib, $\frac{9}{16} \leq y \leq 1$ va $(*)$ tenglama $y=\sqrt[3]{2\cdot\sqrt[3]{2y-1}-1}$ $(**)$ ko‘rinishni oladi.

Agar $f(z)=\sqrt[3]{2z-1}$ bo‘lsa (**) tenglama $u=f(f(y))$ ko‘rinishdagi funksional tenglama bo‘lib, $f(z)=\sqrt[3]{2z-1}$ funksiya oz o‘qida o‘suvchi, u holda $y=f(f(y))$ tenglama o‘zgaruvchi qabul qilishi mumkin bo‘lgan qiymatlarida $f(y)=y$ tenglamaga teng kuchli, ya’ni $\frac{9}{16} \leq y \leq 1$ bo‘lganda y=$\sqrt[3]{2y-1}$ tenglikning ikkala tomonini kubga ko‘tarib, $y^3-2y+1=0$ yoki $(y-1)(y^2+y-1)=0$ bundan $\frac{9}{16} \leq y \leq 1$ shartga ko‘ra $y_1=1$ va $y_2=\frac{\sqrt{5}-1}{2}$.

Shuningdek, $\cos^2 x=y$ bo‘lib u holda $\cos x=\pm 1$ va $\cos x=\pm\sqrt{\frac{\sqrt{5}-1}{2}}$ tengliklardan berilgan tenglamaning ildizlari butun n va k uchun $x_1=\pi n$ va $x_2=\pm arccos\sqrt{\frac{\sqrt{5}-1}{2}}+\pi k$ bo‘ladi.

22. $\sqrt[4]{2\sin x-1}+\sqrt[4]{3\sin x-2}=2$ tenglamani yeching.

Yechish. $\sin x \leq 1$ bo‘lib $2\sin x-1 \leq 1$ va $3\sin x-2 \leq 1$ bo‘lganligidan $\sqrt[4]{2\sin x-1} \leq 1$, $\sqrt[4]{3\sin x-2} \leq 1$ va $\sqrt[4]{2\sin x-1}+\sqrt[4]{3\sin x-2} \leq 2$.

Demak, berilgan tenglama faqat va faqat $\sqrt[4]{2\sin x-1}=1$ va $\sqrt[4]{3\sin x-2}=1$ tengliklar bajarilgandagina o‘rinli bo‘ladi.

Demak, $\sin x=1$ yoki $x_1=\frac{\pi}{2}+2\pi n$ bu yerda n- butun son .

23. $32cos^6x-\cos 6x=1$ tenglamani yeching.

Yechish.

$$cos^6x=(\frac{1+\cos 2x}{2})^3=\frac{1}{8}(1+3\cos 2x+3cos^2 2x+cos^3 2x)$$

va

$$\cos 6x=4cos^3 2x-3\cos 2x$$

ayniyatlardan foydalanib berilgan tenglamani quyidagi ko‘rinishda yozish mumkin:

$$4cos^2 2x+5\cos 2x+1=0$$

yoki

$$(\cos 2x)_1=-1, (\cos 2x)_2=-\frac{1}{4}$$

bo‘lib,

$$x_1=\left(k+\frac{1}{2}\right)\pi, x_2=\pm\frac{1}{2}\arccos(-\frac{1}{4})+\pi k, k\epsilon Z$$

24. $2\sin^2 x+2\sin^2 y=\sin x+\sin y+\sin x\cdot\sin y-1$ tenglamani yeching.

Yechish. Tenglama ikkala tomonini 2 ga ko‘paytirib, almashtirishlar yordamida:

$$2\sin^2 x+2\sin^2 y-2\sin x-2\sin y-2\sin x\cdot\sin y+2=0,$$

$$(\sin^2 x - 2\sin x + 1) + (\sin^2 y - 2\sin y + 1) + (\sin^2 x - 2\sin x \cdot \sin y + \sin^2 y) = 0,$$

$$(\sin x - 1)^2 + (\sin y - 1)^2 + (\sin x - \sin y)^2 = 0$$

tenglamani hosil qilish mumkin. Tenglama chap tomonidagi ko‘paytuvchilar nomanfiy bo‘lib tenglik $\sin x = \sin y = 1$ bo‘lganda bajariladi.

Demak, $x_1 = \frac{\pi}{2} + 2\pi n$ va $y_1 = \frac{\pi}{2} + 2\pi k$ bu yerda n, k- butun sonlar.

25. $2 + 2(\sin y + \cos y) \cdot \sin x = \cos 2x$ tenglamani yeching.

Yechish. $\cos 2x = 1 - 2\sin^2 x$ formuladan foydalanib berilgan tenglamani $\sin x$ ga nisbatan kvadrat tenglama ko‘rinishida yozish mumkin:

$$2sin^2 x + 2(\sin y + \cos y)\sin x + 1 = 0 \qquad (*)$$

Ushbu tenglama ildizga ega bo‘lishi uchun uning diskriminanti nomanfiy bo‘lishi kerak, ya’ni D=($(\sin y + \cos y)^2$ −2≥0.

Ammo, $-\sqrt{2} \le \sin y + \cos y \le \sqrt{2}$ ekanligidan $|\sin y + \cos y| = \sqrt{2}$ tenglik o‘rinli bo‘ladi.

Agar $\cos y + \sin y = \sqrt{2}$ bo‘lsa, u holda $y_1 = \frac{\pi}{4} + 2\pi n$ bo‘lib (∗) tenglamadan n va k butun sonlar uchun $2\sin^2 x + 2\sqrt{2}\sin x + 1 = 0$, $\left(\sqrt{2}\sin x + 1\right)^2 = 0$, $\sin x = -\frac{\sqrt{2}}{2}$ va $x_1 = (-1)^{k+1}\frac{\pi}{4} + \pi k$ bo‘ladi.

Agar $\cos y + \sin y = -\sqrt{2}$ bo‘lsa, u holda $y_2 = \frac{5\pi}{4} + 2\pi l (l - butunson)$ bo‘lib, tenglama

$$2\sin^2 x - 2\sqrt{2}\sin x + 1 = 0$$

ko‘rinishni oladi. Bundan $\left(\sqrt{2}\sin x - 1\right)^2 = 0$, $\sin x = \frac{\sqrt{2}}{2}$ va $x_2 = (-1)^m\frac{\pi}{4} + \pi m$ bu yerda m – butun son.

26. $x^2 + 2\sin \pi x = 3x + \cos^2 \pi x - \frac{17}{4}$ tenglamani yeching.

Yechish. Berilgan tenglamani quyidagicha shakl almashtirish mumkin:

$$x^2 + 2\sin \pi x = 3x + 1 - \sin^2 \pi x - \frac{17}{4}, x^2 + 2\sin \pi x - 3x + \sin^2 \pi x + \frac{13}{4} = 0,$$

$$(\sin \pi x + 1)^2 + \left(x - \frac{3}{2}\right)^2 = 0.$$

Oxirgi tenglama $\begin{cases} \sin \pi x + 1 = 0, \\ x - \frac{3}{2} = 0 \end{cases}$ tenglamalar sistemasiga teng kuchli bo‘lib, u yagona $x_1 = \frac{3}{2}$ ildizga ega.

27. $1 - 2\sin^2 x + \sqrt{3} \cdot \sin 2x = 12x^2 - 4\pi x + \frac{\pi^2}{3} + 2$ tenglamani yeching.

Yechish. Tenglamaning o‘ng tomonida quyidagicha shakl almashtirishlarni bajarish mumkin:

$$1 - 2\sin^2 x + \sqrt{3}\sin 2x =$$

$$= \cos 2x + \sqrt{3} \cdot \sin 2x = 2 \cdot \left(\frac{1}{2}\cos 2x + \frac{\sqrt{3}}{2}\sin 2x\right)$$

$$= 2 \cdot \left(\sin\frac{\pi}{6} \cdot \cos 2x + \cos\frac{\pi}{6} \cdot \sin 2x\right) = 2\sin\left(2x + \frac{\pi}{6}\right).$$

Natijada, $12x^2 - 4\pi x + \frac{\pi^2}{3} + 2 = 2\sin\left(2x + \frac{\pi}{6}\right)$ tenglama hosil bo‘ladi va bundan:

$12x^2 - 4\pi x + \frac{\pi^2}{3} + 2 \leq 2,\ 36x^2 - 12\pi x + \pi^2 \leq 0$ yoki $(6x - \pi)^2 \leq 0$ va $(6x - \pi)^2 = 0$ yoki $x_1 = \frac{\pi}{6}$. x- ning bu qiymatini berilgan tenglamaga qo‘yib ko‘rib, $x_1 = \frac{\pi}{6}$ berilgan tenglama ildizi ekanligini ko‘rish mumkin.

28. $(\sin^4 x + \cos^2 2x) \cdot \left(\frac{1}{\sin^4 x} + \frac{1}{\cos^2 2x}\right) = 4\cos^2\sqrt{\frac{\pi^2}{4} - x^2}$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi $-\frac{\pi}{2} \leq x < 0$ va $0 < 0 \leq \frac{\pi}{2}$ oraliqlardan iborat.

$a > 0, b > 0$ bo‘lganda $(a + b) \cdot \left(\frac{1}{a} + \frac{1}{b}\right) \geq 4$ $(*)$ tengsizlikdan foydalanib tenglamaning chap tomoni $(\sin^4 x + \cos^2 2x)\left(\frac{1}{\sin^4 x} + \frac{1}{\cos^2 2x}\right) \geq 4$ bo‘lib, o‘ng tomoni esa $4\cos^2\sqrt{\frac{\pi^2}{4} - x^2} \leq 4$ bo‘ladi.

Demak, $\begin{cases} (\sin^4 x + \cos^2 2x) \cdot \left(\frac{1}{\sin^4 x} + \frac{1}{\cos^2 2x}\right) = 4, \\ 4\sin^2\sqrt{\frac{\pi^2}{3} - x^2} = 4 \end{cases}$ bo‘lib, ushbu sistemadan tenglamaning aniqlanish sohasini hisobga olib $x_1 = \frac{\pi}{2}$ va $x_2 = -\frac{\pi}{2}$ ildizlarni topish mumkin. Ularni berilgan tenglamaga qo‘yib ko‘rib haqiqatan, ular berilgan tenglama ildizlari ekanligiga ishonch hosil qilish mumkin.

Mustaqil yechish uchun misollar

1. $sin^3x\cos x-\sin x\,cos^3x=\frac{1}{4}$, x=$-\frac{\pi}{8}+\frac{\pi k}{2}, k\epsilon Z$;

2. $\frac{1-tgx}{1+tgx}=1+\sin 2x$ x=$\pi k, k\epsilon Z$;

3. $1+\sin x+\cos x+\sin 2x+\cos 2x=0, x_1=-\frac{\pi}{4}+\pi k, x_2=\pm\frac{2\pi}{3}+2\pi k$;

4. 1+ $\sin x+\cos 3x=\cos x+\sin 2x+\cos 2x\; x_1=\pi k, x_2=(-1)^{k+1}\frac{\pi}{6}+\pi k, x_3=\pm\frac{\pi}{3}+2\pi k$;

5. $(\sin 2x+\sqrt{3}\cos 2x)^2-5=\cos\left(\frac{\pi}{6}-2x\right)$, x=$\frac{7\pi}{12}+\pi k$;

6. $2\sin 17x+\sqrt{3}\cos 5x+\sin 5x=0, x_1=-\frac{\pi}{66}+\frac{\pi k}{11}, x_2=\frac{\pi}{36}+\frac{(2k+1)\pi}{12}$;

7. $sin^2x(tgx+1)=3\sin x(\cos x-\sin x)+3$, $x_1=-\frac{\pi}{4}+\pi k, x_2=\frac{\pi}{3}+\pi k, x_3=-\frac{\pi}{3}+\pi k$;

8. $sin^3x+cos^3x=1-\frac{1}{2}\sin 2x, x_1=2\pi k, x_2=\frac{\pi}{2}+2\pi k$;

9. $\frac{1}{sin^2x}-\frac{1}{cos^2x}-\frac{1}{tg^2x}-\frac{1}{ctg^2x}-\frac{1}{sec^2x}-\frac{1}{cosec^2x}=-3$, x=$\frac{1}{4}\pi+\frac{\pi k}{2}$;

10. $sin^4\frac{x}{3}+cos^4\frac{x}{3}=\frac{5}{8}$, x= $\frac{3n\pm1}{2}\pi$;

11. $\frac{1}{2}(sin^4x+cos^4x)=sin^2xcos^2x+\sin x\cos x$, x=$(-1)^k\frac{1}{2}arcsin\frac{\sqrt{5}-1}{2}+\frac{\pi k}{2}$;

12. $(1+k)\frac{\cos x\cos(2x-\propto)}{\cos(x-\propto)}=1+k\cos 2x$, x=$\frac{\alpha}{2}+(-1)^n\frac{1}{2}\arcsin(k\sin\alpha)+\frac{\pi n}{2}, |\, k\sin\propto|\le 1$;

13. 2+ $\cos x=2tg\frac{x}{2}$, x=$\frac{\pi}{2}+2\pi k$;

14. ctgx$-2\sin 2x=1 x_1=\frac{\pi}{8}+\frac{\pi k}{2}, x_2=\frac{3\pi}{4}+\pi k$;

15. $2\cos x\cos(\beta-x)=\cos\beta$, x=$\pm\frac{\pi}{4}+\frac{\beta}{2}+\pi k$,

16. $\sin x+\sin(\varphi-x)+\sin(2\varphi+x)=\sin(\varphi+x)+\sin(2\varphi-x)\; va\,\varphi$ burchak koordinata tekisligining uchinchi choragida yotishi ma’lum bo‘lsa $\cos\varphi$ ni hisoblang

$$\cos\varphi=\frac{1-\sqrt{5}}{4}.$$

17. $\sin 2x-12(\sin x-\cos x)+12=0$, $x_1=\pi+2\pi, x_2=\frac{\pi}{2}+2\pi$k;

18. 1+2cosecx=$-\frac{sec^2\frac{x}{2}}{2}$, x=$-\frac{\pi}{2}+2\pi k$;

19. $ctg^2x=\frac{1+\sin x}{1+\cos x}$, $x_1=\frac{\pi}{4}+k\pi, x_2=-\frac{\pi}{2}+2k\pi$;

20. $2tg3x-3tg2x=tg^2 2xtg3x$, x=kπ;

21. 6tgx+5ctg3x=tg2x x=$\pm\frac{1}{2}arccos\frac{1}{3}+\pi k$, x=$\pm\frac{1}{2}(\pi-arccos\frac{1}{4})+\pi k$

22. $sin^5x-cos^5x=\frac{1}{\cos x}-\frac{1}{\sin x}$ x=$\frac{\pi}{4}+\pi n$

23. $tg(x-\frac{\pi}{4})$tgxtg(x+$\frac{\pi}{4}$) $=\frac{4cos^2x}{tg\frac{x}{2}-ctg\frac{x}{2}}$ $\emptyset$

24. $\cos\pi\frac{x}{31}\cos2\pi\frac{x}{31}\cos4\pi\frac{x}{31}\cos8\pi\frac{x}{31}\cos16\pi\frac{x}{31}=\frac{1}{32}$ $x_1=2n,n\neq 31l,x_2=\frac{31}{33}(2n+1),n\neq 33l+16$;l$\epsilon Z$

25. $\cos7x-\sin5x=\sqrt{3}(\cos5x-\sin7x)$ x=$\frac{\pi}{12,}(12k+1)$,x=$\frac{\pi}{24}(4m+1)$ (k,mϵZ)

26. $2-(7+\sin2x)\,sin^2x+(7+\sin2x)sin^4x=0$ x=$\frac{\pi}{4}+\pi k$,kϵZ

27. $\begin{cases}\sin(x+y)=0,\\ \sin(x-y)=0,\\ 0\le x\le\pi,0\le x\le\pi.\end{cases}$ (0;0),($\frac{\pi}{2};\frac{\pi}{2}$)$,(0;\pi),(\pi;0),(\pi;\pi)$

28. $\begin{cases}\sin x=cosecx+\sin y,\\ \cos x=secx+\cos y.\end{cases}$ $x=\frac{2k+2n+1}{4}\pi,y=\frac{2k-6n}{4}\pi$ (k,nϵZ)

29. $\begin{cases}sin^3x=\frac{1}{2}\sin y,\\ cos^3x=\frac{1}{2}\cos y.\end{cases}$ $x_1=\frac{\pi}{4}+2k\pi,y_1=\frac{\pi}{4}+2l\pi;x_2=\frac{\pi}{4}+(2k+1)\pi,y_2=\frac{\pi}{4}+(2l+1)\pi$

30. $x_3=\frac{3\pi}{4}+2k\pi,y_3=\frac{3\pi}{4}+2l\pi;x_4=\frac{3\pi}{4}+(2k+1)\pi,y_4=\frac{3\pi}{4}+(2l+1)\pi$ (k,lϵ)

31. $\begin{cases}tgx+tgy=1,\\ \cos x\cos y=\frac{1}{\sqrt{2}}.\end{cases}$ $x_1=\frac{\pi}{4}+(k+l)\pi,y_1=(k-l)\pi;x_2=(k+l)\pi,y_2=\frac{\pi}{4}+(k-l)\pi(k,l\epsilon Z)$

32. $\begin{cases}\sin x\sin y=\frac{1}{4\sqrt{2}},\\ tgxtgy=\frac{1}{3}.\end{cases}$ $x_{1,2}=(k+l)\pi+\frac{1}{2}arccos\frac{1}{2\sqrt{2}}\pm\frac{\pi}{8},y_{1,2}=(l-k)\pi+\frac{1}{2}arccos\frac{1}{2\sqrt{2}}\mp\frac{\pi}{8}$;

$$x_{3,4}=(k+l)\pi-\frac{1}{2}arccos\frac{1}{2\sqrt{2}}\pm\frac{\pi}{8},y_{3,4}=(l-k)\pi-\frac{1}{2}arccos\frac{1}{2\sqrt{2}}\mp\frac{\pi}{8}$$

33. $\begin{cases}tgx+\frac{1}{tgx}=2\sin(y+\frac{\pi}{4}),\\ tgy+\frac{1}{tgy}=2\sin(x-\frac{\pi}{4}).\end{cases}$ X=$\frac{3}{4}\pi+2k\pi,y=-\frac{3}{4}\pi+2n\pi$

34. $\begin{cases}tgx=tg^3y,\\ \sin x=\cos2y.\end{cases}$ $x_1=\frac{\pi}{2}-2arctg\frac{1}{\sqrt{2}}+2m\pi,y_1=arctg\frac{1}{\sqrt{2}}+n\pi;\ x_2=\frac{\pi}{2}+2arctg\frac{1}{\sqrt{2}}+2m\pi,y_2=-arctg\frac{1}{\sqrt{2}}+n\pi(m,n\epsilon Z)$

35. $\begin{cases}\sin x+\sin y=\sin(x+y),\\ \mid x\mid+\mid y\mid=1.\end{cases}$ (1;0),(-1;0), (0;1), (0;-1)

36. $\begin{cases}\sin(u-3x)=2sin^3x,\\ \cos(u-3x)=2cos^3x\end{cases}$ x=$\frac{\pi}{4}(2n+1), y=\pi(2m+1)\ \ (m,n\epsilon Z)$

Trigonometrik tengsizliklar

1. Ixtiyoriy x($x\neq\frac{\pi}{2}k$,k butun son)lar uchun

$$\left(1+\frac{1}{\sin^2 x}\right)\left(1+\frac{1}{\cos^2 x}\right)\geq 9.$$

tengsizlikni isbotlang.

Yechish. Tengsizlikning chap tomonida quyidagi almashtirishlarni bajarish mumkin:

$$\left(1+\frac{1}{\sin^2 x}\right)\left(1+\frac{1}{\cos^2 x}\right)=\left(1+\frac{\sin^2 x+\cos^2 x}{\sin^2 x}\right)\left(1+\frac{\sin^2 x+\cos^2 x}{\cos^2 x}\right)$$

$$=(2+ctg^2x)(2+tg^2x)=$$

$$=5+2(tg^2x+ctg^2x)=9+2(tg^2x-2tgx\cdot ctgx+ctg^2x)=9+2(tgx-ctgx)^2\geq 9.$$

2. Ixtiyoriy haqiqiy x lar uchun quyidagi tengsizlikni isbotlang

$$(\sin^2 x)^{\cos^2 x}+(\cos^2 x)^{\sin^2 x}\leq 1+\frac{1}{2}\sin^2 2x.$$

Yechish. Berilgan tengsizlik n- butun son uchun $x=\frac{\pi}{2}n$ bo‘lganda to‘g‘ri.

Agar $x\neq\frac{\pi}{2}n$ bo‘lsa, u holda $0<\sin^2 x<1$ va $0<\cos^2 x<1$.

$\sin^2 x+\cos^2 x=1$ ayniyatning chap tomoniga Bernulli (8) tengsizligini qo‘llab, ko‘zlangan natijaga erishish mumkin:

$$(\sin^2 x)^{\cos^2 x}+(\cos^2 x)^{\sin^2 x}=(1-\cos^2 x)^{\cos^2 x}+(1-\sin^2 x)^{\sin^2 x}$$

$$\leq 1-cos^4x+1-sin^4x=$$

$$=2-(cos^4x+sin^4x)=2-((cos^2x+sin^2x)^2-2cos^2xsin^2x)=2-\left(1-\frac{1}{2}sin^2 2x\right)=1+\frac{1}{2}sin^2 2x.$$

3. (0;π) oraliqda quyidagi tengsizlikni isbotlang

$$x-\frac{x^3}{4}<\sin x.$$

Yechish.

$$\sin x=2tg\frac{x}{2}cos^2\frac{x}{2}=tg\frac{x}{2}(1-sin^2\frac{x}{2})$$

bo‘lib, avvaldan ma’lum:

$$\sin x\leq x\leq tgx$$

ayniyatdan

$$tg\frac{x}{2}\geq\frac{x}{2}\ va\ 1-sin^2\frac{x}{2}\geq 1-\frac{x^2}{4}$$

ayniyatlarni hosil qilish mumkin. Ushbu tengsizliklardan berilgan tengsizlikning to‘g‘riligi kelib chiqadi.

6. Tengsizlikni yeching

$$(\sin^2(x+y)+2\sin(x+y)+2)\cdot\log_2(3^x+3^{-x})\leq\ 1\ .$$

Yechish. Berilgan shartga ko‘ra: $((sin(x+y)+1)^2+1)\cdot\log_2(3^x+3^{-x})\leq 1$ $(*)$ bo‘lib, $(\sin(x+u)+1)^2+1\geq 1$ Koshi (3) tengsizligiga ko‘ra $3^x+3^{-x}=3^x+\frac{1}{3^x}\geq 2,$ u holda:

$$\log_2(3^x+3^{-x})\geq 1.$$

Natijada,

$$((sin(x+y)+1)^2+1)\cdot\log_2(3^x+3^{-x})\geq 1$$

tengsizlik hosil bo‘lib, $(*)$ tegnsizlikka asosan:

$$((sin(x+y)+1)^2+1)\cdot\log_2(3^x+3^{-x})=1$$

tenglikni yozish mumkin.

Ushbu tenglik, $sin(x+y)=-1$ va $3^x=1$ bo‘lganda bajariladi.

Demak $x_1=0$ va $y_1=-\frac{\pi}{2}+2\pi n$ bu yerda n- butun son .

7. Agar $\sin x+\sin y+\sin z\geq\sqrt{5}$ bo‘lsa, u holda quyidagi ttengsizlikni isbotlang

$$\cos x+\cos \mathrm{y}+\cos z\ \leq 2.$$

Yechish. Teskarisidan faraz qilib, ya’ni $\sin x+\sin y+\sin z\geq\sqrt{5}$ bo‘lganda tengsizlik bajarilmasin. U holda: $\begin{cases}\sin x+\sin y+\sin z\geq\sqrt{5},\\ \cos x+\cos\cos y+\cos z>2\end{cases}$ sistema o‘rinli bo‘lib, bundan

$$(\sin x+\sin y+\sin z)^2+(\cos x+\cos y+\cos z)^2>9,$$

$$3+2(\sin x\cdot\sin y+\cos x\cdot\cos y)+2(\sin y\cdot\sin z+\cos y\cdot\cos z)+$$
$$2(\sin x\cdot\sin z+\cos x\cdot\cos z)>9$$

yoki

$$\cos(x-y)+\cos(y-z)+\cos(x-z)>3 \qquad (*)$$

Ammo $(*)$ tengsizlik noto‘g‘ri, chunki

$$\cos(x-y)\leq 1,\ \cos(y-z)\leq 1,\ \cos(x-z)\leq 1$$

bo‘lib,

$$\cos(x-y)+\cos(y-z)+\cos(x-z)\leq 3.$$

Ushbu qarama-qarshilik olingan teskari faraz noto‘g‘riligini va $\cos x+\cos \mathrm{y}+\cos z\ \leq 2$ tengsizlik x, y, z lar uchun $\sin x+\sin y+\sin z\geq\sqrt{5}$ shartda bajarilishini ko‘rsatadi .

8. tg$\alpha=ntg\beta$ $(n>0)$ tenglik to‘g‘ri bo‘lsa,

$$tg^2(\alpha-\beta)\leq\frac{(n-1)^2}{4n}$$

tengsizlikni isbotlang.

Yechish.

$$tg(\alpha-\beta)=\frac{tg\alpha-tg\beta}{1+tg\alpha tg\beta}\ va\ tg\alpha=ntg\beta$$

tengliklardan:

$$tg^2(\alpha-\beta)=\frac{(n-1)^2tg^2\beta}{(1+ntg^2\beta)^2}=\frac{(n-1)^2}{(ctg\beta+ntg\beta)^2}$$

bo‘lib, Koshi tengsizligiga ko‘ra:

$$(ctg\beta+ntg\beta)^2=\frac{(1+ntg^2\beta)^2}{tg^2\beta}\geq\frac{(2\sqrt{1\cdot ntg^2\beta})^2}{tg^2\beta}=4n$$

Demak, berilgan shartda berilgan tengsizlik isbotlandi.

9. Trigonometrik tengsizlikni yeching

$$\frac{sin^2x-\frac{1}{4}}{\sqrt{3}-(\sin x+\cos x)}>0.$$

Yechish.

$$|\sin x+\cos x|=|\sqrt{2}(\sin(x+\frac{\pi}{4}))|\leq\sqrt{2}$$

bo‘lib, kasrning maxraji har doim musbat. Berilgan tengsizlik o‘rinli bo‘lishi uchun kasrning surati ham musbat bo‘lishi kerak,ya’ni

$$sin^2x>\frac{1}{4},|\sin x|>\frac{1}{2},$$

$$\frac{\pi}{6}+\pi k<x<\frac{5\pi}{6}+\pi k, k\epsilon Z.$$

10. Trigonometrik tengsizlikni yeching

$$5+2\cos 2x\leq 3\ |\ 2\sin x-1\ |.$$

Yechish. Berilgan tengsizlik quyidagi ikkita tengsizliklar sistemasuga teng kuchli bo‘ladi:

$$\begin{cases}5+2\cos 2x\leq 3(2\sin x-1),\\ 2\sin x-1\geq 0\end{cases}\ va\ \begin{cases}5+2\cos 2x\leq 3(-2\sin x+1),\\ 2\sin x-1<0\end{cases}$$

Birinchi sistemadan $\sin x\geq 1$ yoki $\sin x=1$ bo‘lib, x=$\frac{\pi}{2}+2\pi k, k\epsilon Z$. Ikkinchi sistemadan esa, $\sin x\leq-\frac{1}{2}$ bo‘lib,

$$-\frac{5\pi}{6}+2\pi n\leq x\leq-\frac{\pi}{6}+2\pi n, k\epsilon Z.$$

Mustaqil yechish uchun misollar

1. α vaβ o‘tkir burchaklar uchun quyidai tengsizliklarni isbotlang

$$a)\ \sin(\alpha+\beta)<\sin\alpha+\sin\beta,$$

$$b)\sin\frac{\alpha+\beta}{2}\geq\frac{\sin\alpha+\sin\beta}{2},$$

$$s)\ tg\frac{\alpha+\beta}{2}\leq\frac{tg\alpha+tg\beta}{2}.$$

2. 1+$tgx+tg^2x+tg^2x=\frac{\sqrt{2}\cos(\frac{\pi}{4}-x)}{cos^3x}$

3. $\frac{1-2sin^2\alpha}{1+\sin 2\alpha} = \frac{1-tg\alpha}{1+tg\alpha}$

4. $\frac{3-4\cos 2\alpha+\cos 4\alpha}{3+4\cos 2\alpha+\cos 4\alpha} = tg^4\alpha$

5. $\frac{\cos 6\alpha-\cos 7\alpha-\cos 8\alpha+\cos 9\alpha}{\sin 6\alpha-\sin 7\alpha-\sin 8+\sin 9\alpha} = ctg\frac{15}{2}\alpha$

6. $ctg\frac{\alpha}{2} > 1 + ctg\alpha, 0 < \alpha < \frac{\pi}{2}$

7. $(1+tg^2x)(1-3tg^2x)(1+tg2xtg3x) > 0$

8. $\frac{\sin x-1}{\sin x-2} + \frac{1}{2} \geq \frac{2-\sin x}{3-\sin x}$

9. $\cos\sin \propto > \sin\cos \propto$, $0 \leq \propto \leq \frac{\pi}{2}$

10. $sin^3 \propto - sin^6 \propto \leq \frac{1}{4}$

11. $\frac{\sin(\alpha+\beta)}{2\sin\alpha\sin\beta} \geq ctg\frac{\alpha+\beta}{2}$ $(\alpha > 0, \beta > 0, \alpha+\beta < \pi)$

12. $\cos x < 1 - \frac{x^2}{2} + \frac{x^4}{16}$ $(0 < x < \pi)$

13. $x - \frac{x^2}{2} < tgx\ 0 < x < \frac{\pi}{2}$

14. $1 - \cos x \leq \frac{x^2}{2}$

Tengsizliklarni yeching(15-22)

15. $5sin^2x + sin^2 2x > 4\cos 2x\frac{\pi}{6} + \pi k < x < \frac{5\pi}{6} + \pi k, k\epsilon Z$

16. $|\ tgx + ctgx\ | < \frac{4}{\sqrt{3}}\frac{\pi}{6} + \frac{\pi k}{2} < x < \frac{\pi}{3} + \frac{\pi k}{2}, k\epsilon Z$

17. $\sqrt{\frac{3(\sin x+\cos x)-\sqrt{2}}{2\sqrt{2}-(\sin x+\cos x)}} > 1 - \frac{\pi}{4} + arcsin\frac{3}{4} + 2\pi k < x < -\frac{\pi}{4} - arcsin\frac{3}{4} + (2k+1)\pi, k\epsilon Z$

Xulosa

Ushbu uslubiy ko‘rsatmada umumiy o‘rta ta’lim maktablari, akdemik litsey va kasb-hunar kollejlari matematika fani o‘quvchilariga mo‘ljallangan darslik, o‘quv qo‘llanmalar chuqur tahlil qilinib “Trigonometrik tenglama” bo‘limining matematika kursidagi o‘rni va ahamiyati yoritilgan.

Trigonometrik funksiyalarni o‘rganish asosan umumiy o‘rta ta’lim maktablarining 9-sinfidan boshlangani bois ularni o‘quvchilarga o‘rgatishni qanday hajmda tashkil etish kerakligi haqida tavsiyalar berilgan. Bundan tashqari uslubiy ko‘rsatmada tigonometrik tenglama va tengsizliklarni yechishda muhim ekanligi ta’kidlangan hamda asosiy tushunchalar yetarlicha misollar yordamida mustahxkamlangan.

Trigonometrik tenglama va tengsizliklarni yechishda ularning xossalari muhim ekanligi bois uslubiy ko‘rsatmada barcha trigonometrik funksiyalarning xossalari mukammal yoritilgan va ularni grafiklari keltirilgan.

Uslubiy ko‘rsatmada trigonometrik tenglama va tengsizliklar sistemasini qanday o‘rganish mumkinligi bo‘yicha tavsiyalar bilan yakunlangan.

Foydalanilgan adabiyotlar ro‘yxati

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MUNDARIJA

Trigonometrik tenglamalar va tengsizliklar

Z.OCHILOV, S. UMAROV

TRIGONOMETRIK TENGLAMALAR VA TENGSIZLIKLARNI O‘RGANISH USULLARI

Terishga berildi:____________
Bosishga ruxsat etildi:__________
Ofset bosma qog`ozi. Qog`oz bichimi 60x80 1/16
«Times» garniturasi. Ofset bosma usuli
1,75 bosma taboq
Adadi: 25 nusxa
Buyurtma №_____

Samarqand viloyati pedagoglarni yangi metodikalarga o‘rgatish milliy markazi bosmaxonasida chop etildi.