O'ZBEKISTON RESPUBLIKASI
QISHLOQ VA SUV XO'JALIGI VAZIRLIGI
SAMARQAND QISHLOQ XO'JALIK INSTITUTI

P.Z.DAVRONOV

OLIY MATEMATIKA:

OLIY MATEMATIKA FANIDAN MASALALAR YECHISH BO'YICHA USLUBIY QO'LLANMA

S A M A R Q A N D – 2013

Uslubiy qo'llanma Samarqand qishloq xo'jalik instituti Ilmiy kengashida tasdiqlangan va nashrga tavsiya etilgan.
«*30* » *mart* 2013 y. № *9-sonli* bayonnoma

Taqrizchilar:
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Uslubiy qo'llanma O'zbekiston Respublikasi Oliy va O'rta maxsus ta'lim Vazirligi tomonidan 2012 yil 14 martda №107-buyruq bilan tasdiqlangan "Oliy matematika" fanidan o'quv dasturga muvofiq yozilgan. Undan mutaxassis professor-o'qituvchilar, oliy o'quv yurtlari talabalari, kollej va litsey o'quvchilari foydalanishlari mumkin.

SO’Z BOSHI

Uslubiy qo’llanma O’zROO’MTV tomonidan 2006 yilda tasdiqlangan «Oliy matematika» fanidan dastur asosida tayyorlangan. Unda analitik geometriyaning: nuqtani yasash, ikki nuqta orasidagi masofa, kesmani berilgan nisbatda bo’lish, uchburchak va ko’pburchak yuzlari, to’g’ri chiziq tenglamalari, koordinatalarni almashtirish, Dekart, qutb, silindrik va sferik koordinatalar sistemalari, ikkinchi tartibli chiziqlar, tekislik, ikkinchi tartibli sirtlar, vektorlar; oliy va chiziqli algebraning: kompleks sonlar, algebraning asosiy teoremasi, kubik tenglamalar, Kardano formulasi, determinantlar, chiziqli tenglamalar sistemalari, matritsalar; matematik tahlilning: to’plam, sonlar ketma-ketligi va funksiya limiti, hosila, differensial, funksiyani hosilalar yordamida tekshirish, aniqmas va aniq integrallar, ratsional kasrlar va trigonometrik funksiyalarni integrallash, aniq integralning tatbiqi, ko’p argumentli funksiyalar, karrali va egri chiziqli integrallar, differensial tenglamalar masalalari qaralgan.

Kitob 47 ta amaliy mashg’ulot mavzularidan iborat. Har bir mavzuda yetarli hajmdagi nazariy ma’lumotlar, misol-masalalarning yechimlari, talabalarning mustaqil bajarishlari uchun topshiriqlar mavjud.

Uslubiy qo’llanmaga 1182 ta misol-masalalar javoblari bilan kiritilgan, ulardan 296 tasining yechimlari bayon qilingan. Foydalanilgan adabiyotlar ro’yxati, mundarija berilgan.

Kitob haqida o’zlarining fikrlarini bildirgan hamkasblarga oldindan minnatdorchilik bildiramiz.

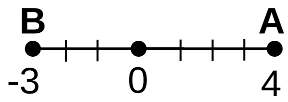
1- MAVZU. ANALITIK GEOMETRIYANING SODDA MASALALARI

Nuqtani yasash

To'g'ri chiziqning faqat uzunligi bor, ya'ni o'lchovi bitta, shuning uchun unda joylashgan nuqtaning koordinatasi (aniqlovchisi) ham bitta bo'ladi.
$A(x)$ ni koordinatasi x ga teng bo'lgan **A** nuqta deb o'qiladi.

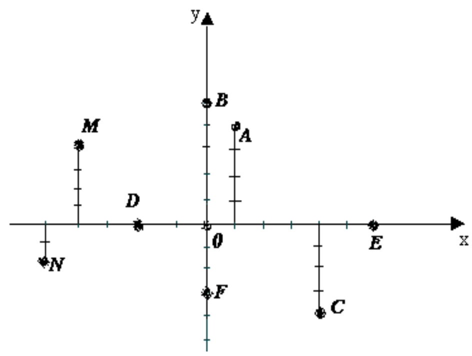
Misol 1. To'g'ri chiziqda $A(4)$ va $B(-3)$ nuqtalarni yasang.

Yechilishi.



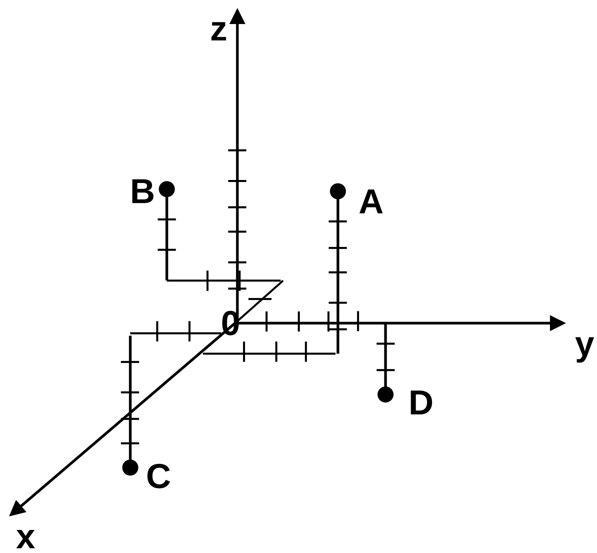
Tekislikning eni va bo'yi bor, ya'ni o'lchovi ikkita, shuning uchun undagi nuqtaning koordinatalari ham ikkita bo'ladi. Ulardan birinchisi abssissa, ikkinchisi ordinata deyiladi. $A(x;y)$ ni abssissasi x ga, ordinatasi y ga teng bo'lgan A nuqta deb o'qiladi.

Misol 2. Tekislikda $A(1;4)$, $B(0;5)$, $C(4;-4)$, $D(-2;0)$, $E(6;0)$, $F(0;-3)$, $O(0;0)$, $M(-4;4)$, $N(-5;-2)$ nuqtalarni yasang.



Fazoning eni, bo'yi va balandligi bor, ya'ni o'lchovi uchta, shuning uchun undagi nuqtaning koordinatalari ham uchta bo'ladi. Ulardan birinchisi abssissa, ikkinchisi ordinata, uchinchisi aplikata deyiladi. $A(x;y;z)$ ni abssissasi x ga, ordinatasi y ga, aplikatasi z ga teng **A** nuqta deb o'qiladi.

Misol 3. Fazoda ***A(2;4;6), B(-2;-3;3), C(1;-3;-5), D(0;5;-3)*** nuqtalarni yasang.



Ikki nuqta orasidagi masofa

A va B nuqtalar orasidagi masofani $|AB|$ ko'rinishida belgilanadi.

To'g'ri chiziqdagi ikkita $A(x_1)$ va $B(x_2)$ nuqtalar orasidagi masofa

$$|AB| = \sqrt{(x_2 - x_1)^2} = |x_2 - x_1| \quad \textbf{(1)}$$

formula yordamida topiladi. Bunda x_1 birinchi o'rinda turgan, x_2 esa ikkinchi o'rinda turgan nuqtaning koordinatasi.

Misol 4. $A(-2)$ va $B(3)$ nuqtalar orasidagi masofani toping.

Yechilishi.

$$|AB| = \sqrt{(x_2 - x_1)^2} = \sqrt{[3-(-2)]^2} = \sqrt{(3+2)^2} = \sqrt{5^2} = 5.$$

Tekislikdagi ikkita $A(x_1; y_1)$ va $B(x_2; y_2)$ nuqtalar orasidagi masofa

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \textbf{(2)}$$

formula yordamida topiladi.

Bunda x_2 ikkinchi o'rinda turgan B nuqtaning abssissasi, x_1 birinchi o'rinda turgan A nuqtaning abssissasi, y_2 ikkinchi o'rinda turgan B nuqtaning ordinatasi, y_1 birinchi o'rinda turgan A nuqtaning ordinatasi.

Misol 5.

C(-1;1) va ***D(3;4)*** nuqtalar orasidagi masofani toping.

Yechilishi.

$$|CD| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{[3-(-1)]^2 + (4-1)^2} = \sqrt{16+9} = 5.$$

Fazodagi ikkita $A(x_1; y_1; z_1)$ va $B(x_2; y_2; z_2)$ nuqtalar orasidagi masofa

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1) + (z_2 - z_1)^2} \qquad \textbf{(3)}$$

formula yordamida topiladi.

Bunda y_2 ikkinchi o'rinda turgan B nuqtaning ordinatasi, z_1 birinchi o'rinda turgan A nuqtaning aplikatasi va hokazo ko'rinishda o'qiladi.

Misol 6. $M(6;4;4)$ va $N(5;2;6)$ nuqtalar orasidagi masofani toping.

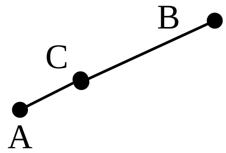
Yechilishi.

$$|MN| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{(5-6)^2 + (2-4)^2 + (6-4)^2} =$$

$$= \sqrt{(-1)^2 + (-2)^2 + 2^2} = \sqrt{1+4+4} = \sqrt{9} = 3.$$

Kesmani λ nisbatda bo'lish

$A(x_1; y_1; z_1)$ va $B(x_2; y_2; z_2)$ nuqtalardan o'tuvchi



[AB] kesmani $\lambda = \dfrac{|AC|}{|CB|}$ nisbatda bo'luvchi $C(x; y; z)$ nuqtaning koordinatalarini topish formulalari:

$$\begin{cases} x = \dfrac{x_1 + \lambda x_2}{1+\lambda}; \\ y = \dfrac{y_1 + \lambda y_2}{1+\lambda}; \\ z = \dfrac{z_1 + \lambda z_2}{1+\lambda}. \end{cases} \qquad \textbf{(4)}$$

Misol 7. Uchlari $A(2;7;3)$ va $B(2;4;-6)$ nuqtalarda bo'lgan kesmani $\lambda = \dfrac{1}{2}$ nisbatda bo'luvchi $C(x; y; z)$ nuqtaning koordinatalarini toping.

Yechilishi.

$$x=\frac{x_1+\lambda x_2}{1+\lambda}=\frac{2+\frac{1}{2}*2}{1+\frac{1}{2}}=\frac{3}{\frac{3}{2}}=\frac{6}{3}=2; \qquad y=\frac{y_1+\lambda y_2}{1+\lambda}=\frac{7+\frac{1}{2}*4}{1+\frac{1}{2}}=\frac{9}{\frac{3}{2}}=\frac{6}{3}=6;$$

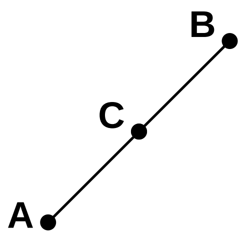
$$z=\frac{z_1+\lambda z_2}{1+\lambda}=\frac{3+\frac{1}{2}*(-6)}{1+\frac{1}{2}}=\frac{0}{\frac{3}{2}}=0.$$

Demak, $C(x;y;z)=C(2;6;0)$ bo'ladi.

Uchlari $A(x_1;y_1;z_1)$ va $B(x_2;y_2;z_2)$ nuqtalarda bo'lgan [AB] kesmani teng ikkiga bo'luvchi $C(x;y;z)$ nuqtaning koordinatalarini topish formulalari:

$$|AC|=|CB|;\ \lambda=\frac{|AC|}{|CB|}=1.$$

$$\begin{cases} x=\dfrac{x_1+x_2}{2}; \\ y=\dfrac{y_1+y_2}{2}; \\ z=\dfrac{z_1+z_2}{2}. \end{cases} \qquad (5)$$



Misol 8. Uchlari ***A(-2;1)*** va ***B(10;5)*** nuqtalarda bo'lgan kesmani teng ikkiga bo'luvchi ***C(x;y)*** nuqtaning koordinatalarini toping.

Yechilishi.

$$x=\frac{x_1+x_2}{2}=\frac{-2+10}{2}=\frac{8}{2}=4; \quad y=\frac{y_1+y_2}{2}=\frac{1+5}{2}=\frac{6}{2}=3.$$

Demak, $C(x;y)=C(4;3)$ bo'ladi.

UCHBURCHAK VA KO'PBURCHAK YUZLARI

Uchlari $A(x_1,y_1), B(x_2;y_2), C(x_3;y_3)$ nuqtalarda bo'lgan uchburchakning yuzini hisoblash formulasi:

$$S_\Delta=\frac{1}{2}*\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=\frac{1}{2}\left[(x_1*y_2-y_1x_2)+(x_2y_3-y_2x_3)+(x_3y_1-y_3x_1)\right] \qquad (6)$$

Misol 9. Uchlari $A(4;-2), B(6;4), C(5;6)$ nuqtalarda yotgan uchburchakni yasang va yuzini toping.

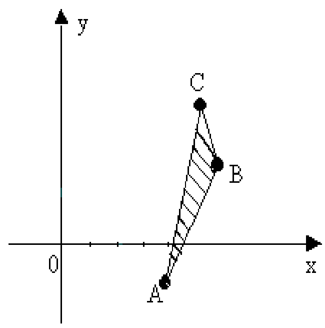
Yechilishi.

$$S_\Delta=\frac{1}{2}*\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=\frac{1}{2}\left[(x_1y_2-y_1x_2)+(x_2y_3-y_2x_3)+(x_3y_1-y_3x_1)\right]=$$

$$=\frac{1}{2}*\left[(4*4-(-2)*6)+(6*6-4*5)+(5*(-2)-6*4)\right]=$$

$$=\frac{1}{2}(16+12+36-20-10-24)=\frac{1}{2}(64-54)=\frac{1}{2}*10=\frac{10}{2}=5.$$

$s_\Delta=5$ kv. birlik.



Ko'pburchakning yuzini hisoblash formulasi:

$$S = \frac{1}{2}\left[(x_1 * y_2 - y_1 x_2) + (x_2 y_3 - y_2 x_3) + (x_3 y_4 - y_3 x_4) + ... + (x_n y_1 - y_n x_1)\right]. \quad (7)$$

Misol 10. Uchlari ***A(-6;-3), B(-5;2), C(0;5), D(5;0) va E(6;-5)*** nuqtalarda yotgan beshburchakni yasang va uning yuzini toping.

Yechilishi.

$$S = \frac{1}{2}\left[(x_1 y_2 - y_1 x_2) + (x_2 y_3 - y_2 x_3) + (x_3 y_4 - y_3 x_4) + (x_4 y_5 - y_4 x_5) + (x_5 y_1 - y_5 x_1)\right] =$$

$$= \frac{1}{2}\left[(-6 * 2 - (-3) * (-5)) + \right.$$

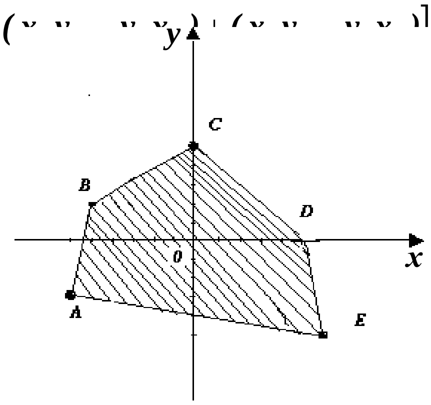
$$+ (-5 * 5 - 2 * 0) + (0 * 0 - 5 * 5) +$$

$$+ (5 * (-5) - 0 * 6) +$$

$$\left. + (6 * (-3) - (-5 * (-6))\right] =$$

$$= \frac{1}{2}(-27 - 25 - 25 - 25 - 48) = -75.$$

$$S = 75 \ \textbf{\textit{kv. birlik.}}$$



MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

To'g'ri chiziqda quyidagi nuqtalarni yasang va ular orasidagi masofani toping:

11. A (2) va B (-3);
Javobi: 5.

12. C (-4) va D (-3);
Javobi: 1.

13. E (-4) va F (2);
Javobi: 6.

14. M (0) va N (-2);
Javobi: 2.

Tekislikda quyidagi nuqtalarni yasang va ular orasidagi masofani toping:

15. A (5;2) va B (1;-1); Javobi: 5.

16. C(-6;3) va D (0;-5); Javobi: 10.

17. O (0;0) va P(-3;4); Javobi: 5.

18. M (9;-7) va N (4;5); Javobi: 13.

Fazoda quyidagi nuqtalarni yasang va ular orasidagi masofani toping:

19. A(3;2;l) va B (4; -1;-2); Javobi: $\sqrt{19}$.

20. C (-5;3;4) va D (1;4;-3); Javobi: $\sqrt{86}$.

21. E (-3;2;-1) va F (0;5;-2); Javobi: $\sqrt{19}$.

22. M (-1;-3;0) va N (2;0;-1) Javobi: $\sqrt{19}$.

Quyidagi nuqtalar orasidagi kesmani λ nisbatda bo'luvchi nuqtaning koordinatalarini toping:

23. A(1;7), B(4;-2), $\lambda = \frac{1}{2}$; Javobi: (2;4).

24. C(2;-3), D(-2;2), $\lambda = 2$; Javobi: $\left(-\frac{2}{3};\frac{1}{3}\right)$.

25. E(-7;3), F(0;2), $\lambda = 1$; Javobi: (-3,5;2,5).

26. P(-7;3;-2), Q(0;2;1), $\lambda = 3$; Javobi: $\left(-1\frac{3}{4};2\frac{1}{4};\frac{1}{4}\right)$.

27. M(-2;1;3), N(0;-1;2), $\lambda = \frac{1}{3}$; Javobi: $\left(-1\frac{1}{2};\frac{1}{2};2\frac{3}{4}\right)$.

Quyida ko'rsatilgan nuqtalar orasidagi kesmani teng ikkiga bo'luvchi nuqtaning koordinatalarini toping:

28. A(3;-7) va B(5;2); Javobi: (4;-2,5).

29. C(3;-2) va D(-1;4); Javobi: (1;1).

29. E(2;1;1) va F(-2;0;3); Javobi: $\left(0;\frac{1}{2};2\right)$.

30. M(-3;1;1) va N(-1;-1;5); Javobi: (-2;0;3).

31. Uchlari A(1;5), B(-5;0), C(-2;1) nuqtalarda bo'lgan uchburchak medianalarining kesishish nuqtasini toping.

Eslatma: $x = \frac{x_1 + x_2 + x_3}{3}$ *va* $y = \frac{y_1 + y_2 + y_3}{3}$ formulalardan foydalaning.

Uchlari quyidagi nuqtalarda bo'lgan uchburchak yuzini toping:

32. A(4;-2), B(6;4), C(5;6); Javobi: 5.
33. D(3;0), E(7;8), F(0;2); Javobi: 16.
34. M(0;-3), N(0;8), K(-4;-6); Javobi: 22.

Uchlari quyidagi nuqtalarda bo'lgan to'rtburchakning yuzini toping

35. A(0;5), B(0;-3), C(6;-3), D(6;5); Javobi: 48.
36. E(-3;5), F(6;6), M(5;3), N(0;0); Javobi: 30.
37. K(-4;1), Q(0;5), G(3;5), L(6;-2); Javobi: 36,5.

38. Uchlari quyidagi nuqtalarda bo'lgan beshburchakning yuzini toping:
A(-8;5), B(-4;11), C(3;9), D(4;2), E(-2;-3). Javobi: 104,5.

2-MAVZU. TO'G'RI CHIZIQ TENGLAMALARI

To'g'ri chiziq matematikaning ta'riflanmaydigan asosiy tushunchalardan biri.

1. $\boldsymbol{u : Ax + By + C} = 0$ - to'g'ri chiziqning umumiy tenglamasi

$\vec{\boldsymbol{u}} = \{-\boldsymbol{B}; \boldsymbol{A}\}$- to'g'ri chiziqning yo'naltiruvchi vektori.

$\vec{\boldsymbol{n}} = \{\boldsymbol{A}; \boldsymbol{B}\}$- to'g'ri chiziqning normal vektori.

$\mu = \frac{1}{\pm\sqrt{\boldsymbol{A}^2 + \boldsymbol{B}^2}}$- to'g'ri chiziqning normallovchisi.

C>0 bo'lsa normallovchi manfiy ishora, ***C<0*** bo'lsa musbat ishora bilan olinadi.

Misol 39. Umumiy tenglamasi bilan berilgan ***5x+2y+13=0*** to'gri chiziqning normal va yo'naltiruvchisi vektorlari, normallovchisini yozing.

Yechilishi. ***A=5 , B=2, C = 13>0***

$\vec{\boldsymbol{u}} = \{-\boldsymbol{B}; \boldsymbol{A}\} = \{-2;5\}$; $\vec{\boldsymbol{n}} = \{\boldsymbol{A}; \boldsymbol{B}\ \ ) = \{5;2\}$;

$\boldsymbol{\mu} = \frac{1}{\pm\sqrt{A^2 + B^2}} = \frac{1}{-\sqrt{5^2 + 2^2}} = -\frac{1}{\sqrt{29}}$.

2.Umumiy tenglamasi bilan berilgan to'g'ri chiziqni normal shaklga keltirish uchun tenglikning ikkala tamoni normollvchiga hadma-had ko'paytiriladi.

u: Ax+By+C=0;

$$\frac{A}{\pm\sqrt{A^2+B^2}}x+\frac{B}{\pm\sqrt{A^2+B^2}}y+\frac{C}{\pm\sqrt{A^2+B^2}}=0.$$

Misol 40. To'g'ri chiziqning *2x-3y-6=0* umumiy tenglamasini normal shalkga keltiring.

Yechilishi.

1. ***A = 2; B = - 3; C = - 6 < 0;***

2. Normallovchisi topiladi:

$$\mu=\frac{1}{\pm\sqrt{A^2+B^2}}=\frac{1}{\sqrt{2^2+(-3)^2}}=\frac{1}{\sqrt{4+9}}=\frac{1}{\sqrt{13}}.$$

3. ***2x-3y-6=0*** to'g'ri chiziqning har bir hadi $\mu=\frac{1}{\sqrt{13}}$ normallovchiga ko'paytiriladi:

$$2\cdot\frac{1}{\sqrt{13}}\cdot x-3\frac{1}{\sqrt{13}}\cdot y-6\cdot\frac{1}{\sqrt{13}}=0.$$

Kasrning oldida yoki orqasida unga ko'paytirilib turgan son, kasrning suratiga ko'paytirilib quyidagicha yoziladi:

$$\frac{2}{\sqrt{13}}\cdot x-\frac{3}{\sqrt{13}}\cdot y-\frac{6}{\sqrt{13}}=0.$$

3. $\frac{x}{a}+\frac{y}{b}=1$ -to'g'ri chiziqning koordinata o'qlaridan kesgan kesmalar bo'yicha tenglamasi.

Misol 41. ***2x - 3y – 6 = 0*** to'g'ri chiziqning koordinata o'qlaridan kesgan kesmalar bo'yicha tenglamasini tuzing va yasang.

Yechilishi. ***2x - 3y = 6*** tenglikning ikkala tomoni hadma-had 6 ga bo'linadi:

1-usul: $\frac{2x}{6}-\frac{3y}{6}=\frac{6}{6};\quad \frac{x}{3}+\frac{y}{-2}=1;$

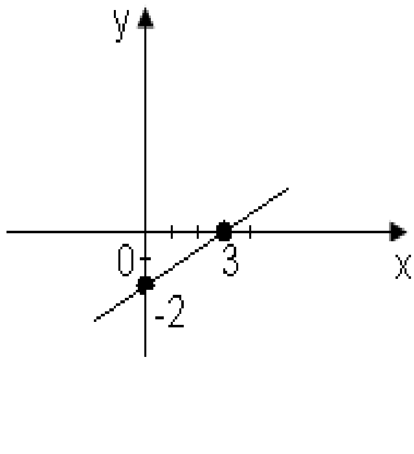
a=3; b=-2.

2-usul: Berilgan tenglamada

$$y=0\ \textit{deb},\ 2x-3\cdot0=6\Rightarrow 2x=6\Rightarrow x=3;$$

$$x=0\ \textit{deb},\ 2\cdot0-3y=6\Rightarrow -3y=6\Rightarrow y=-2.$$

$$\textit{Bundan}\ a=3,\ b=-2$$



topiladi va tenglama tuziladi.

4.Nuqtaning to'g'ri chiziqqa tegishli yoki tegishli emasligini aniqlash uchun nuqtaning koordinatalarini to'g'ri chiziq tenglamasiga qo'yish kerak. Bunda tenglik bajarilsa nuqta to'g'ri chiziqda, tenglik buzilsa to'g'ri chiziqdan tashqarida yotadi.

Misol 42. ***A(-2;-3)*** nuqtaning ***x-5y+7=0*** to'g'ri chiziqqa tegishliligini tekshiring.

Yechilishi. **-2-5·(-3)+7=0**$\Rightarrow -2+15+7=0\Rightarrow 20\neq0.$

Demak, nuqta to'g'ri chiziqda yotmaydi.

Misol 43. *A(1;3)* nuqtaning **y = - 2x + 5** to'g'ri chiziqda yotishini tekshiring.

Yechlishi. $3 = -2\cdot 1 + 5 \Rightarrow 3 = -2 + 5 \Rightarrow 3 = 3.$ Demak, nuqta to'g'ri chiziqda yotadi.

5. $u_1: A_1x+B_1y+C_1=0$

$u_2: A_2x+B_2y+C_2=0$ - ikkita u_1 va u_2 to'g'ri chiziqlarning umumiy tenglamalari.

$\frac{A_1}{A_2} = \frac{B_1}{B_2}$ - ikki to'g'ri chiziqning parallellik sharti, $u_1 \parallel u_2$.

$\frac{A_1}{A_2} \neq \frac{B_1}{B_2}$ - ikki to'g'ri chiziqning kesishish sharti, $u_1 \cap u_2$.

$A_1 \cdot A_2 + B_1 \cdot B_2 = 0$ - ikki to'g'ri chiziqning perpendikulyarlik sharti, $u_1 \perp u_2$.

$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$ - ikki to'g'ri chiziqning ustma –ust tushish sharti, $u_1 = u_2$.

$$Cos(u_1 \hat{;} u_2) = Cos(\vec{n}_1 \hat{;} \vec{n}_2) = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \cdot |\vec{n}_2|} = \frac{A_1 \cdot A_2 + B_1 \cdot B_2}{\sqrt{A_1^2 + B_1^2} \cdot \sqrt{A_2^2 + B_2^2}}$$ – umumiy tenglamalari bilan berilgan ikki to'g'ri chiziq orasidagi burchakni topish formulasi.

Misol 44. $u_1 : 3x + y = 0$ *va* $u_2 : -2x + y - 6 = 0$ to'g'ri chiziqlar orasidagi burchakni toping.

Yechilishi. $A_1 = 3;\ B_1 = 1;\ C_1 = 0;\ A_2 = -2;\ B_2 = 1;\ C_2 = -6.$

$$\cos(u_1 \hat{;} u_2) = \cos(\vec{n}_1 \hat{;} \vec{n}_2) = \frac{A_1 \cdot A_2 + B_1 \cdot B_2}{\sqrt{A_1^2 + B_1^2} \cdot \sqrt{A_2^2 + B_2^2}} = \frac{3\cdot(-2) + 1\cdot 1}{\sqrt{3^2+1^2}\cdot\sqrt{(-2)^2+1^2}} =$$

$$= \frac{-5}{\sqrt{10}\sqrt{5}} = \frac{-5}{\sqrt{50}} = \frac{-5}{\sqrt{25}\cdot\sqrt{2}} = \frac{-5}{5\cdot\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}; \qquad (u_1 \hat{;} u_2) = 135^0.$$

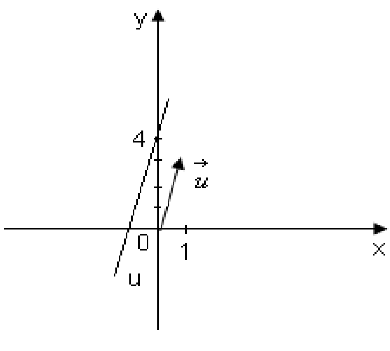
$$\rho(M_0; u) = \left| \frac{Ax_0 + By_0 + C}{\pm\sqrt{A^2 + B^2}} \right|$$ – $M_0(x_0; y_0)$ nuqtadan **u** to'g'ri chiziqqacha bo'lgan masofani topish formulasi.

Misol 45. *A(2;3)* nuqtadan *3x+4y+2=0* to'g'ri chiziqqacha bo'lgan masofani toping.

Yechilishi. $x_0 = 2;\ y_0 = 3;\ A = 3;\ B = 4;\ C = 2 > 0.$

$$\rho(A; u) = \left| \frac{Ax_0 + By_0 + C}{\pm\sqrt{A^2 + B^2}} \right| = \left| \frac{3*2 + 4*3 + 2}{-\sqrt{3^2 + 4^2}} \right| = \left| \frac{20}{-5} \right| = 4.$$

Agar to'g'ri chiziq normal tenglamasi bilan berilgan bo'lsa, nuqtadan bu to'g'ri chiziqqacha bo'lgan masofani topish uchun nuqtaning koordinatalarini to'g'ri chiziq tenglamasiga qo'yib hisoblash kifoya.



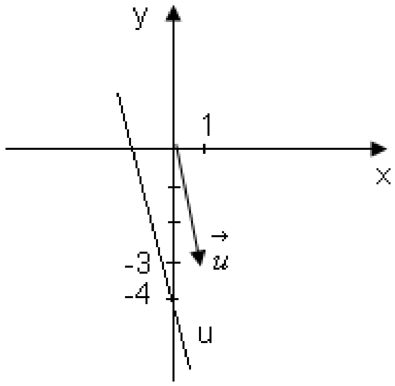
6. ***u: y=kx+b*** - to'g'ri chiziqning burchak koeffitsientli tenglamasi.

k=tga - burchak koeffitsienti.

b - to'g'ri chiziqning **(0y)** o'qini kesgan nuqtasi, ya'ni boshlang'ich ordinatasi.

$\vec{u}=\{1;k\}$ – to'g'ri chiziqning yo'naltiruvchi vektori.

Misol 46. ***u:*** ***y=3x+4*** to'g'ti chiziqni yo'naltiruvchi vektori asosida yasang va boshlang'ich ordinatasini ko'rsating.



Yechilishi. $k = 3 > 0;$ $\vec{u}=\{1;k\}=\{1;3\};$ $b = 4.$

Misol 47. ***u:*** $y = -3x - 4$ to'g'ri chiziqni yo'naltiruvchi vektori asosida yasang va boshlang'ich ordinatasini ko'rsating.

Yechilishi. $k = -3 < 0;$ $\vec{u}=\{1;k\}=\{1;-3\};$

$b=-4.$

4. $u_1 : y = k_1 x + b_1$, $u_2 : y = k_2 x + b_2$ - ikkita u_1 va u_2 to'g'ri chiziqning burchak koeffitsientli tenglamalari.

$k_1 = k_2$ - ikki to'g'ri chiziqning parallellik sharti, $u_1 \| u_2$

$k_1 = -\frac{1}{k_2}$ - ikki to'g'ri chiziqning perpendikulyarlik sharti, $u_1 \perp u_2$

$tg\alpha = \frac{k_2 - k_1}{1 + k_1 k_2}$ - burchak koeffitsientli tenglamalari bilan berilgan ikki to'g'ri chiziq orasidagi burchakni topish formulasi.

Misol 48. $u_1 : y = 3x$ va $u_2 : y = -2x + 5$ to'g'ri chiziqlar orasidagi burchakni toping.

Yechilishi. $k_1 = 3,$ $k_2 = -2.$

$$tga = \frac{k_2 - k_1}{1 + k_1 k_2} = \frac{-2-3}{1+3\cdot(-2)} = \frac{-5}{-5} = 1; \qquad \alpha = 45^0.$$

Misol 49. Koordinatalar boshidan o'tib, abscissa o'qi bilan 135^0 burchak hosil qilgan to'g'ri chiziq tenglamasini tuzing.

Yechilishi.

$$b=0, \quad tg135^0 = tg(180^0 - 45) = -tg45^0 = -1; \quad tg\alpha = k \Rightarrow k = -1;$$

Demak, $y = -x$ bo'ladi.

Misol 50. ***M(0;5)*** nuqtadan o'tuvchi va abscissa o'qi bilan $\mathbf{135^0}$ burchak tashkil etuvchi to'g'ri chiziq tenglamasini tuzing.

Yechilishi. $k = tga = tg135^0 = -1;$ $b = 5.$ Demak, $y = -x + 5$ bo'ladi.

7. Ikkita $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi quyidagi ko'rinishda bo'ladi:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \qquad yoki \qquad \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

Misol 51. $A(1;2)$ *va* $B(-2;5)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Yechilishi.

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}\Rightarrow\frac{x-1}{-2-1}=\frac{y-2}{5-2}\Rightarrow\frac{x-1}{-3}=\frac{y-2}{3}\Rightarrow 3x-3=-3y+6\Rightarrow$$

$$\Rightarrow 3x+3y-9=0\Rightarrow x+y-3=0.$$

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Quyidagi umumiy tenglamasi bilan berilgan to'g'ri chiziqlarning yo'naltiruvchi vektorini, normal vektorini va normalovchisini yozing:

52. *8x-3y-1=0*; **53.** *4x+y-13=0;* **54.** *3x+7y-15=0;*

55. *9x+21y-32=0;* **56.** *5x-2y-13=0;* **57.** *x+3y-11=0*

Quyidagi to'g'ri chiziqlarni normal shaklga keltiring:

58. *2x-3y-5=0;* **59.** *x+y+1=0;* **60.** *y=2x+5;*

61. *y=-3x+2;* **62.** *3x-4y-15=0;* **63.** *5x+12y-26=0.*

64. ***A(-1;1), B(2;-3), C(4;0), D(3;1), E(2;7), O(0;0)*** nuqtalarning ***2x-y+3=0*** to'g'ri chiziqda yotishini tekshiring.

Quyidagi umumiy tenglamalari bilan berilgan to'g'ri chiziqlar orasidagi burchakni toping:

65. *3x-2y+7=0* va *4x+6y-3=0;* Javobi: 90^0

66. *4x-y-8=0* va *x+4y-3=0;* Javobi: 90^0

67. *-x+2y+3=0* va *2x+4y-2=0;* Javobi: $\arccos\frac{3}{5}$.

68. Quyidagi to'g'ri chiziqlardan o'zaro parallel, perpendikulyar, ustma-ust tushadiganlarini aniqlang:

3x-2y-7=0; *6x-4y-9=0;* *6x+4y-5=0;* *2x+3y-6=0;*

x-y+8=0; *x+y-12=0;* *-x+y-3=0.*

Nuqtadan to'g'ri chiziqqacha bo'lgan masofani toping:

69. *A(4;-2),* *8x-15y-11=0;* Javobi: 3.

70. *B(2;7),* *12x+5y-7=0;* Javobi: 4.

71. *C(-3;5),* *9x-21y-3=0;* Javobi: 6.

72. *D(-3;2),* *3x-4y+16=0;* Javobi: 0.

73. *E (8;5),* *3x-4y+16=0;* Javobi: 4.

Quyidagi burchak koeffitsientli tenglamasi bilan berilgan to'g'ri chiziqni yo'naltiruvchi vektori asosida yasang va boshlang'ich ordinatasini ko'rsating:

74. *y=4x;* 75. *y=-4x;* **76.** *y=4x+3;* 77. *y=-4x+3;*

78. *y=4x-3;* **79**. *y=-4x-3;* **80**. $y=\frac{1}{2}x+4;$ **81**. $y=-\frac{1}{2}x+4;$

82. $y=-\frac{1}{2}x+4;$

Quyidagi to'g'ri chiziqlar orasidagi burchakni toping:

83. *y=-3x* va *y=2x+6;* Javobi: 135^0.

84. *3x-2y+7=0* va *4y-6x-3=0;* Javobi: $\frac{12}{13}$.

85. *2y-x+3=0* va *2x+4y-2=0;* Javobi: 90^0

86. *2y-x+3=0* va *2x+4y-3=0;* Javobi: $\frac{3}{5}$.

87. ***x=0*** va ***y=3;*** Javobi: 90^0.

88. ***x=-3*** va ***y=2;*** Javobi: 90^0.

89.Koordinatalar boshidan o'tuvchi va (***Ox***) o'qi bilan $30^0, 45^0, 60^0, 120^0, 150^0$ burchak hosil qiluvchi to'g'ri chiziqlarning tenglamalarini tuzing, ularni yasang, normal va yo'naltiruvchi vektorlarini, burchak koeffitsientlarini, boshlang'ich ordinatalarini ko'rsating.

Quyidagi nuqtalardan o'tuvchi to'g'ri chiziq tenglamalarini tuzing:

90. **A(-3;2)** va **B(5;7)** **91. C(-2;6)** va **D(0;4);**

92. M(3;0) va F**(-4;-7)** **93. M(3;0)** va **N(-3;6);**

94. M(3;0) va **N(-3;6)** **95. O(0;0)** va **K(2;7).**

Quyidagi to'g'ri chiziqlarning parallelligi, perpendikulyarligi, ustma-ust tushishini hamda kesishish nuqtasining koordinatalarini aniqlang:

96. ***8x-3y-1=0*** va ***4x+y-13=0;*** **97.** ***3x+7y-15=0*** va ***9x+21y-45=0;***

98. ***5x-2y+13=0*** va ***x+3y-11=0;*** **99.*2x-3y+4=0*** va ***4x-6y-8=0.***

100. ***a*** ning qanday qiymatlarida ***ax+2y=3*** va ***2x-y=-1*** to'g'richiziqlar kesishadi.

Javobi***:*** $a \neq -4;$

101. ***a*** ning qanday qiymatlarida ***ax+y=3*** va ***2x+y=-1*** to'g'ri chiziqlar parallel bo'ladi.

Javobi: ***a=2.***

102. ***a*** va ***b*** ning qanday qiymatlarida ***ax+2y=4*** va ***by-x=4*** to'g'ri chiziqlar ustma-ust tushadi.

Javobi;***a=-1; b=2.***

103. *k>0; b>0* bo'lsa, *y=kx+b* to'g'ri chiziqning grafigi koordinatalar tekisligining qaysi choraklarida yotadi?

Javobi: I, II, III, choraklarda.

104. *k>0; b<0* bo'lsa, *y=kx+b* to'g'ri chiziqning grafigi koordinatalar tekisligining qaysi choraklarida yotadi?

Javobi: I, III, IV choraklarda.

105. *k<o; b<0* bo'lsa, *y=kx+b* to'g'ri chiziqning grafigi koordinatalar tekisligining qaysi choraklarida yotadi.

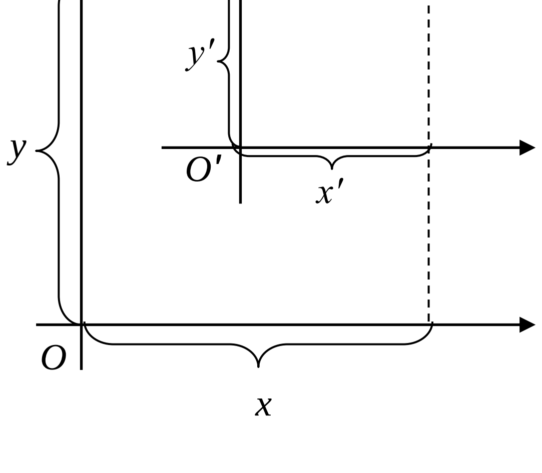
Javobi: II, III, IV choraklarda.

106. *k<0; b>0* bo'lsa, *y=kx+b* to'g'ri chiziqning grafigi koordinatalar tekisligining qaysi choraklarida yotadi.

Javobi: I, II, IV choraklarda.

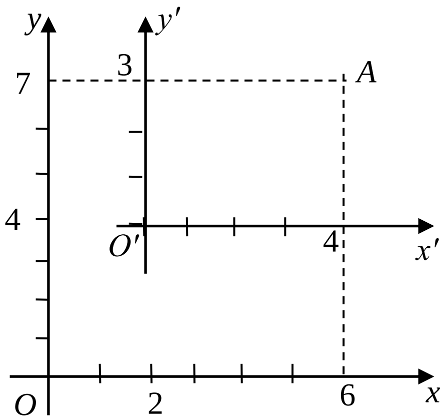
3-MAVZU. KORDINATALARNI ALMASHTIRISH

O'qlar parallel va koordinatalar boshi ***O'(a;b)*** nuqtaga ko'chgan hol.



$$\begin{cases} x = x' + a; \\ y = y' + b. \end{cases}$$

Misol 107. (*xOy*) koordinatalar sistemasiga nisbatan *A(x;y)=A(6;7)* nuqta berilgan, bu *A* nuqnaning *O′(2;4)* bo'lgandagi, x' va y' koordinatalarini toping va yasang.

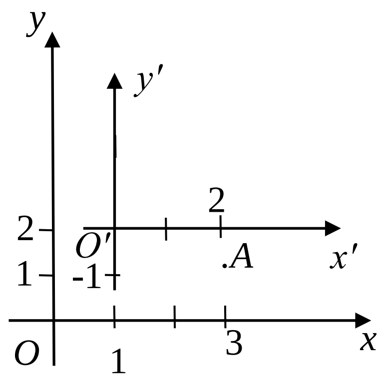


Yechilishi.

$$\begin{cases} 6 = x' + 2 \\ 7 = y' + 4 \end{cases} \Rightarrow \begin{cases} x' = 6 - 2 \\ y' = 7 - 4 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x' = 4 \\ y' = 3 \end{cases} \Rightarrow \quad A(x'; y') = A(4;3).$$

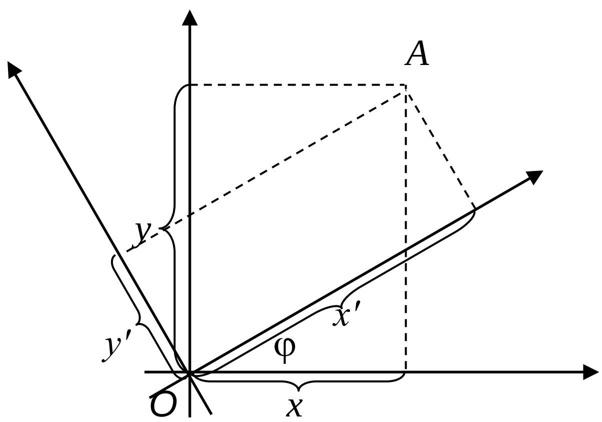
Misol 108. Koordinata o'qlarini parallel ko'chirganda *A(x;y)=A(3;1)* nuqta yangi A′(x′;y′)=A(2;-1) koordinatalarga ega bo'ladi. Eski va yangi koordinatalar sistemalari va *A* nuqtani yasang.



Yechilishi.

$$\begin{cases} x = x' + a \\ y = y' + b \end{cases} \Rightarrow \begin{cases} 3 = 2 + a \\ 1 = -1 + b \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} a = 1 \\ b = 2 \end{cases} \Rightarrow \; O'(1;2).$$



Koordinatalar boshi qo'zg'almagan, o'qlar φ burchakka burilgan hol.
$$\begin{cases} x = x' \cos\varphi - y' \sin\varphi; \\ y = x' \sin\varphi - y' \cos\varphi. \end{cases}$$

Misol 109. (*xOy*) koordinatalar sistemasiga nisbatan *A(x;y)=A(6;7)* nuqta berilgan, bu *A* nuqnaning koordinatalar boshi o'zgarmasdan qolib, o'qlar $\varphi = 30^o$ ga burilgandagi koordinatalarini toping.

Yechilishi.

$$\begin{cases}6 = x'\cos 30^o - y'\sin 30^o\\ 7 = x'\sin 30^o - y'\cos 30^o\end{cases} \Rightarrow \begin{cases}6 = x'\cdot\dfrac{\sqrt{3}}{2} - y'\cdot\dfrac{1}{2}\\ 7 = x'\cdot\dfrac{1}{2} - y'\cdot\dfrac{\sqrt{3}}{2}\end{cases} \Rightarrow \begin{cases}12 = \sqrt{3}x' - y'\\ 14 = x' - \sqrt{3}y'\end{cases}\Bigg|^{\sqrt{3}} \Rightarrow$$

$$\Rightarrow\quad - \begin{cases}12\sqrt{3} = 3x' - \sqrt{3}y'\\ 14 = x' - \sqrt{3}y'\end{cases} \Rightarrow\ 12\sqrt{3} - 14 = 2x' \ \Rightarrow\ x' = \frac{12\sqrt{3} - 14}{2} = 6\sqrt{3} - 7.$$

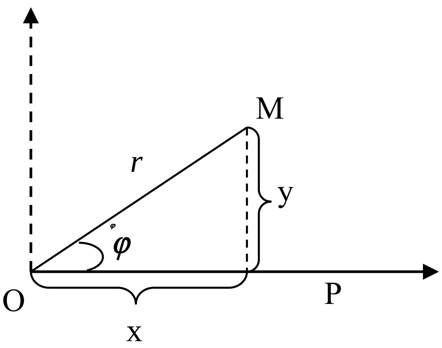
$x' = 6\sqrt{3} - 7$.

$12 = \sqrt{3}\cdot\left(6\sqrt{3} - 7\right) - y' \ \Rightarrow\ y' = 18 - 7\sqrt{3} - 12 = 6 - 7\sqrt{3}.$ $\qquad y' = 6 - 7\sqrt{3}.$

$A(x'; y') = A\left(6\sqrt{3} - 7; 6 - 7\sqrt{3}\right).$

Tekislikda *O* nuqta (qutb) va *OP* nur (qutb o'qi) berilgan bo'lsin.

Tekislikda *M* nuqtaning o'rni quyidagicha aniqlanadi:



1) Qutb burchagi $\varphi = \angle \boldsymbol{POM}$ yasaladi;

2) Radius-vektor *r=OM* yasaladi.

Agar φ musbat bo'lsa qutb burchagi soat mili yo'nalishiga teskari yo'nalishda bo'ladi, agar *r* manfiy bo'lsa radius vektor qutb o'qiga teskari yo'nalishda yasaladi.

Agar qutb to'g'ri burchakli Dekart koordinatalar sistemasining koordinatalar boshida va *OP* qutb o'qi abssissa o'qi (*Ox*) bilan ustma-ust tushsa, qutb koordinatalar sistemasining to'g'ri burchakli Dekart koordinatalar sistemasi bilan bog'lanishi quyidagicha bo'ladi:

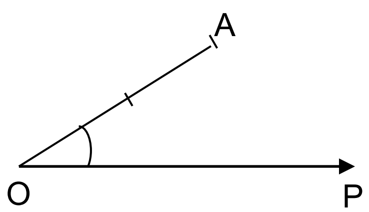
$$\boldsymbol{x = r\cos\varphi}, \qquad \boldsymbol{y = r\sin\varphi}$$

$$\boldsymbol{r = \sqrt{x^2 + y^2}}, \qquad \boldsymbol{tg\varphi = \frac{y}{x}} \ \textbf{\textit{ёки}}\ \boldsymbol{\varphi = arctg\frac{y}{x}}$$

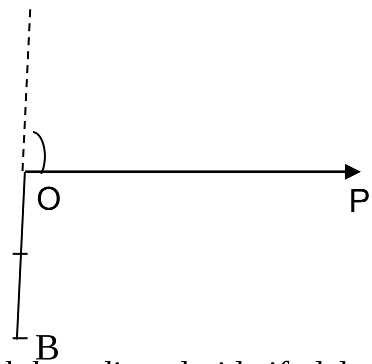
Misol 110. ($r;\varphi$) qo'tb koordinatalar sistemasida $A(2;\frac{\pi}{4})$ va $\boldsymbol{B}\left(-2;\frac{\pi}{2}\right)$ nuqtalarni yasang.

Yechilishi.

$\varphi = \dfrac{\pi}{4}$;
$\boldsymbol{r} = 2.$



$\varphi = \dfrac{\pi}{2}$;
$\boldsymbol{r} = -2.$



Misol 111. $\boldsymbol{x^2 - y^2 = a^2}$ egri chiziq tenglamasini qutb koordinatalarida ifodalang.
Yechilishi. $\boldsymbol{x = r\cos\varphi}$ va $\boldsymbol{y = r\,sin}\varphi$ o'tish formulalaridan foydalanamiz:

$$(r\cos\varphi)^2-(r\sin\varphi)^2=a^2\Rightarrow r^2\cos^2\varphi-r^2\sin^2\varphi=a^2\Rightarrow r^2\left(\cos^2\varphi-\sin^2\varphi\right)=a^2.$$

$$r^2\cos 2\varphi=a^2.$$

Misol 112. $r\sin\varphi=a$ chiziq tenglamasini Dekart koordinatalar sistemasida ifodalang.

Yechilishi. $r=\sqrt{x^2+y^2}$ *va* $tg\varphi=\frac{y}{x}$ o'tish formulalaridan foydalanamiz:

$$\sqrt{x^2+y^2}\cdot\sin(arc\quad tg\frac{y}{x})=a.$$

$$\cos arc\,tgx=\frac{1}{\sqrt{1+x^2}};\quad \sin arctgx=\frac{x}{\sqrt{1+x^2}}\ \ ga\qquad asosan$$

$$\sqrt{x^2+y^2}\cdot\frac{\frac{x}{y}}{\sqrt{1+\left(\frac{x}{y}\right)^2}}=a\Rightarrow\sqrt{x^2+y^2}\cdot\frac{x}{\sqrt{x^2+y^2}}=a,$$

$$x=a.$$

Misol 113. Qutb koordinatalari bilan berilgan $M_1(r_1;\ \varphi_1)$ va $M_2(r_2;\ \varphi_2)$ nuqtalar orasidagi masofani hisoblash formulasini keltirib chiqaring.

Yechilishi. M_1 va M_1 nuqtalarning dekart koordinatalari $M_1(x_1;\ y_1)$ va $M_1(x_2;\ y_1)$ bo'lsin.

Dekart koordinatalardan qutb koordinatalarga o'tish formulalariga asosan

$$\begin{cases}x_1=r_1\cos\varphi_1,\\ y_1=r_1\sin\varphi_1\end{cases}\quad va\quad \begin{cases}x_2=r_2\cos\varphi_2,\\ y_2=r_2\sin\varphi_2.\end{cases}$$

Ikki nuqta orasidagi masofani topish formulasiga asosan

$$|M_1M_2|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(r_2\cos\varphi_2-r_1\cos\varphi_1)^2+(r_2\sin\varphi_2-r_1\sin\varphi_1)^2}=$$

$$=\sqrt{r_2^2\cos^2\varphi_2+r_1^2\cos\varphi_1^2-2r_1r_2\cos\varphi_2\cos\varphi_1+r_2^2\sin^2\varphi_2+r_1^2\sin\varphi_1^2-2r_1r_2\sin\varphi_2\sin\varphi_1}=$$

$$=\sqrt{r_1^2\left(\sin^2\varphi_1+\cos^2\varphi_1\right)+r_2^2(\sin^2\varphi_2+\cos^2\varphi_2)-2r_1r_2(\cos\varphi_2\cos\varphi_1+\sin\varphi_2\sin\varphi_1)}=$$

$$=\sqrt{r_1^2+r_2^2-2r_1r_2\cos(\varphi_2-\varphi_1)}.$$

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114. (*xOy*) koordinatalar sistemasiga nisbatan *A(x;y)=A(3;5)* nuqta berilgan, bu *A* nuqnaning *O'(-1;3)* bo'lgandagi, x' va y' koordinatalarini toping va yasang. ***Javobi:*** $A'(4;2)$

115. Koordinata o'qlarini parallel ko'chirganda *A(x;y)=A(5;3)* nuqta yangi A′(x′;y′)=A(-1;5) koordinatalarga ega bo'ladi. Eski va yangi koordinatalar sistemalari va *A* nuqtani yasang. ***Javobi:*** $O'(6;-2)$

116. Koordinata o'qlarini parallel ko'chirganda *A(x;y)=A(5;3)* nuqta yangi A′(x′;y′)=A(-1;5) koordinatalarga ega bo'ladi. Eski va yangi koordinatalar sistemalari va *A* nuqtani yasang. ***Javobi:*** $O'(6;-2)$

117. Koordinata o’qlarini parallel ko’chirganda *A(x;y)=A(3;-2)* nuqta yangi *A'(x';y')= A' (8;-1)* koordinatalarga ega bo’ladi. Eski va yangi koordinatalar sistemalari va *A* nuqtani yasang. ***Javobi:*** $O'(-5;1)$

118. *(xOy)* koordinatalar sistemasiga nisbatan *A(x;y)=A(4;5)* nuqta berilgan, bu *A* nuqnaning koordinatalar boshi o’zgarmasdan qolib, o’qlar $\varphi=30^o$ ga burilgandagi koordinatalarini toping. ***Javobi:*** $A'(x';y')=A'(4\sqrt{3}-5;4-5\sqrt{3})$.

119. *(xOy)* koordinatalar sistemasiga nisbatan *A(x;y)=A(4;3)* nuqta berilgan, bu *A* nuqnaning koordinatalar boshi o’zgarmasdan qolib, o’qlar $\varphi=60^o$ ga burilgandagi koordinatalarini toping. ***Javobi:*** $A'(x';y')=A'(3\sqrt{3}-4;3-4\sqrt{3})$.

120. *(xOy)* koordinatalar sistemasiga nisbatan *A(x;y)=A(3;4)* nuqta berilgan, bu *A* nuqnaning koordinatalar boshi o’zgarmasdan qolib, o’qlar $\varphi=60^o$ ga burilgandagi koordinatalarini toping. ***Javobi:*** $A'(x';y')=A'(4\sqrt{3}-3;4-3\sqrt{3})$.

$(r;\varphi)$ qutb koordinatalar sistemasida quyidagi nuqtalarni yasang:

121) $A(3;0)$; 122) $B\left(3;\frac{\pi}{2}\right)$; 123) $C(2;\pi)$; 124) $D\left(3;\frac{3\pi}{2}\right)$; 125) $E\left(2;\frac{2\pi}{3}\right)$;

126) $A\left(3;-\frac{\pi}{2}\right)$; 127) $B\left(-3;\frac{\pi}{3}\right)$; 128) $C\left(-2;-\frac{\pi}{2}\right)$; 129) $D\left(-3;\frac{3\pi}{2}\right)$; 130) $E\left(1;-\frac{2\pi}{3}\right)$

Quyidagi chiziq tenglamalarini qutb koordinatalari orqali ifodalang:

131) $x^2+y^2=4$; 132) $x+y=1$; 133) $y=x$; 134) $x^2+y^2=2x$;

Quyidagi chiziq tenglamalarini Dekart koordinatalari orqali ifodalang:

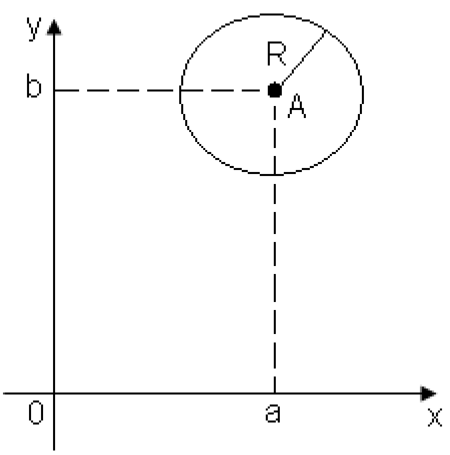
135) $r=2\sin\varphi$; 136) $r^2\sin 2\varphi=8$; 137) $r\sin\left(\varphi+\frac{\pi}{4}\right)=\sqrt{2}$; 138) $r=1+\cos\varphi$.

Qutb koordinatalari bilan berilgan quyidagi nuqtalar orasidagi masofani toping:

139) $M_1(3;15^0)$ va $M_2(2;75^0)$. Javobi: $\sqrt{7}$. **140)** $A(1;\frac{\pi}{12})$ va $B(\sqrt{3};\frac{\pi}{4})$. Javobi:1.

4 – MAVZU. IKKINCHI TARTIBLI CHIZIQLAR

Tekislikdagi markaz deb atalgan nuqtadan teng masofada joylashgan nuqtalarning geometrik o’rniga aylana deyiladi.



$$x^2+y^2+Ax+By+C=0 \qquad (1)$$

aylananing umumiy tenglamasi.

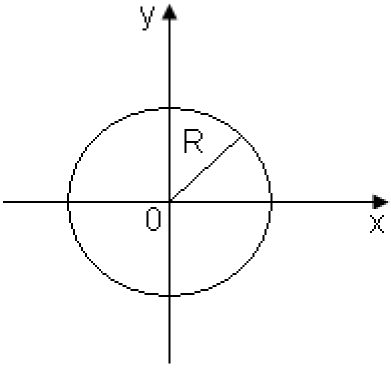
Markazi ***A(a;b)*** nuqtada, radiusi ***R*** ga teng bo’lgan aylananing normal tenglamasi:

$$(x-a)^2+(y-b)^2=R^2 \qquad (2)$$

Markazi koordinatalar boshi ***O(0;0)*** da, radiusi ***R*** ga teng bo’lgan aylana tenglamasi:

$$x^2+y^2=R^2. \qquad (3)$$

$(x-a)^2+(y-b)^2=R^2$ aylananing $M_0(x_0;y_0)$ nuqtasiga o'tkazilgan urinma to'g'ri chiziq tenglamasi:



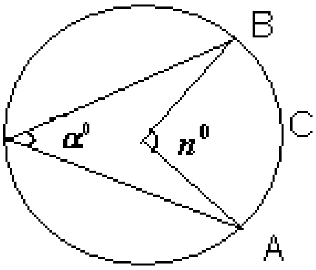
$$(x-a)(x_0-a)+(y-b)(y_0-b)=R^2. \quad (4)$$

$x^2+y^2=R^2$ aylananing $M_0(x_0;y_0)$ nuqtasiga o'tkazilgan urinma to'g'ri chiziq tenglamasi:

$$x\cdot x_0+y\cdot y_0=R^2. \quad (5)$$

Doiraning yuzini topish formulasi: $S=\pi R^2$. (6)

Aylananing uzunligini topish formulasi: $\ell=2\pi R.$ (7)



Aylana yoyi uzunligini topish formulasi: $\ell=\dfrac{\pi R n^0}{180^0}.$ (8)

Bunda n^0 - markaziy burchak.

$$n^0=A\breve{C}B. \quad (9)$$

$$\alpha^0=\frac{1}{2}*n^0. \quad (10)$$

Misol 141. $x^2+y^2+4x-6y-3=0$ umumiy tenglamasi bilan berilgan aylananing normal tenglamasini tuzing, markazining koordinatalari va radiusini toping.

Yechilishi. x bilan y oldidagi koeffitsientlar ikkiga bo'linib, olingan natijaning kvadrati berilgan tenglamaga ham qo'shiladi, ham ayriladi:

$$x^2+4x+2^2-2^2+y^2-6y+3^2-3^2-3=0;$$

$$(x+2)^2+(y-3)^2=16.$$

Bu (2) bilan taqqoslanadi:

$$\begin{cases}-a=2\\ -b=-3,\\ R^2=16.\end{cases}\Leftrightarrow\begin{cases}a=-2\\ b=3\\ R^2=4^2\end{cases}\Rightarrow\begin{cases}A(a;b)=A(-2;3);\\ R=4.\end{cases}$$

Misol 142. Markazi ***C(-4; 3)*** nuqtada, radiusi ***R=2*** bo'lgan aylananing umumiy tenglamasini yozing.

Yechilishi. $a=-4;\ b=3;\ R=2$ ma'lumotlar (1) ga qo'yiladi:

$$[x-(-4)]^2+(y-3)^2=2^2\Leftrightarrow (x+4)^2+(y-3)^2=4\Leftrightarrow$$

$$\Leftrightarrow x^2+2\cdot x\cdot 4+4^2+y^2-2\cdot y\cdot 3+3^2-4=0\Leftrightarrow x^2+y^2+8x-6y+21=0.$$

Misol 143. $x^2+y^2+2x-4y-20=0.$ aylananing ***x-y-4=0*** to'g'ri chiziq bilan kesishish nuqtalarini toping.

Yechilishi. Ikkita figuraning kesishish nuqtalarini topish uchun ularning tenglamalarini sistema qilib birgalikda yechish kerak.

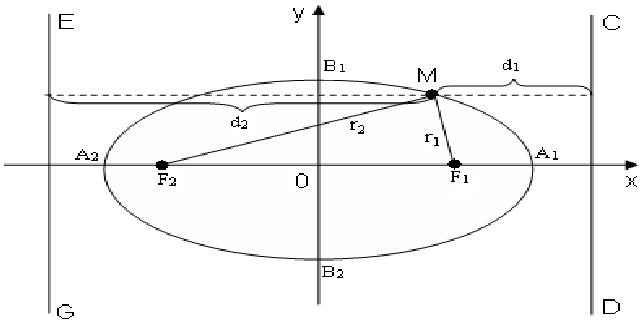
$$\begin{cases}x^2+y^2+2x-4y-20=0\\ x-y-4=0\end{cases}\Leftrightarrow\begin{cases}(y+4)^2+y^2+2(y+4)-4y-20=0\\ x=y+4\end{cases}\Rightarrow$$

$$\begin{cases} y^2+8y+16+y^2+2y+8-4y-20=0 \\ x=y+4 \end{cases} \Rightarrow \begin{cases} 2y^2+6y+4=0 \\ x=y+4 \end{cases} \Rightarrow \begin{cases} y^2+3y+2=0 \\ x=y+4 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} y_1=-1 \\ y_2=-2 \\ x=y+4 \end{cases} \Rightarrow \left[\begin{array}{l} \begin{cases} x_1=3 \\ y_1=-1 \end{cases} \Rightarrow \quad A(3;\ -1); \\ \begin{cases} x_2=2 \\ y_2=-2 \end{cases} \Rightarrow \quad B(2;\ -2). \end{array} \right.$$

E L L I P S

Ellips tekislikdagi shunday nuqtalarning geometrik o'rniki, bu nuqtalarning har biridah ikkita o'zgarmas nuqtagacha, ya'ni ellipsning fokuslarigacha bo'lgan masofalarning yi'g'indisi $2a$ ga teng o'zgarmas miqdordir.



Ellipsning kanonik (sodda) tenglamsi:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1. \tag{11}$$

Bunda

$$b^2=a^2-c^2. \tag{12}$$

Katta o'q: $A_1A_2=2a.$ (13)

Kichik o'q: $B_1B_2=2b.$ (14)

Fokuslar orasidagi masofa: $F_1F_2=2c.$ (15)

Ekssentrisitet: $e=\frac{c}{a}<1.$ (16)

Fokal radius-vektorlar:

$$\begin{cases} r_1=a-ex; \\ r_2=a+ex. \end{cases} \tag{17}$$

$$r_1+r_2=2a. \tag{18}$$

Direktrisalar tenglamalari:

$$x=\frac{a}{e}; \quad x=-\frac{a}{e}. \tag{19}$$

Ellipsning $M_0(x_0;\ y_0)$ nuqtasiga o'tkazilgan urinma to'g'ri chiziq tenglamasi:

$$\frac{x\,x_0}{a^2}+\frac{y\,y_0}{b^2}=1. \tag{20}$$

Misol 144. $25x^2 + 169y^2 = 4225$ ellips berilgan:

1) kanonik shaklga keltiring;
2) katta va kichik o'qlarini toping;
3) fokuslari orasidagi masofani toping;
4) ekssentrisitetini toping;
5) fokal radius-vektorlarini toping;
6) Direktrisalari tenglamalarini yozing.

Yechilishi.

1) berilgan tenglama hadlab 4225 ga bo'linadi:

$$\frac{25x^2}{4225} + \frac{169y^2}{4225} = 1 \Leftrightarrow \frac{x^2}{169} + \frac{y^2}{25} = 1;$$

2) $a^2 = 169 \Rightarrow a = 13 \Rightarrow 2a = 26;$ $b^2 = 25 \Rightarrow b = 5 \Rightarrow 2b = 10;$

3) $(2) \Rightarrow c^2 = a^2 - b^2 = 13^2 - 5^2 = 144 \Rightarrow c = 12 \Rightarrow 2c = 24;$ 4) $e = \frac{c}{a} = \frac{12}{13} < 1;$

5) $r_1 = a - ex \Rightarrow r_1 = 13 - \frac{12}{13}x;$ $r_2 = a + ex \Rightarrow r_2 = 13 + \frac{12}{13}x;$

6) $x = \frac{a}{e} \Rightarrow x = 13 : \frac{12}{13} = \frac{169}{12};$ $x = -\frac{a}{e} \Rightarrow x = -13 : \frac{12}{13} = -\frac{169}{12}.$

Misol 145. $x^2 + y^2 - 2x + 6y - 5 = 0$ ellipsni kanonik ko'rinishga keltiring.

Yechilishi. $x^2 - 2x + 1^2 - 1^2 + y^2 + 6y + 3^2 - 3^2 - 5 = 0;$ $(x-1)^2 + (y+3)^2 = 15;$

$$\frac{(x-1)^2}{15} + \frac{(y+3)^2}{15} = 1.$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi umumiy tenglamasi bilan berilgan aylananing normal tenglamasini tuzing, markazi koordinatalarini va radiusini toping:

146. $x^2 + y^2 - 6x - 6y + 17 = 0;$ *Javobi* :(3; 3); $R = 1.$
147. $x^2 + y^2 - 10x + 4y + 20 = 0;$ *Javobi* :(5;–2); $R = 3.$
148. $x^2 + y^2 - 8x - 4y + 16 = 0;$ *Javobi* :(4;2); $R = 2.$
149. $x^2 + y^2 - 6x + 4y - 23 = 0;$ *Javobi* :(3;–2); $R = 6.$
150. $x^2 + y^2 - 4x + 2y + 1 = 0;$ *Javobi* :(2;–1); $R = 2.$
151. $x^2 + y^2 - 4x + 6y - 3 = 0;$ *Javobi* :(2;–3); $R = 4.$
152. $x^2 + y^2 + 4x - 6y = 0;$ *Javobi* :(–2; 3); $R = \sqrt{13}.$
153. $x^2 + y^2 - 12x + 27 = 0;$ *Javobi* :(6;0); $R = 3.$
154. $x^2 + y^2 - 6y + 5 = 0;$ *Javobi* :(0; 3); $R = 2.$
155. $x^2 + y^2 - 8x = 0;$ *Javobi* :(4; 0); $R = 4.$
156. $x^2 + y^2 + 4y = 0;$ *Javobi* :(0; – 2); $R = 2.$

Markazi va radiusiga asosan aylananing umumiy tenglamasini tuzing va yasang:

157. A(-3;0) va R=3; 158. B(3;0) va R=3; 159. C(0;-3) va R=3;
160. D(0;3) va R=3; 161. O(0;0) va R=5; 162. E(4;5) va R=1;
163. F(-4;5) va R=2; 164. M(4;-5) va R=1; 165. N(-4;-5) va R=2.

Aylanaga berilgan nuqtada o'tkazilgan urinma to'g'ri chiziq tenglamasini tuzing:

166. A(4;3); $x^2 + y^2 - 6x - 6y + 17 = 0;$ 167. B(-2; -3); $x^2 + y^2 - 4x + 6y - 3 = 0;$
168. C(3;0); $x^2 + y^2 - 12x + 27 = 0;$ 169. D(-2;3); $x^2 + y^2 - 6y + 5 = 0;$

170. Berilgan ma'lumotlardan foydalanib ellipsning kanonik tenglamasini tuzing:

Yarim o'qlar 4 va 2 ga teng. Fokuslar orasidagi masofa 6 ga va katta yarim o'qi 5 ga teng. Katta yarim o'qi 10ga va ekssentrisiteti e=0,8 ga teng. Kichik yarim o'qi 3 ga va ekssentrisiteti $e=\frac{\sqrt{2}}{2}$ ga teng. Yarim o'qlarining yig'indisi 8 ga, fokuslari orasidagi masofa ham 8 ga teng.

171. $5x^2+20y^2=180$ tenglamasi bilan berilgan ellipsning kanonik tenglamasini, fokuslarining koordinatalarini va ekssentrisitetini toping.

Javobi **:** $\frac{x^2}{36}+\frac{y^2}{9}=1;\ F_1(5;0);\ F_2(-5;0);\ e=\frac{5}{6}<1.$

172. $x=\pm 8$ to'g'ri chiziqlar kichik o'qi 8 ga teng bo'lgan ellipsning direktrisalari. Shu ellips tenglamasini toping. ***Javobi*** **:** $\frac{x^2}{32}+\frac{y^2}{16}=1.$

173. $\frac{x^2}{36}+\frac{y^2}{20}=1$ ellipsning direktrisalari tenglamalarini yozing. ***Javobi*** **:** $x=\pm 9.$

174. $\frac{x^2}{48}+\frac{y^2}{36}=1$ ellipsga nisbatan A(6; -3), B(-2; 5), C(3; -6), D($\sqrt{50}$; 0), E(-4; 2$\sqrt{6}$), F(1; $\sqrt{26}$) nuqtalarning vaziyatini aniqlang.

Javobi **:** A va E nuqtalar ellipsda, B va F nuqtalar ellips ichida, C va D nuqtalar ellipsdan tashqarida yotadi.

175. $\frac{x^2}{36}+\frac{y^2}{12}=1$ ellipsning 2x-y-9=0 to'g'ri chiziq bilan kesishish nuqtalarini toping.

Quyidagi ellipsni kanonik ko'rinishga keltiring

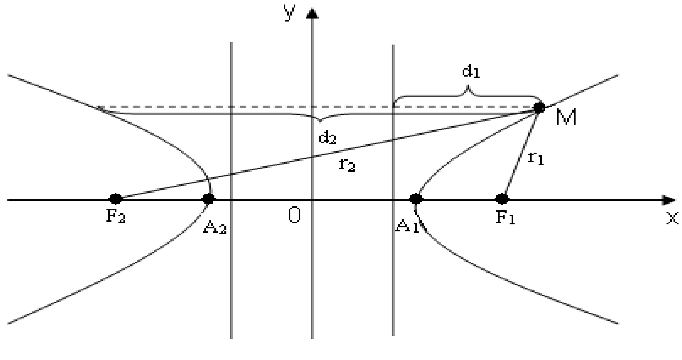
176. $x^2+4y^2+4x-16y-8=0.$ ***Javobi*** **:** $\frac{(x+2)^2}{16}+\frac{(y-1)^2}{4}=1.$

177. $x^2+2y^2+8x-4=0.$ ***Javobi*** **:** $\frac{(x+4)^2}{20}+\frac{y^2}{10}=1.$

5-MAVZU. IKKINCHI TARTIBLI CHIZIQLAR VA ULARNING KLASSIFIKATSIYASI

G I P E R B O L A

Giperbola tekislikdagi shunday nuqtalarning geometrik o'rniki, bu nuqtalarning har biridan ikkita o'zgarmas nuqtagacha, ya'ni giperbolaning fokuslarigacha bo'lgan masofalar ayirmasi o'zgarmas miqdor bo'lib 2*a* ga teng.



Giperbolaning kanonik tenglamasi:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1. \qquad (21)$$

Bunda

$$b^2=c^2-a^2. \qquad (22)$$

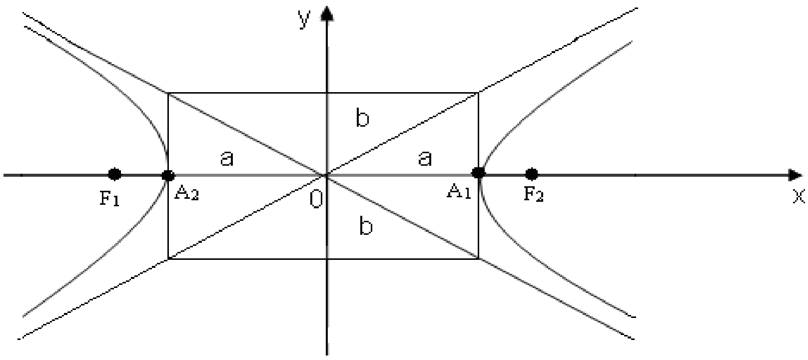
Haqiqiy o'q: $AA_1=2a.$ (23)

Mavhum o'q: $BB_1=2b.$ (24)

Fokuslar orasidagi masofa: $F_1F_2=2c.$ (25)

Qo'shma giperbolalar: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ va $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$ (26)

Teng tomonli giperbolalar: $\frac{x^2-y^2}{a^2}=1$ va $\frac{x^2-y^2}{b^2}=1$,(a=b). (27)



Ekssentrisitet: $e=\frac{c}{a}>1$ (28)

O'ng tarmoq nuqtalari uchun fokal radius-vektorlar:

$$\begin{cases} r_1=ex-a; \\ r_2=ex+a; \\ r_2-r_1=2a. \end{cases} \qquad (29)$$

Chap tarmoq nuqtalari uchun fokal radius-vektorlar:

$$\begin{cases} r_1=-ex+a; \\ r_2=-ex-a; \\ r_1-r_2=2a. \end{cases} \qquad (30)$$

Direktrisalar tenglamalari:

$$x=\frac{a}{e}; \qquad x=-\frac{a}{e}. \qquad (31)$$

Asimptotalar tenglamalari:

$$\begin{cases} y=\frac{b}{a}x; \\ y=-\frac{b}{a}x. \end{cases} \qquad (32)$$

Giperbolaning $M_0(x_0;y_0)$ nuqtasiga o'tkazilgan urinma to'g'ri chiziq tenglamasi:

$$\frac{x\cdot x_0}{a^2}-\frac{y\cdot y_0}{b^2}=1. \qquad (33)$$

Misol 178. $36x^2-25y^2=900$ giperbola berilgan:

1) kanonik shaklga keltiring;
2) qo'shmasini yozing;
3) haqiqiy va mavhum o'qlarini toping;
4) fokuslari orasidagi masofani toping;
5) ekssentrisitetini toping;
6) fokal radius-vektorlarini toping;
7) direktrisalari tenglamalarini yozing;
8) asimptotalari tenglamalarini yozing.

Yechilishi.

1) Berilgan tenglama hadlab **900** ga bo'linadi:

$$\frac{36x^2}{900}-\frac{25y^2}{900}=1 \Leftrightarrow \frac{x^2}{25}-\frac{y^2}{36}=1;$$

2) $-\frac{x^2}{25}+\frac{y^2}{36}=1;$

3) $a^2=25 \Rightarrow a=5 \Rightarrow 2a=10;$

$b^2=36 \Rightarrow b=6 \Rightarrow 2b=12.$

4) (2) $\Rightarrow c^2=a^2+b^2=25+36=61 \Rightarrow c=\sqrt{61} \Rightarrow 2c=2\sqrt{61};$

5) $e=\frac{c}{a}=\frac{\sqrt{61}}{5}>1;$

6) $\begin{cases} r_1=\sqrt{61}x-5; \\ r_2=\sqrt{61}x+5; \\ r_2-r_1=10; \end{cases}$ va $\begin{cases} r_1=-\sqrt{61}x+5; \\ r_2=-\sqrt{61}x-5; \\ r_1-r_2=10. \end{cases}$

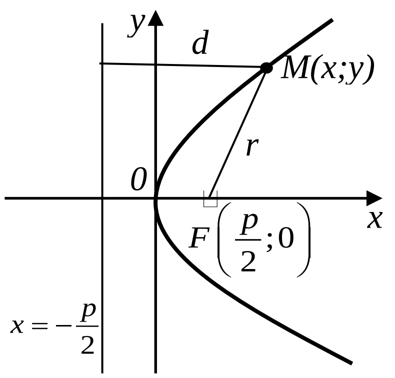
7) $x=\frac{25}{\sqrt{61}}$ **va** $x=-\frac{25}{\sqrt{61}};$

8) $\begin{cases} y=\frac{6}{5}x; \\ y=-\frac{6}{5}x. \end{cases}$

Misol 179. $\frac{x^2}{8}-\frac{y^2}{4}=1$ giperbolaning **(4; 2)** nuqtasiga o'tkazilgan urinma to'g'ri chiziq tenglamasini tuzing.

Yechilishi. $x_0=4;\ \ y_0=2.$

$$(33) \Rightarrow \frac{4x}{8}-\frac{2y}{4}=1 \Rightarrow \frac{x}{2}-\frac{y}{2}=1 \Rightarrow x-y=2 \Rightarrow x-y-2=0 \ .$$



PARABOLA

Parabola tekislikdagi shunday nuqtalarning geometrik o'rniki, bu nuqtalarning har biridan o'zgarmas bitta nuqtagacha, ya'ni parabolaning fokusigacha va o'zgarmas bitta to'g'ri chiziqqacha, ya'ni parabolaning direktikasigacha bo'lgan masofalar tengdir.

Parabolaning kanonik tenglamasi: $y^2 = 2px.$ (34)

Direktrisa tenglamasi: $x = -\frac{p}{2}.$ (35)

Parabola ixtiyoriy nuqtasining fokal radius-vektori:

$$r = x + \frac{p}{2}. \qquad (36)$$

$$\frac{r}{d} = 1. \qquad (37)$$

Parabolaning $M_0(x_0; y_0)$ nuqtasiga o'tkazilgan urinma to'g'ri chiziq tenglamasi:

$$\boldsymbol{yy_0 = p(x + x_0).} \qquad (38)$$

$x^2 = 2py$ bo'lsa, $\boldsymbol{y = -\frac{p}{2}, r = y + \frac{p}{2}, x * x_0 = p(y + y_0), F(0; \frac{p}{2})}$ bo'ladi.

Misol 180. $y^2 = 8x$ parabola berilgan;

1) p- parametrini aniqlang;
2) fokus nuqtasini koordinatalari bilan yozing;
3) direktrisasi tenglamasini yozing;
4) fokal radius-vektorini toping.

Yechilishi.

1) $y^2 = 8x \Rightarrow y^2 = 2 \cdot 4 \cdot x \Rightarrow p = 4;$ 2) $\frac{p}{2} = 2 \Rightarrow F(2;0);$

3) $x = -2;$ 4) $r = x + 2.$

Misol 181. $y^2 = 8x$ parabolaning $\boldsymbol{x - 2y + 8 = 0}$ to'g'ri chiziq bilan kesishish nuqtasini toping.

Yechilishi. Ikki tenglama sistema qilinib birgalikda yechiladi.

$$\begin{cases} y^2 = 8x \\ x - 2x + 8 = 0 \end{cases} \Leftrightarrow \begin{cases} y^2 - 8x = 0 \\ x = 2y - 8 \end{cases} \Leftrightarrow \begin{cases} y^2 - 8(2y - 8) = 0 \\ x = 2y - 8 \end{cases} \Rightarrow \begin{cases} y^2 - 16y + 64 = 0 \\ x = 2y - 8 \end{cases} \Rightarrow$$

$$\Leftrightarrow \begin{cases} (y - 8)^2 = 0 \\ x = 2y - 8 \end{cases} \Rightarrow \begin{cases} y - 8 = 0 \\ x = 2y - 8 \end{cases} \Rightarrow \begin{cases} y = 8 \\ x = 2*8 - 8 \end{cases} \Rightarrow \begin{cases} y = 8 \\ x = 8 \end{cases} \Rightarrow M(x; y) = M(8; 8).$$

Misol 182. $y^2 = 8x$ parabolaning $M_0(8; 8)$ nuqtasiga o'tkazilgan urinma to'g'ri chiziq tenglamasini tuzing.

Yechilishi.

$x_0 = 8; \quad y_0 = 8;$

$2p = 8 \Rightarrow p = 4.$

$(38) \Rightarrow y \cdot 8 = 4(x + 8) \Leftrightarrow 8y = 4x + 32 \Leftrightarrow 2y = x + 8 \Leftrightarrow x - 2y + 8 = 0.$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

183. $36x^2 - 49y^2 = 1764$ **giperbola berilgan**

1) kanonik shaklga keltiring;
2) qo'shmasini yozing;
3) uchlarini koordinatalari bilan yozing;
4) haqiqiy va mavhum o'qlarini toping;
5) fokus nuqtalarini koordinatalari bilan yozing;
6) fokuslari orasidagi masofani toping;
7) ekssentrisitetini toping;
8) fokal radius-vektorlarini toping;
9) direktrisalari tenglamalarini yozing;
10) asimptotalari tenglamalarini yozing.

184. $\frac{x^2}{9} - \frac{y^2}{16} = 1$ **giperbola berilgan**

1) fokuslarining koordinatalarini toping; ***Javobi*** **:** $c = \pm 5.$

2) ekssentrisitetini hisoblang; ***Javobi*** **:** $e = \frac{5}{3} > 1.$

3) asimptotalarining va direktrisalarining tenglamalamalarini yozing; ***Javobi*** **:** $y = \pm \frac{4}{3}x;$ $x = \pm \frac{9}{5}.$

4) qo'shma giperbolaning tenglamasini yozing va uning ekssentrisitetini hisoblang.
Javobi **:** $-\frac{x^2}{9} + \frac{y^2}{16} = 1;$ $e = \frac{5}{3} > 1.$

185. $\frac{x^2}{5} - \frac{y^2}{4} = 1$ giperbolaga N(5; -4) nuqtada urinuvchi to'g'ri chiziqning tenglamasini tuzing. ***Javobi*** **:** $x + y - 1 = 0.$

186. $y^2 = 12x$ **parabola berilgan:**

1) p- parametrini aniqlang;
2) fokus nuqtasini koordinatalari bilan yozing;
3) direktrisasi tenglamasini yozing;
4) fokal radius- vektorini toping.

187. Quyidagilarni bilgan holda parabolaning tenglamasini tuzing:

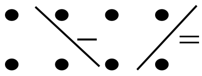
1) parabolaning uchidan fokusigacha bo'lgan masofa 3 ga teng;
2) fokus nuqtasi F(5; 0) bo'lib, ordinata o'qi direktrisa vazifasini o'taydi;
3) parabola (0x) o'qiga nisbatan simmetrik bo'lib, M(1; -4) nuqtadan va koordinatalar boshidan o'tadi.
4) parabola (0y) o'qiga nisbatan simmetrik bo'lib, fokusi (0; 2) nuqtada va uchi koordinatalar boshida yotadi;
5) parabola (0y) o'qiga nisbatan simmetrik bo'lib, M(6; -2) nuqtadan va koordinatalar boshidan o'tadi.

188. $y^2 = 8x$ parabolada fokal radius vektori 20 ga teng bo'lgan nuqtani toping.
Javobi: A(18; 12) va B(18; -12).

6-MAVZU. DETERMINANTLAR

IKKINCHI TARTIBLI DETERMINANT

Ikkinchi tartibli determinantni to'g'ridan –to'g'ri hisoblash:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} =$$  $$= a_{11}a_{22} - a_{12}a_{21}. \qquad (1)$$

Misol 189.

$$\begin{vmatrix} 2 & 3 \\ -1 & 3 \end{vmatrix} = 2 \cdot 3 - 3 \cdot (-1) = 6 + 3 = 9 \quad .$$

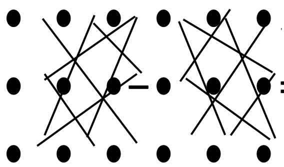
Ikkinchi tartibli determinantni minorlarga yoyish orqali hisoblash:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = (-1)^{1+1} \cdot a_{11} \cdot a_{22} + (-1)^{1+2} \cdot a_{12} \cdot a_{21} \quad . \qquad (2)$$

Misol 190. $\begin{vmatrix} 2 & 3 \\ -1 & 3 \end{vmatrix} = (-1)^{1+1} \cdot 2 \cdot 3 + (-1)^{1+2} \cdot 3 \cdot (-1) = 9 \quad .$

UCHINCHI TARTIBLI DETERMINANT

Uchinchi tartibli determinantni hisoblashning uchburchak usuli:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$  $$= a_{11} \cdot a_{22} \cdot a_{33} + a_{12} \cdot a_{23} \cdot a_{31} + a_{21} \cdot a_{32} \cdot a_{13} -$$

$$-a_{13} \cdot a_{22} \cdot a_{31} - a_{12} \cdot a_{21} \cdot a_{33} - a_{23} \cdot a_{32} \cdot a_{11} \quad . \qquad (3)$$

Misol 191.

$$\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix} = 2 \cdot 2 \cdot 6 + (-3) \cdot (-4) \cdot 5 + 1 \cdot (-4) \cdot 1 - 1 \cdot 2 \cdot 5 - (-3) \cdot 1 \cdot 6 - (-4) \cdot (-4) \cdot 2 =$$

$$= 24 + 60 - 4 - 10 + 18 - 32 = 24 + 60 + 18 - 4 - 10 - 32 = 102 - 46 = 56 \quad .$$

Uchinchi tartibli determinantni minorlarga yoyish orqali hisoblash:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} \cdot a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} \cdot a_{12} \cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} \cdot a_{13} \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \,. \qquad (4)$$

Misol 192. Birinchi satr bo'yicha minorlarga yoyish:

$$\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix} = (-1)^{1+1}\cdot 2\cdot \begin{vmatrix} 2 & -4 \\ -4 & 6 \end{vmatrix} + (-1)^{1+2}\cdot(-3)\cdot \begin{vmatrix} 1 & -4 \\ 5 & 6 \end{vmatrix} + (-1)^{1+3}\cdot 1\cdot \begin{vmatrix} 1 & 2 \\ 5 & -4 \end{vmatrix} =$$

$$= 2\cdot\left[2\cdot 6-(-4)\cdot(-4)\right]-(-3)\cdot\left[1\cdot 6-(-4)\cdot 5\right]+1\cdot(-4)-2\cdot 5 =$$

$$= 2\cdot(12-16)+3(6+20)-4-10 = -8+78-14 = 78-22 = 56.$$

Misol 193. Uchinchi satr bo'yicha minorlarga yoyish:

$$\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix} = (-1)^{3+1}\cdot 5\cdot \begin{vmatrix} -3 & 1 \\ 2 & -4 \end{vmatrix} + (-1)^{3+2}\cdot(-4)\cdot \begin{vmatrix} 2 & 1 \\ 1 & -4 \end{vmatrix} + (-1)^{3+3}\cdot 6\cdot \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} =$$

$$= 5\cdot(12-2)+4\cdot(-8-1)+6\cdot(4+3) = 50-36+42 = 50+42-36 =$$

$$= 92-36 = 56.$$

Uchunchi tartibli determinantni Sarrius usuli yordamida hisoblash:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{matrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{matrix} = a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}- \quad (5)$$

$$-a_{13}a_{22}a_{31}-a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}\quad .$$

Misol 194.

$$\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix} = \begin{matrix} 2 & -3 & 1 & 2 & -3 \\ 1 & 2 & -4 & 1 & 2 \\ 5 & -4 & 6 & 5 & -4 \end{matrix} = 2\cdot 2\cdot 6+(-3)\cdot(-4)\cdot 5+1\cdot 1\cdot(-4)-$$

$$-5\cdot 2\cdot 1-(-4)\cdot(-4)\cdot 2-6\cdot 1\cdot(-3) = 24+60-4-10-32+18 =$$

$$= 24+60+18-4-10-32 = 102-46 = 56.$$

DETERMINANTNING XOSSALARI

1^0. Determinantning barcha satrlari o'ziga mos ustunlar bilan almashtirilsa determinantning qiymati o'zgarmaydi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} \quad (7)$$

Misol 195.

$$\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 5 \\ -3 & 2 & -4 \\ 1 & -4 & 6 \end{vmatrix} = 56$$

2^0. Determinantda ikkita satri (yoki ikkita ustun) ning o'rinlari almashtirilsa, determinantning ishorasi o'zgaradi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad (8)$$

Misol 196.

$\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix} = 56$ determinantning ikkita satri va ikkita ustuni o'rinlarini almashtirib hisoblang.

Yechilishi.

$$\begin{vmatrix} 1 & 2 & -4 \\ 2 & -3 & 1 \\ 5 & -4 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 1 & -3 \\ 1 & -4 & 2 \\ 5 & 6 & -4 \end{vmatrix} = -56.$$

3^0. Determinantning bitta satri yoki bitta ustunidagi barcha elementlar noldan farqli biror songa ko'paytirilsa, determinantning qiymati shu son marta o'zgaradi:

$$\begin{vmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \lambda \cdot \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Misol 197.

$\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & =4 & 6 \end{vmatrix} = 56$. Determinantning bitta satri va bitta ustunidagi barcha elementlarni $\frac{1}{2}$ ga ko'paytirib hisoblang.

Yechilishi .

$$\begin{vmatrix} \frac{1}{2}\cdot 2 & \frac{1}{2}\cdot(-3) & \frac{1}{2}\cdot 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix} = \begin{vmatrix} 2 & -3 & \frac{1}{2}\cdot 1 \\ 1 & 2 & \frac{1}{2}\cdot(-4) \\ 5 & -4 & \frac{1}{2} * 6 \end{vmatrix} = \frac{1}{2}\cdot 56 = 28.$$

4^0. Determenantning ikkita satr (yoki ikkita ustun)elementlari proporsional bo'lsa, bunday determinantning qiymati nolga teng bo'ladi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & \mu a_{11} \\ a_{21} & a_{22} & \mu a_{21} \\ a_{31} & a_{32} & \mu a_{31} \end{vmatrix} = 0. \qquad 10)$$

Misol 198.

$$\begin{vmatrix} 2 & -3 & 1 \\ 2*2 & 2*(-3) & 2*1 \\ 5 & -4 & 6 \end{vmatrix} - \begin{vmatrix} 2 & 3*2 & 1 \\ 1 & 3*1 & -4 \\ 5 & 3*5 & 6 \end{vmatrix} = 0.$$

5⁰.Agar determinantning biror satri (yoki biror ustuni)dagi elementlar ikkita qo'shuluvchidan iborat bo'lsa, bunday determinant ikkita determinantning yigindisiga teng bo'ladi.

$$\begin{Vmatrix} a_{11}+a & a_{12} & a_{13} \\ a_{21}+b & a_{22} & a_{23} \\ a_{31}+c & a_{32} & a_{33} \end{Vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a & a_{12} & a_{13} \\ b & a_{22} & a_{23} \\ c & a_{32} & a_{33} \end{vmatrix} : \qquad (11)$$

yoki

$$\begin{vmatrix} a_{11}+a & a_{12}+b & a_{13}+c \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{3} \end{vmatrix} + \begin{vmatrix} a & b & c \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad (12)$$

6⁰ Determinantning bitta satr (ustun)idagi barcha elementlar noldan farqli biror songa ko'paytirilib, boshqa istalgan satr (ustun) ning mos elementlariga qo'shilsa, determinantning qiymati o'zgarmaydi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21}+\lambda a_{11} & a_{22}+\lambda a_{12} & a_{23}+\lambda a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} : \qquad (13)$$

yoki

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13}+\mu a_{11} \\ a_{21} & a_{22} & a_{23}+\mu a_{21} \\ a_{31} & a_{32} & a_{33}+\mu a_{31} \end{vmatrix}. \qquad (14)$$

Misol 199.

$\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix}$ determinantni hisoblang.

Yechilishi.

1) ikkinchi satr elementlari -2 ga ko'paytirilib, biirinchi satrning mos elementlariga qo'shiladi;

2) ikkinchi satr elementlari –5 ga ko'paytirilib, uchunchi satrning mos elementlariga qo'shiladi;
3) ustunlar o'ziga mos satrlar bilan almashtiiladi;
4) birinchi satr bo'yicha minorlarga yoyiladi;

$$\begin{vmatrix} 1 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix} = \begin{vmatrix} 0 & -7 & 9 \\ 1 & 2 & -4 \\ 0 & -14 & 26 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ -7 & 2 & -14 \\ 9 & -4 & 26 \end{vmatrix} - (-1)^{1+1} * 0 * \begin{vmatrix} 2 & -14 \\ -4 & 26 \end{vmatrix} + (-1)^{1+2} * 1 * \begin{vmatrix} -7 & -14 \\ 9 & 26 \end{vmatrix} +$$

$$+ (-1)^{1+3} * 0 * \begin{vmatrix} -7 & 2 \\ 9 & -4 \end{vmatrix} = -(-7) * \begin{vmatrix} 1 & 2 \\ 9 & 26 \end{vmatrix} = 7 * 2 * \begin{vmatrix} 1 & 1 \\ 9 & 13 \end{vmatrix} = 14 * (13 - 9) = 56$$

Misol 200.

$\begin{vmatrix} 3 & 6 & 0 \\ 1 & 2 & 9 \\ 10 & 8 & 18 \end{vmatrix} = 324$ tenglikning to'g'riligini isbotlang.

Yechilishi. Berilgan determinantning birinchi satridan umumiy ko'paytiruvchi 3, uchinchi satridan 2 chiqariladi.

$$\begin{vmatrix} 3 & 6 & 0 \\ 1 & 2 & 9 \\ 10 & 8 & 18 \end{vmatrix} = 3 * 2 * \begin{vmatrix} 1 & 2 & 0 \\ 1 & 2 & 9 \\ 5 & 4 & 9 \end{vmatrix} =$$

Hosil bo'lgan determinantning ikkinchi ustunidan umumiy ko'paytuvchi 2, uchinchi ustundan 9 chiqariladi.

$$= 6 * 2 * 9 * \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{vmatrix} =$$

Birinchi satrning yana bir elementini nolga aylantirish uchun birinchi ustun elemenlari –1 ga ko'paytirilib ikkinchi ustunining mos elementlariga qo'shiladi.

$$= 108 * \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 5 & -3 & 1 \end{vmatrix} =$$

Birinchi satr bo'yicha minorlarga yoyiladi.

$$= 108 * (-1)^{1+1} * 1 * \begin{vmatrix} 0 & 1 \\ -3 & 1 \end{vmatrix} = 108 * (0 + 3) = 324.$$

Misol 201.

$\begin{vmatrix} 2 & 1 & 0 & 2 \\ 3 & 2 & 1 & 0 \\ -1 & 0 & 1 & 3 \\ -2 & 2 & 2 & 6 \end{vmatrix}$ determinantni hisoblang.

Yechilishi. Uchunchi satr elementlari –2 ga ko'paytirilib, to'rtinchi satrning mos elementlariga qo'shiladi.

$$\begin{vmatrix} 2 & 1 & 0 & 2 \\ 3 & 2 & 1 & 0 \\ 2 & 0 & -2 & -6 \\ -2 & 2 & 2 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 0 & 2 \\ 3 & 2 & 1 & 0 \\ -1 & 0 & 1 & 3 \\ 0 & 2 & 0 & 0 \end{vmatrix} =$$

Hosil bo'lgan determinant to'rtinchi satr elementlari bo'yicha minorlarga yoyiladi

$$= (-1)^{4+2} * 2 * \begin{vmatrix} 2 & 0 & 2 \\ 3 & 1 & 0 \\ -1 & 1 & 3 \end{vmatrix} =$$

Determinantning birinchi ustuni elementlari –1 ga ko'paytirilib uchinchi ustunning mos elementlariga qo'shiladi .

$$= 2 * \begin{vmatrix} 2 & 0 & 0 \\ 3 & 1 & -3 \\ -1 & 1 & 4 \end{vmatrix} =$$

Determinant birinchi satr elementlari bo'yicha minorlarga yoyiladi.

$$= 2 * (-1)^{1+1} * 2 * \begin{vmatrix} 1 & -3 \\ 1 & 4 \end{vmatrix} = 4 * (4+3) = 28.$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi ikkinchi tartibli determinantlarni hisoblang:

202. $\begin{vmatrix} -2 & 3 \\ 1 & 4 \end{vmatrix}$; Javobi:-11,

203. $\begin{vmatrix} 0 & 4 \\ -5 & 8 \end{vmatrix}$; Javobi:20,

204. $\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix}$; . Javobi:5,

205. $\begin{vmatrix} 2 & -3 \\ 1 & -1 \end{vmatrix}$; Javobi: 1.

Quyidagi uchinchi tartibli determinantlarni uchburchak, minorlarga yoyish va Sarrius usullari yordamida hisoblang:

206. $\begin{vmatrix} 2 & -5 & 6 \\ 2 & 20 & 1 \\ 2 & 8 & 5 \end{vmatrix}$; Javobi:80,

207. $\begin{vmatrix} 3 & 4 & 5 \\ 6 & 8 & 10 \\ 1 & -2 & 3 \end{vmatrix}$; Javobi: 0,

208. $\begin{vmatrix} 2 & 4 & 5 \\ 2 & 4 & 5 \\ -1 & 3 & -4 \end{vmatrix}$; Javobi:0.

209. $\begin{vmatrix} 3 & -8 & 2 \\ 1 & -4 & 3 \\ 2 & -10 & 1 \end{vmatrix}$; Javobi:34,

210. $\begin{vmatrix} 0 & 1 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & 7 \end{vmatrix}$; Javobi:4,

211. $\begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & 1 \end{vmatrix}$; Javobi:-2,

212. $\begin{vmatrix} 2 & 3 & 4 \\ 4 & 9 & 8 \\ 6 & 15 & 16 \end{vmatrix}$. Javobi:24.

Quyidagi yuqori tartibli determinantlarni hisoblang:

213. $\begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -2 \\ 6 & 0 & 3 & -2 \\ 1 & -5 & 3 & -3 \end{vmatrix}$; 214. $\begin{vmatrix} 2 & 4 & -1 & 8 \\ 4 & 8 & -2 & 16 \\ 5 & -1 & -3 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix}$; 215. $\begin{vmatrix} 1 & 1 & 2 & 3 \\ 2 & 1 & 3 & 4 \\ 3 & 2 & 5 & 5 \\ 4 & 3 & 6 & 6 \end{vmatrix}$:

Javobi: 240. Javobi: 0, Javobi: 6,

216. $\begin{vmatrix} 1 & 2 & 2 & 2 \\ -2 & 2 & 2 & 2 \\ -2 & 2 & 3 & 2 \\ -2 & -2 & 2 & 4 \end{vmatrix}$; 217. $\begin{vmatrix} 1 & 2 & 2 & 3 \\ 3 & -1 & -4 & -6 \\ 1 & 0 & 0 & 1 \\ 1 & 2 & -1 & -7 \end{vmatrix}$; 218. $\begin{vmatrix} 3 & -1 & 4 & 2 \\ 5 & 2 & 0 & 1 \\ 9 & 2 & 1 & -2 \\ 6 & -2 & 9 & 8 \end{vmatrix}$;

Javobi: 36. Javobi: 12. Javobi:-31.

219. $\begin{vmatrix} 3 & -1 & 4 & 2 \\ 5 & 2 & 0 & -6 \\ 9 & 2 & 1 & -6 \\ 6 & -2 & 9 & 6 \end{vmatrix}$;

Javobi: 0.

7- MAVZU. N - O'ZGARUVCHILI N TA CHIZIQLI TENGLAMALAR SISTEMASINI YECHISH

CHIZIQLI TENGLAMALAR SISTEMASINI YECHISHNING KRAMER QOIDASI

Darajasi birga teng bo'lgan noma'lumlar qatnashgan tenglamani chiziqli tenglama deyiladi. Agar bunday tenglamalar bir nechta bo'lsa, ular sistema tashkil qiladi va uni chiziqli tenglamalar sistemasi deyiladi.

Ikki yoki uch noma'lumli tenglamalar sistemasi yechilganda, undagi har bir tenglama bilan aniqlanadigan geometrik figuralarning kesishishidan hosil bo'ladigan bitta nuqtaning koordinatalari (x-abssissasi, y- ordinatasi, z-aplekatasi) topiladi.

Ikki noma'lumli ikkita chiziqli tenglamalar sistemasi:

$$\begin{cases} a_{11}x + a_{12}y = b_1, \\ a_{21}x + a_{22}y = b_2. \end{cases} \qquad (1)$$

Yechilishi. Sistemadagi x va y noma'lumlarning darajalari birga teng bo'lganligi uchun har bir tenglama geometrik nuqtai nazaridan bittadan to'g'ri chiziqni aniqlaydi. Demak, sistema ikkita to'g'ri chiziq tenglamalaridan tuzilgan. Bunday chiziqli tenglamalar sistemasi yechilsa, undagi to'g'ri chiziqlarning kesishishidan hosil bo'ladigan bitta nuqtaning koordinatalari (x-abssissasi, y-ordinatasi) topiladi.

Ikki yoki uch noma'lumli chiziqli tenglamalar sistemasini Kramer qoidasi yordamida yechish uchun quyidagi tartibda ish yuritiladi:

1) Sistemadagi x va y noma'lumlar oldidagi koeffitsientlardan asosiy determinant tuziladi:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}. \tag{2}$$

2) Sistemadagi x yoki y oldidagi koeffitsientlarni ozod sonlar bilan almashtirish orqali yordamchi determinantlar tuziladi. Bunda Δ_x determinantni tuzish uchun x ning koeffitsientlari, Δ_y determinantni tuzish uchun esa y ning koeffitsientlari ozod sonlar bilan almashtiriladi:

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}; \qquad \Delta_y = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}. \tag{3}$$

Kramer formulalari:

$$x = \frac{\Delta_x}{\Delta}; \qquad y = \frac{\Delta_y}{\Delta}. \tag{4}$$

Misol 220.

$$\begin{cases} 3x + 5y = 2; \\ 5x + 9y = 4. \end{cases}$$

Kramer qoidasi yordamida yechilishi:

$$\Delta = \begin{vmatrix} 3 & 5 \\ 5 & 9 \end{vmatrix} = 27 - 25 = 2;$$

$$\Delta_x = \begin{vmatrix} 2 & 5 \\ 4 & 9 \end{vmatrix} = 18 - 20 = -2; \qquad \Delta_y = \begin{vmatrix} 3 & 2 \\ 5 & 4 \end{vmatrix} = 12 - 10 = 2;$$

$$\begin{cases} x = \dfrac{\Delta_x}{\Delta} = \dfrac{-2}{2} = -1 \\ y = \dfrac{\Delta_y}{\Delta} = \dfrac{2}{2} = 1 \end{cases} \Rightarrow \begin{cases} x = -1; \\ y = 1. \end{cases} \quad \Rightarrow \quad A(x; y) = A(-1;1).$$

Uch noma'lumli uchta chiziqli tenglamalar sistemasi:

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1; \\ a_{21}x + a_{22}y + a_{23}z = b_2; \\ a_{31}x + a_{32}y + a_{33}z = b_3. \end{cases} \tag{5}$$

Kramer qoidasi yordamida yechish.

Asosiy determinant tuziladi:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}. \qquad (6)$$

Yordamchi determinantlar tuziladi:

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}; \quad \Delta_y = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}; \quad \Delta_z = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}. \qquad (7)$$

Kramer formulalari:

$$x = \frac{\Delta_x}{\Delta}; \quad y = \frac{\Delta_y}{\Delta}; \quad z = \frac{\Delta_z}{\Delta}. \qquad (8)$$

Misol 221.

$$\begin{cases} 2x + y - 5z - 3 = 0; \\ 3x - 5y + 2z - 1 = 0; \\ 5x - 6y + 3z - 6 = 0. \end{cases} \Leftrightarrow \begin{cases} 2x + y - 5z = 3; \\ 3x - 5y + 2z = 1; \\ 5x - 6y + 3z = 6. \end{cases}$$

Kramer qoidasi yordamida yechish.

$$\Delta = \begin{vmatrix} 2 & 1 & -5 \\ 3 & -5 & 2 \\ 5 & -6 & 3 \end{vmatrix} = 2*(-5)*3 + 1*2*5 + 3*(-6)*(-5) - (-5)*(-5)*5 - 1*3*3 -$$

$$-2*(-6)*2 = -30 + 10 + 90 - 125 - 9 + 24 = -30 - 125 - 9 + 10 + 90 + 24 =$$

$$= -164 + 124 = -40; \qquad \Delta = -40.$$

$$\Delta_x = \begin{vmatrix} 3 & 1 & -5 \\ 1 & -5 & 2 \\ 6 & -6 & 3 \end{vmatrix} = (-1)^{1+1} * 3 * \begin{vmatrix} -5 & 2 \\ -6 & 3 \end{vmatrix} + (-1)^{1+2} * 1 * \begin{vmatrix} 1 & 2 \\ 6 & 3 \end{vmatrix} + (-1)^{1+3} * (-5) * \begin{vmatrix} 1 & -5 \\ 6 & -6 \end{vmatrix} =$$

$$= 3*(-15+12) - (3-12) - 5*(-6+30) = -9 + 9 - 120 = -120;$$

$$\Delta_x = -120.$$

$$\Delta_y = \begin{vmatrix} 2 & 3 & -5 \\ 3 & 1 & 2 \\ 5 & 6 & 3 \end{vmatrix} = (-1)^{3+1} * 5 * \begin{vmatrix} 3 & -5 \\ 1 & 2 \end{vmatrix} + (-1)^{3+2} * 6 * \begin{vmatrix} 2 & -5 \\ 3 & 2 \end{vmatrix} + (-1)^{3+3} * 3 * \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} =$$

$$= 5*(6+5) - 6*(4+15) + 3*(2-9) = 55 - 114 - 21 = 55 - 135 = -80;$$

$$\Delta_y = -80.$$

$$\Delta_z=\begin{vmatrix}2&1&3\\3&-5&1\\5&-6&6\end{vmatrix}=\begin{matrix}2&1&3&2&1\\3&-5&1&3&-5\\5&-6&6&5&-6\end{matrix}=-60+5-54+75+12-18=$$

$$=-60-54-18+5+75+12=-132+92=-40;$$

$$\Delta_z=-40.$$

$$\begin{cases}x=\dfrac{\Delta_x}{\Delta}=\dfrac{-120}{-40}=3\\ y=\dfrac{\Delta_y}{\Delta}=\dfrac{-80}{-40}=2\\ z=\dfrac{\Delta_z}{\Delta}=\dfrac{-40}{-40}=1\end{cases}\Rightarrow\begin{cases}x=3;\\ y=2;\\ z=1;\end{cases}\Rightarrow A(x;y;z)=A(3;2;1).$$

CHIZIQLI TENGLAMALAR SISTEMASINI YECHISHNING GAUSS USULI

Chiziqli tenglamalar sistemasida noma'lumlarni ketma-ket yo'qotib yechishni Gauss usuli deyiladi.

Misol 222.

$$\begin{cases}3x+5y-2=0;\\5x+9y-4=0.\end{cases}$$

Gauss usulida yechilishi: Berilgan misol (1) ga moslab yoziladi

$$\begin{cases}3x+5y=2;\\5x+9y=4.\end{cases}$$

Tenglamalar sistemasidagi bir xil noma'lumlar, masalan, **x** lar oldidagi koeffitsientlar tenglashtiriladi. Buning uchun birinchi tenglama 5 yoki –5 ga, ikkinchi tenglama 3 yoki –3 ga ko'paytiriladi. Agar teng koeffitsientli noma'lumlar oldidagi ishoralar bir xil bo'lsa, tenglamalardan biri ikkinchisidan hadlab ayriladi, ishoralar har xil bo'lsa, tenglamalar hadlab qo'shiladi.

$$\begin{cases}3x+5y=2\\5x+9y=4\end{cases}\begin{vmatrix}-5\\3\end{vmatrix}\Leftrightarrow\begin{cases}-15x-25y=-10\\15x+27y=12\end{cases}\Rightarrow\begin{cases}3x+5y=2\\2y=2\end{cases}\Rightarrow$$

$$\begin{cases}3x+5*1=2\\y=1\end{cases}\Rightarrow\begin{cases}3x=2-5\\y=1\end{cases}\Rightarrow\begin{cases}3x=-3;\\y=1.\end{cases}\Rightarrow\begin{cases}x=-1;\\y=1.\end{cases}$$

Demak, sistemadagi tenglamalari bilan berilgan ikkita to'g'ri chiziq bitta A nuqtada kesishadi:

$$A(x;y)=A(-1;1).$$

Misol 223. $$\begin{cases}2x+y-5z-3=0\\3x-5y+2z-1=0\\5x-6y+3z-6=0\end{cases}\Leftrightarrow\begin{cases}2x+y-5z=3;\\3x-5y+2z=1;\\5x-6y+3z=6.\end{cases}$$

Gauss usulida yechish.

$$\begin{cases} 2x+y-5z=3 \\ 3x-5y+2z=1 \\ 5x-6y+3z=6 \end{cases}\Bigg| 5 \Leftrightarrow \begin{cases} 10x+5y-25z=15 \\ 3x-5y+2z=1 \\ 5x-6y+3z=6 \end{cases} \Leftrightarrow \begin{cases} 2x+y-5z=3 \\ 13x-23z=16 \\ 5x-6y+3z=6 \end{cases}\Bigg| 6 \Leftrightarrow$$

$$\begin{cases} 12x+6y-30z=18 \\ 13x-23z=16 \\ 5x-6y+3z=6 \end{cases} \Leftrightarrow \begin{cases} 2x+y-5z=3 \\ 13x-23z=16 \\ 17x-27z=24 \end{cases}\begin{matrix} \\ -17 \\ 13 \end{matrix} \Leftrightarrow \begin{cases} 2x+y-5z=3 \\ -17\cdot 13x+17\cdot 23z=-17\cdot 16 \\ 17\cdot 13x-13\cdot 27z=24\cdot 13 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 2\boldsymbol{x}+\boldsymbol{y}-5\boldsymbol{z}=3 \\ -221\boldsymbol{x}+391\boldsymbol{z}=-271 \\ 221\boldsymbol{x}-351\boldsymbol{z}=312 \end{cases} \Leftrightarrow \begin{cases} 2\boldsymbol{x}+\boldsymbol{y}-5\boldsymbol{z}=3 \\ 13\boldsymbol{x}-23\boldsymbol{z}=16 \\ 40\boldsymbol{z}=40 \end{cases} \Rightarrow \begin{cases} 2\boldsymbol{x}+\boldsymbol{y}-5\boldsymbol{z}=3 \\ 13\boldsymbol{x}=39 \\ \boldsymbol{z}=1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 2\cdot 3+\boldsymbol{y}-5\cdot 1=3 \\ \boldsymbol{x}=3 \\ \boldsymbol{z}=1 \end{cases} \Rightarrow \begin{cases} \boldsymbol{x}=3; \\ \boldsymbol{y}=2; \\ \boldsymbol{z}=1. \end{cases}$$

Demak, sistemadagi tenglamalari bilan berilgan uchta tekislik bitta A nuqtada kesishadi:

$$A(x;y;z)=A(3;2;1).$$

Misol 224. Fermer 35 gektar yerga paxta va bug'doy ekish uchun qilinadigan xarajatlarga 16000000 so'm ajratdi. Ekiladigan paxta va bug'doydan kutiladigan hosil mos ravishda 2,5t va 4t. Har bir gektar paxta ekiladigan erga 610000 so'm, bug'doy ekiladigan erga 250000 so'm sarflansin.

Barcha yer va pul resurslaridan foydalanadigan variant yechimni toping.

Yechilishi. $\boldsymbol{x}$ bilan $\boldsymbol{y}$ mos ravishda yetishtiriladigan paxta va bug'doy hajmi bo'lsin.

$$\begin{cases} \frac{x}{2,5}+\frac{y}{4}=35 \\ \frac{x}{2,5}*610000+\frac{y}{4}*250000=16000000 \end{cases} \Rightarrow \begin{cases} 4x+2,5y=350; \\ 2440000x+625000y=16000000:5000 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 4x+2,5y=350; \\ 488x+125y=32000. \end{cases}$$

Chiziqli tegnlamalar sistimasini yechishning biror usulidan foydalanib $x\approx 50,4$ *va* $y\approx 59,4$ yechim topiladi. Demak, $x\approx 50,4$ t paxta $y\approx 59,4$ t bug'doy yetishtirilar ekan.

Buning uchun

Paxtaga $\boldsymbol{\frac{x}{2,5}=\frac{50,4}{2,5}\approx 20,2}$***ga,*** bug'doyga $\boldsymbol{\frac{y}{4}=\frac{59,4}{4}\approx 14,8}$ ga yer ajratish kerak.

Shuningdek paxta yetishtirishga $\boldsymbol{\frac{x}{2,5}*610000=\frac{50,4}{2,5}*610000\approx 12300000}$ so'm,

bug'doy yetishtirishga $\boldsymbol{\frac{y}{4}*250000=\frac{59,4}{4}*250000\approx 370000}$ so'm ajratilishi kerak.

Misol 225. Dorixona uch xildagi xom ashyoni ishlatib, uch turdagi dori ishlab chiqaradi, ishlab chiqarish xarakteristikalari jadvalda berilgan:

Xom ashyo xillari	Dori turlari bo'yicha xom ashyo sarflari			Xom ashyo zahirasi
	1	2	3	
1	5	12	7	2000
2	10	6	8	1660
3	9	11	4	2070

Mavjud xom ashyo zaxirasini ishlatib, dori turlari bo'yicha ishlab chiqarish hajmini aniqlang.

Yechilishi. Ishlab chiqarilishi rejalashtirilgan dorilar hajmini mos ravishda x, y, z bilan belgilaymiz.U holda

$$\begin{cases} 5\boldsymbol{x}+12\boldsymbol{y}+7\boldsymbol{z}=2000; \\ 10\boldsymbol{x}+6\boldsymbol{y}+8\boldsymbol{z}=1660; \\ 9\boldsymbol{x}+11\boldsymbol{y}+4\boldsymbol{z}=2070. \end{cases}$$

Chiziqli tenglamalar sistemasini yechishning biror usulidan foydalanib *x=70, y=120, z=30* yechim topiladi.

BIR JINSLI TENGLAMALAR SISTEMASI

Uch noma'lumli ikkita bir jinsli tenglamar sistemasi .

$$\begin{cases} a_{11}x+a_{12}y+a_{13}z=0; \\ a_{21}x+a_{22}y+a_{23}z=0. \end{cases}$$

ning yechimi

$$x=\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}*k; \qquad y=\begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix}*k; \qquad z=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}*k$$

formula yordamida topiladi. Bunda *k*- ixtiyoriy son.

Misol 226

$$\begin{cases} 2x-3y+5z=0; \\ x+2y-6z=0. \end{cases}$$

Bir jinsli tenglamalar sistemasini yeching.

Yechilishi.

$$x=\begin{vmatrix} -3 & 5 \\ 2 & -6 \end{vmatrix}*\kappa=(18-10)*\kappa=8*\kappa; \qquad y=\begin{vmatrix} 5 & 2 \\ -6 & 1 \end{vmatrix}\cdot k=(5+12)\cdot k=17\cdot k;$$

$$z=\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix}*k=(4+3)*k=7*k.$$

Demak, *x=8k; y=17k; z=7k; k*-ixtiyoriy son.

Misol 227.

$$\begin{cases} 3x - 2y + z = 0; \\ x + 3y - z = 0; \\ 2x - y + 3z = 0 \end{cases}$$

bir jinsli chiziqli tenglamalar sistemasini yeching.

Yechilishi.

$$\Delta = \begin{vmatrix} 3 & -2 & 1 \\ 1 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix} = 9 \neq 0.$$

Demak, sistema yechimga ega. Javobi: x=0; y=0; z=0.

Misol 228.

$$\begin{cases} 3x + 2y + z = 0; \\ 7x + 8y + 9z = 0; \\ 2x + 3y + 4z = 0. \end{cases}$$

bir jinsli tenglamalar sistemasini yeching.

Yechilishi.

$$\Delta = \begin{vmatrix} 3 & 2 & 1 \\ 7 & 8 & 9 \\ 2 & 3 & 4 \end{vmatrix}$$

determinantning uchinchi satri 2ga ko’paytirilib birinchi satriga qo’shiladi.

$$\Delta = \begin{vmatrix} 7 & 8 & 9 \\ 7 & 8 & 9 \\ 2 & 3 & 4 \end{vmatrix} = 0.$$

Demak, sistema cheksiz ko’p yechimga ega.

Berilgan sistemaning uchinchi tenglamasi 2 ga ko’paytirilib, birinchi tenglamaga qo’shiladi va oldingi sistemaga teng kuchli bo’lgan

$$\begin{cases} 7x + 8y + 9z = 0; \\ 2x + 3y + 4z = 0. \end{cases}$$

sistema hosil qilinadi. U holda

$$x = \begin{vmatrix} 8 & 9 \\ 3 & 4 \end{vmatrix} * k = (32 - 27) * k = 5k;$$

$$y = \begin{vmatrix} 9 & 7 \\ 4 & 2 \end{vmatrix} * k = (18 - 28) * k = -10k;$$

$$z = \begin{vmatrix} 7 & 8 \\ 2 & 3 \end{vmatrix} * k = (21 - 16) * k = 5k.$$

Bunda *k*-ixtiyoriy son

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi ikki no'malumli ikkita chiziqli tenglamalar sistemasini Gauss usuli va Kramer qoidasi yordamida yeching:

229. $\begin{cases} 2x+3y+1=0; \\ 5x+4y-1=0. \end{cases}$ Javobi: x=1, y=-1, **230.** $\begin{cases} 2x-3y-5=0; \\ 4x-5y-7=0. \end{cases}$ Javobi:x=-2, y=-3

231. $\begin{cases} 4x+9y=-7; \\ x+3y=5. \end{cases}$ Javobi:x=-22, y=9. **232.** $\begin{cases} \mathbf{2x-3y=22;} \\ \mathbf{-3x-4y=1.} \end{cases}$ Javobi: x=5, y=-4.

Quyidagi uch no'malumli uchta chiziqli tenglamalar sistemasini Gauss usuli va Kramer qoidasi yordamida yeching:

233. $\begin{cases} 2x-11y+3z=-2; \\ 3x+13y+5z=-4; \\ 4x-17y-2z=12. \end{cases}$ Javobi: x=2, y=0, z=-2.

234. $\begin{cases} \mathbf{3x-5y+7z=9;} \\ \mathbf{2x-4y-5z=6;} \\ \mathbf{-5x+2y-3z=-15.} \end{cases}$ Javobi: x=3, y=0, z=0.

235. $\begin{cases} 3x+4y+7z=-11; \\ -2x-5y-15z=4; \\ 4x-3y+5z=6. \end{cases}$ Javobi: x=-2, y=-3, z=-1.

236. $\begin{cases} -5x+7y-4z+5=0; \\ 3x-8y+3z+2=0; \\ 2x+5y+5z-3=0. \end{cases}$ Javobi: x=4, y=1. z=-2.

237. $\begin{cases} 2\mathbf{x}-3\mathbf{y}+\mathbf{z}=-5; \\ \mathbf{x}+2\mathbf{y}-4\mathbf{z}=-9; \\ 5\mathbf{x}-4\mathbf{y}+6\mathbf{z}=5. \end{cases}$ Javobi: x=-1, y=2, z=3.

238. $\begin{cases} \mathbf{3x_1+4x_2+2x_3+x_4=16;} \\ \mathbf{x_1+7x_2+x_3+x_4=23;} \\ \mathbf{2x_1+x_2+4x_3+6x_4=10;} \\ \mathbf{4x_1-3x_2+4x_3+6x_4=1.} \end{cases}$ Javobi; $\mathbf{x_1=1,\ x_2=3,\ x_3=0,\ x_4=1.}$

Quyidagi bir jinsli tenglamalar sistemalarini yeching:

239. $\begin{cases} 3x+5y+2z=0; \\ x+2y-z=0. \end{cases}$

Javobi: $x=k; y=5k; z=11k;$ $k-ixtiyoriy \quad son.$

240. $\begin{cases} 3x-2y+5z=0; \\ x+2y-3z=0. \end{cases}$

Javobi: $x=-4; y=14k; z=8k.$ $k-ixtiyoriy \quad son.$

241. $\begin{cases} 3x-2y+z=0; \\ 6x-4y+z=0. \end{cases}$

242. $\begin{cases} x-3y+z=0; \\ 2x-9y+3z=0. \end{cases}$

Javobi: $\begin{matrix} x=2k; y=3k; z=0; \\ k-ixtiyoriy \quad son. \end{matrix}$

Javobi: $\begin{matrix} x=0; y=-k; z=-3k; \\ k-ixtiyoriy \quad son. \end{matrix}$

243. $\begin{cases} 3x-2y+z=0; \\ x+2y+z=0. \end{cases}$

Javobi: $\begin{matrix} x=0; y=4k; z=8k; \\ k-ixtiyoriy \quad son. \end{matrix}$

244. $\begin{cases} 3x-2y+z=0; \\ x+2y-3z=0. \end{cases}$

Javobi: $\begin{matrix} x=4k; y=10kz=8k; \\ k-ixtiyoriy \quad son. \end{matrix}$

245. $\begin{cases} 2x-y-2z=0; \\ x-5y+2z=0. \end{cases}$

Javobi: $\begin{matrix} x=-12k; y=-6k; z=-9k; \\ k-ixtiyoriy \quad son. \end{matrix}$

246. $\begin{cases} x+2y-z=0; \\ 3x-5y+2z=0. \end{cases}$

Javobi: $\begin{matrix} x=-k; y=-5kz=-11k \\ k-ixtiyoriy \quad son. \end{matrix}$

247. $\begin{cases} 3x+2y-z=0; \\ 2x-y+3z=0; \\ x+y-z=0. \end{cases}$

Javobi: $\boldsymbol{\Delta \neq 0;\ x=y=z=0.}$

248. $\begin{cases} 3x+2y-z=0; \\ 2x-y+3z=0; \\ x+3y-4z=0. \end{cases}$

Javobi: $\begin{matrix} \Delta=0; x=-5k; y=11k; z=7k; \\ k-ixtiyoriy \quad son. \end{matrix}$

249. $\begin{cases} 3x-y+2z=0; \\ 2x+3y-5z=0; \\ x+y+z=0. \end{cases}$

Javobi: $\Delta \neq 0; x=y=z=0;$

250. $\begin{cases} 2x+y+3z=0; \\ x+2y-5z=0; \\ 3x+y-2z=0. \end{cases}$

Javobi: $\begin{matrix} \Delta=0; x=k; y=-13k; z=-5k; \\ k-ixtiyoriy \quad son. \end{matrix}$

8- MAVZU. MATRITSALAR. MATRITSALAR USTIDA AMALLAR

MATRITSALARNI QO'SHISH

$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ va $B=\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ yoki $A=\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}$ va $B=\begin{Vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{Vmatrix}$.

$$A+B=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}+\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}=\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}. \qquad (1)$$

Misol 251. $A=\begin{pmatrix} 3 & 4 \\ 0 & 2 \end{pmatrix}$ va $B=\begin{pmatrix} -3 & 2 \\ 4 & 1 \end{pmatrix}$ matritsalarni qo'shing va ayiring.

Yechilishi.

$$A+B=\begin{pmatrix} 3+(-3) & 4+2 \\ 0+4 & 2+1 \end{pmatrix}=\begin{pmatrix} 0 & 6 \\ 4 & 3 \end{pmatrix};$$

$$A-B=\begin{pmatrix} 3-(-3) & 4-2 \\ 0-4 & 2-1 \end{pmatrix}=\begin{pmatrix} 6 & 2 \\ -4 & 1 \end{pmatrix}$$

bo'ladi.

Misol 252. $A=\begin{pmatrix}1&2&3\\2&1&4\\5&0&3\end{pmatrix}$ va $B=\begin{pmatrix}4&0&1\\3&2&4\\5&6&0\end{pmatrix}$ matritsalarni qo'shing va ayiring.

Yechilishi.

$$A+B=\begin{pmatrix}1+4&2+0&3+1\\2+3&1+2&4+4\\5+5&0+6&3+0\end{pmatrix}=\begin{pmatrix}5&2&4\\5&3&8\\10&6&3\end{pmatrix};$$

$$A-B=\begin{pmatrix}1-4&2-0&3-1\\2-3&1-2&4-4\\5-5&0-6&3-0\end{pmatrix}=\begin{pmatrix}-3&2&2\\-1&-1&0\\0&-6&3\end{pmatrix}$$

bo'ladi.

MATRITSANI SONGA KO'PAYTIRISH

Matritsani λ soniga ko'paytirish uchun matritsaning har bir elementi shu songa ko'paytiriladi:

$$\lambda A=\lambda\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}=\begin{pmatrix}\lambda a_{11}&\lambda a_{12}\\\lambda a_{21}&\lambda a_{22}\end{pmatrix}. \qquad (2)$$

Misol 253.

$A=\begin{pmatrix}5&-20&40\\25&30&80\\10&60&-35\end{pmatrix}$ matritsani $\lambda=\frac{1}{5}$ songa ko'paytiring.

Yechilishi.

$$\lambda A=\frac{1}{5}\begin{pmatrix}5&-20&40\\25&30&80\\10&60&-35\end{pmatrix}=\begin{pmatrix}\frac{1}{5}*5&\frac{1}{5}*(-20)&\frac{1}{5}*40\\\frac{1}{5}*25&\frac{1}{5}*30&\frac{1}{5}*80\\\frac{1}{5}*10&\frac{1}{5}*60&\frac{1}{5}*(-35)\end{pmatrix}=\begin{pmatrix}1&-4&8\\5&6&16\\2&12&-7\end{pmatrix}.$$

MATRITSALARNI KO'PAYTIRISH

$$A=\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix} \quad \text{va} \quad B=\begin{pmatrix}b_{11}&b_{12}\\b_{21}&b_{22}\end{pmatrix}$$

Matritsalar quyidagicha ko'paytiriladi:

$$\boldsymbol{A*B=\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}*\begin{pmatrix}b_{11}&b_{12}\\b_{21}&b_{22}\end{pmatrix}=\begin{pmatrix}a_{11}*b_{11}+a_{12}*b_{21}&a_{11}*b_{12}+a_{12}*b_{22}\\a_{21}*b_{11}+a_{22}*b_{21}&a_{21}*b_{12}+a_{22}*b_{22}\end{pmatrix}.} \qquad (3)$$

Bundan tashqari

$$A*B\neq B*A. \qquad (4)$$

Misol 254. $A=\begin{Vmatrix}0&0\\1&0\end{Vmatrix}$ va $B=\begin{Vmatrix}1&0\\0&0\end{Vmatrix}$ matritsalarni ko'paytiring.

Yechilishi.

$$\boldsymbol{A}\cdot\boldsymbol{B}=\begin{Vmatrix}0&0\\1&0\end{Vmatrix}=\begin{Vmatrix}1&0\\0&0\end{Vmatrix}=\begin{Vmatrix}0\cdot1+0\cdot0&0\cdot0+0\cdot0\\1\cdot1+0\cdot0&1\cdot0+0\cdot0\end{Vmatrix}=\begin{Vmatrix}0&0\\1&0\end{Vmatrix};$$

$$\boldsymbol{B}\cdot\boldsymbol{A}=\begin{Vmatrix}1&0\\0&0\end{Vmatrix}\cdot\begin{Vmatrix}0&0\\1&0\end{Vmatrix}=\begin{Vmatrix}1\cdot0+0\cdot1&1\cdot0+0\cdot0\\0\cdot0+0\cdot1&0\cdot0+0\cdot0\end{Vmatrix}=\begin{Vmatrix}0&0\\0&0\end{Vmatrix};$$

Misol 255. $A=\begin{Vmatrix}1&0&0\\0&2&1\\3&0&0\end{Vmatrix}$ va $B=\begin{Vmatrix}0&1&0\\2&0&1\\1&0&1\end{Vmatrix}$ matritsalarni ko'paytiring.

Yechilishi.

$$\boldsymbol{A}\cdot\boldsymbol{B}=\begin{Vmatrix}1\cdot0+0\cdot2+0\cdot1&1\cdot1+0\cdot0+0\cdot0&1\cdot0+0\cdot1+0\cdot1\\0\cdot0+2\cdot2+1\cdot1&0\cdot1+2\cdot0+1\cdot0&0\cdot0+2\cdot1+1\cdot1\\3\cdot0+0\cdot2+0\cdot1&3\cdot1+0\cdot0+0\cdot0&3\cdot0+0\cdot1+0\cdot1\end{Vmatrix}=\begin{Vmatrix}0&1&0\\5&0&3\\0&3&0\end{Vmatrix};$$

$$\boldsymbol{B}\cdot\boldsymbol{A}=\begin{Vmatrix}0\cdot1+1\cdot0+0\cdot3&0\cdot0+1\cdot2+0\cdot0&0\cdot0+1\cdot1+0\cdot0\\2\cdot1+0\cdot0+1\cdot3&2\cdot0+0\cdot2+1\cdot0&2\cdot0+0\cdot1+1\cdot0\\1\cdot1+0\cdot0+1\cdot3&1\cdot0+0\cdot2+1\cdot0&1\cdot0+0\cdot1+1\cdot0\end{Vmatrix}=\begin{Vmatrix}0&2&1\\5&0&0\\4&0&0\end{Vmatrix};$$

Haqiqatan $\boldsymbol{A}\cdot\boldsymbol{B}\neq\boldsymbol{B}\cdot\boldsymbol{A}$**.**

Bosh diagonalida turgan elementlari birga, qolgan barcha elementlari nolga teng bo'lgan kvadrat matritsani birlik matritsa deyiladi:

$$E=\begin{pmatrix}1&0\\0&1\end{pmatrix}. \qquad (5)$$

A matritsaning E birlik matritsaga, shuningdek E birlik matritsaning A matritsaga ko'paytmasi A matritsaga teng, ya'ni

$\boldsymbol{A}\cdot\boldsymbol{E}=\boldsymbol{A}$ va $\boldsymbol{E}\cdot\boldsymbol{A}=\boldsymbol{A}$ bo'ladi.

$$\boldsymbol{A}\cdot\boldsymbol{E}=\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}\cdot\begin{pmatrix}1&0\\0&1\end{pmatrix}=\begin{pmatrix}a_{11}\cdot1+a_{12}\cdot0&a_{11}\cdot0+a_{12}\cdot1\\a_{21}\cdot1+a_{22}\cdot0&a_{21}\cdot0+a_{22}\cdot1\end{pmatrix}=\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix};$$

$$\boldsymbol{E}\cdot\boldsymbol{A}=\begin{pmatrix}1&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}=\begin{pmatrix}1\cdot a_{11}+0\cdot a_{21}&1\cdot a_{12}+0\cdot a_{22}\\0\cdot a_{11}+1\cdot a_{21}&0\cdot a_{12}+1\cdot a_{22}\end{pmatrix}=\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}.$$

$A=\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix}$ matritsa $X=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$ ustun matritsaga ko'paytirilsa yana ustun matritsa hosil bo'ladi:

$$\boldsymbol{A}\cdot\boldsymbol{X}=\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix}\cdot\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}a_{11}\cdot x_1+a_{12}\cdot x_2+a_{13}\cdot x_3\\a_{21}\cdot x_1+a_{22}\cdot x_2+a_{23}\cdot x_3\\a_{31}\cdot x_1+a_{32}\cdot x_2+a_{33}\cdot x_3\end{pmatrix}. \qquad (6)$$

Misol 256.

$$A=\begin{pmatrix}1&0&2&3\\4&-3&1&0\\2&5&3&1\end{pmatrix} \quad \text{va} \quad B=\begin{pmatrix}2&7\\0&2\\5&4\\6&0\end{pmatrix}$$

matritsalarni ko'paytiring.

Yechilishi. Ikki ustunli matritsa hosil bo'ladi:

$$A\cdot B=\begin{pmatrix}1 & 0 & 2 & 3\\ 4 & -3 & 1 & 0\\ 2 & 5 & 3 & 1\end{pmatrix}\cdot\begin{pmatrix}2 & 7\\ 0 & 2\\ 5 & 4\\ 6 & 0\end{pmatrix}=$$

$$=\begin{pmatrix}1\cdot 2+0\cdot 0+2\cdot 5+3\cdot 6 & 1\cdot 7+0\cdot 2+2\cdot 4+3\cdot 0\\ 4\cdot 2+(-3)\cdot 0+1\cdot 5+0\cdot 6 & 4\cdot 7+(-3)\cdot 2+1\cdot 4+0\cdot 0\\ 2\cdot 2+5\cdot 0+3\cdot 5+1\cdot 6 & 2\cdot 7+5\cdot 2+3\cdot 4+1\cdot 0\end{pmatrix}=\begin{pmatrix}30 & 15\\ 13 & 26\\ 25 & 36\end{pmatrix}.$$

TESKARI MATRITSA

$A=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}$ matritsaga teskari matritsa

$$A^{-1}=\begin{pmatrix}\frac{A_{11}}{\Delta} & \frac{A_{21}}{\Delta} & \frac{A_{31}}{\Delta}\\ \frac{A_{12}}{\Delta} & \frac{A_{22}}{\Delta} & \frac{A_{32}}{\Delta}\\ \frac{A_{13}}{\Delta} & \frac{A_{23}}{\Delta} & \frac{A_{33}}{\Delta}\end{pmatrix} \qquad (7)$$

ko'rinishda bo'ladi.

Bunda

$$\Delta=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}\neq 0 \quad \text{determinant.}$$

$A_{11},\ A_{12},\ ...,A_{33}$ lar algebraik to'ldiruvchilar.

Masalan,

$$A_{11}=(-1)^{1+1}\begin{vmatrix}a_{22} & a_{23}\\ a_{32} & a_{33}\end{vmatrix};\quad A_{23}=(-1)^{2+3}\begin{vmatrix}a_{11} & a_{12}\\ a_{31} & a_{32}\end{vmatrix} \quad \text{va hakoza.}$$

$$\tilde{A}=\begin{pmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{pmatrix} \qquad (8)$$

matritsani A matritsaga biriktirilgan (tirkalgan) matritsa deyiladi.

$$A^{-1}=\frac{1}{\Delta}*\tilde{A}. \qquad (9)$$

Misol 257.

$A=\begin{pmatrix}2 & 1 & -5\\ 3 & -5 & 2\\ 5 & -6 & 3\end{pmatrix}$ matritsaga teskari matritsani tuzing.

Yechilishi.

$$\Delta=\begin{vmatrix}2 & 1 & -5\\ 3 & -5 & 2\\ 5 & -6 & 3\end{vmatrix}=-30+10+90-125-9+24=-40\neq 0.$$

Ushbu determinantning algebraik to'ldiruvchilari topiladi:

$$A_{11}=(-1)^{1+1}\begin{vmatrix}-5 & 2\\ -6 & 3\end{vmatrix}=-15+12=-3; \qquad A_{12}=(-1)^{1+2}\begin{vmatrix}3 & 2\\ 5 & 3\end{vmatrix}=-(9-10)=1;$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix}3 & -5\\ 5 & -6\end{vmatrix}=-18+25=7; \qquad A_{21}=(-1)^{2+1}\begin{vmatrix}1 & -5\\ -6 & 3\end{vmatrix}=-(3-30)=27;$$

$$A_{22}=(-1)^{2+2}\begin{vmatrix}2 & -5\\ 5 & 3\end{vmatrix}=6+25=31; \qquad A_{23}=(-1)^{2+3}\begin{vmatrix}2 & 1\\ 5 & -6\end{vmatrix}=-(-12-5)=17;$$

$$A_{31}=(-1)^{3+1}\begin{vmatrix}1 & -5\\ -5 & 2\end{vmatrix}=2-25=-23; \qquad A_{32}=(-1)^{3+2}\begin{vmatrix}2 & -5\\ 3 & 2\end{vmatrix}=-(4+15)=-19;$$

$$A_{33}=(-1)^{3+3}\begin{vmatrix}2 & 1\\ 3 & -5\end{vmatrix}=-10-3=-13;$$

(7) ga asosan berilgan A matritsaga teskari matritsa quyidagi ko'rinishni oladi:

$$A^{-1}=\begin{pmatrix}\frac{-3}{-40} & \frac{27}{-40} & \frac{-23}{-40}\\ \frac{1}{-40} & \frac{31}{-40} & \frac{-19}{-40}\\ \frac{7}{-40} & \frac{17}{-40} & \frac{-13}{-40}\end{pmatrix}=\begin{pmatrix}\frac{3}{40} & -\frac{27}{40} & \frac{23}{40}\\ -\frac{1}{40} & -\frac{31}{40} & \frac{19}{40}\\ -\frac{7}{40} & -\frac{17}{40} & \frac{13}{40}\end{pmatrix}=\frac{1}{40}\begin{pmatrix}3 & -27 & 23\\ -1 & -31 & 19\\ -7 & -17 & 13\end{pmatrix}.$$

Matritsa o'ziga teskari matritsaga ko'paytirilsa birlik matritsa hosil bo'ladi.

$$\boldsymbol{E}=\boldsymbol{A}\cdot\boldsymbol{A}^{-1}=\begin{pmatrix}\boldsymbol{a}_{11} & \boldsymbol{a}_{12} & \boldsymbol{a}_{13}\\ \boldsymbol{a}_{21} & \boldsymbol{a}_{22} & \boldsymbol{a}_{23}\\ \boldsymbol{a}_{31} & \boldsymbol{a}_{32} & \boldsymbol{a}_{33}\end{pmatrix}\cdot\begin{pmatrix}\frac{\boldsymbol{A}_{11}}{\boldsymbol{\Delta}} & \frac{\boldsymbol{A}_{21}}{\boldsymbol{\Delta}} & \frac{\boldsymbol{A}_{31}}{\boldsymbol{\Delta}}\\ \frac{\boldsymbol{A}_{12}}{\boldsymbol{\Delta}} & \frac{\boldsymbol{A}_{22}}{\boldsymbol{\Delta}} & \frac{\boldsymbol{A}_{32}}{\boldsymbol{\Delta}}\\ \frac{\boldsymbol{A}_{13}}{\boldsymbol{\Delta}} & \frac{\boldsymbol{A}_{23}}{\boldsymbol{\Delta}} & \frac{\boldsymbol{A}_{33}}{\boldsymbol{\Delta}}\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}. \qquad (10)$$

Misol 258.

$$A=\begin{pmatrix}2 & 1 & -5\\ 3 & -5 & 2\\ 5 & -6 & 3\end{pmatrix}$$ matritsani o'ziga teskari

$$A^{-1}=\frac{1}{40}\begin{pmatrix}3 & -27 & 23\\ -1 & -31 & 19\\ -7 & -17 & 13\end{pmatrix}$$ matritsaga ko'paytiring va tirkalgan matritsani yozing.

Yechilishi:

$$\boldsymbol{E}=\boldsymbol{A}\cdot\boldsymbol{A}^{-1}=\frac{1}{40}\begin{pmatrix}2 & 1 & -5\\ 3 & -5 & 2\\ 5 & -6 & 3\end{pmatrix}\cdot\begin{pmatrix}3 & -27 & 23\\ -1 & -31 & 19\\ -7 & -17 & 13\end{pmatrix}=$$

$$=\frac{1}{40}\begin{pmatrix} 2\cdot3+1\cdot(-1)+(-5)\cdot(-7) & 2\cdot(-27)+1\cdot(-31)+(-5)\cdot(-17) & 2\cdot23+1\cdot19+(-5)\cdot13 \\ 3\cdot3+(-5)\cdot(-1)+2\cdot(-7) & 3\cdot(-27)+(-5)\cdot(-31)+2\cdot(-17) & 3\cdot23+(-5)\cdot19+2\cdot13 \\ 5\cdot3+(-6)\cdot(-1)+3\cdot(-7) & 5\cdot(-27)+(-6)\cdot(-31)+3\cdot(-17) & 5\cdot23+(-6)\cdot19+3\cdot13 \end{pmatrix}=$$

$$=\frac{1}{40}\begin{pmatrix} 6-1+35 & -54-31+85 & 46+19-65 \\ 9+5-14 & -81+155-34 & 69-95+26 \\ 15+6-21 & -135+186-51 & 115-114+39 \end{pmatrix}=\frac{1}{40}\begin{pmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

bo'ladi.

Yoqorida aniqlangan algebraik to'ldiruvchilar (8) ga qo'yilib tirkalgan matritsa hosil qilinadi:

$$\tilde{A}=\begin{pmatrix} -3 & 27 & -23 \\ 1 & 31 & -19 \\ 7 & 17 & -13 \end{pmatrix}.$$

Matritsaning satrlari o'zining mos ustunlar bilan almashtirilsa, hosil bo'lgan matritsani dastlabki matritsaga nisbatan transponirlangan matritsa deyiladi:

$$A=\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{Vmatrix}; \qquad A^*=\begin{Vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \end{Vmatrix}. \qquad (11)$$

Transponirlangan A matritsa A^* ko'rinishda bo'ladi.

Misol 259.

$$A=\begin{pmatrix} 2 & 1 & -5 \\ 3 & -5 & 2 \\ 5 & -6 & 3 \end{pmatrix}$$

matritsaning transponirlangan matritsasini yozing.

Yechilishi.

$$A^*=\begin{pmatrix} 2 & 3 & 5 \\ 1 & -5 & -6 \\ -5 & 2 & 3 \end{pmatrix}$$

bo'ladi.

MATRITSANING RANGI

Matritsaning rangi deb, uning noldan farqli minorining eng yuqori tartibiga aytiladi.

Misol 260.

$$A=\begin{Vmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{Vmatrix}$$ martitsaning rangini toping.

Yechilishi.

$$\begin{vmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}=0-1+0-0+2-1=0;$$

$$\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1 \neq 0.$$

Demak, matritsaning rangi $r(A) = 2$ bo'ladi.

Misol 261.

$$B = \begin{vmatrix} 2 & 3 & 4 \\ 6 & 9 & 12 \end{vmatrix}$$ matritsaning rangini toping.

Yechilishi.

$$\begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} = 18 - 18 = 0; \quad \begin{vmatrix} 2 & 4 \\ 6 & 12 \end{vmatrix} = 24 - 24 = 0; \quad \begin{vmatrix} 3 & 4 \\ 9 & 12 \end{vmatrix} = 36 - 36 = 0;$$

Demak, matritsaning rangi $r(B) = 1$ bo'ladi.

CHIZIQLI TENGLAMALAR SISTEMASINI MATRISA USULIDA YECHISH

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1; \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2; \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3. \end{cases} \qquad \textbf{(1)}$$

chiziqli tenglamalar sestemasi berilgan .

Bundan quyidagi matritsalar tuzuladi:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{pmatrix}; \quad B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}; \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

Bulardan

$$\boldsymbol{X} = \boldsymbol{A}^{-1} \cdot \boldsymbol{B} \qquad \textbf{(2)}$$

bo'ladi.

Misol 262.

$$\begin{cases} 2x_1 + x_2 - 5x_3 - 3 = 0; \\ 3x_1 - 5x_2 + 2x_3 - 1 = 0; \\ 5x_1 - 6x_2 + 3x_3 - 6 = 0 \end{cases}$$

chiziqli tenglamalar sistemasini yeching.

Yechilishi.

$$\begin{cases} 2x_1 + x_2 - 5x_3 = 3; \\ 3x_1 - 5x_2 + 2x_3 = 1; \\ 5x_1 - 6x_2 + 3x_3 = 6. \end{cases}$$

O'zgaruvchilar oldidagi koeffisientlardan asosiy matritsa tuziladi:

$$A = \begin{pmatrix} 2 & 1 & -5 \\ 3 & -5 & 2 \\ 5 & -6 & 3 \end{pmatrix}.$$

Ozod sonlardan ustun matritsa tuziladi:

$$B = \begin{pmatrix} 3 \\ 1 \\ 6 \end{pmatrix}.$$

O’zgaruvchilardan ustun matrisa tuzuladi:

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} .$$

A matrisaning determinanti topiladi:

$$\Delta = \begin{vmatrix} 2 & 1 & -5 \\ 3 & -5 & 2 \\ 5 & -6 & 3 \end{vmatrix} = 2\cdot(-5)\cdot 3 + 1\cdot 2\cdot 5 + 3\cdot(-6)\cdot(-5) - (-5)\cdot(-5)\cdot 5 - 1\cdot 3\cdot 3 -$$

$$-2\cdot(-6)\cdot 2 = -30 + 10 + 90 - 125 - 9 + 24 = 124 - 164 = -40.$$

$\Delta = -40 \neq 0.$

Algebraik to’ldiruvchilar topiladi:

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -5 & 2 \\ -6 & 3 \end{vmatrix} = -15 + 12 = -3;$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & 2 \\ 5 & 3 \end{vmatrix} = -(9 - 10) = 1;$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 3 & -5 \\ 5 & -6 \end{vmatrix} = -18 + 25 = 7;$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 1 & -5 \\ -6 & 3 \end{vmatrix} = -(3 - 30) = 27;$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 2 & -5 \\ 5 & 3 \end{vmatrix} = 6 + 25 = 31;$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 2 & 1 \\ 5 & -6 \end{vmatrix} = -(-12 - 5) = 17;$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 1 & -5 \\ -5 & 2 \end{vmatrix} = 2 - 25 = -23; \qquad A_{32} = (-1)^{3+2}\begin{vmatrix} 2 & -5 \\ 3 & 2 \end{vmatrix} = -(4 + 15) = -19;$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 2 & 1 \\ 3 & -5 \end{vmatrix} = -10 - 3 = -13.$$

A matritsaga teskari matritsa tuzuladi:

$$A^{-1}=\begin{pmatrix}\frac{A_{11}}{\Delta} & \frac{A_{21}}{\Delta} & \frac{A_{31}}{\Delta}\\ \frac{A_{12}}{\Delta} & \frac{A_{22}}{\Delta} & \frac{A_{32}}{\Delta}\\ \frac{A_{13}}{\Delta} & \frac{A_{23}}{\Delta} & \frac{A_{33}}{\Delta}\end{pmatrix}=\frac{1}{\Delta}\begin{pmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{pmatrix}=\frac{1}{-40}\begin{pmatrix}-3 & 27 & -23\\ 1 & 31 & -19\\ 7 & 17 & -13\end{pmatrix}.$$

To'plangan ma'lumotlar (2) formulaga qo'yiladi:

$$\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{B} \Leftrightarrow \begin{pmatrix}\boldsymbol{x}_1\\ \boldsymbol{x}_2\\ \boldsymbol{x}_3\end{pmatrix}=\frac{1}{-40}\begin{pmatrix}-3 & 27 & -23\\ 1 & 31 & -19\\ 7 & 17 & -13\end{pmatrix}\cdot\begin{pmatrix}3\\ 1\\ 6\end{pmatrix}=-\frac{1}{40}\begin{pmatrix}-3\cdot3+27\cdot1+(-23)\cdot6\\ 1\cdot3+31\cdot1+(-19)\cdot6\\ 7\cdot3+17\cdot1+(-13)\cdot6\end{pmatrix}=$$

$$=-\frac{1}{40}\begin{pmatrix}-9+27-138\\ 3+31-114\\ 21+17-78\end{pmatrix}=-\frac{1}{40}\begin{pmatrix}-120\\ -80\\ -40\end{pmatrix}=\begin{pmatrix}-\frac{1}{40}\cdot(-120)\\ -\frac{1}{40}\cdot(-80)\\ -\frac{1}{40}\cdot(-40)\end{pmatrix}=\begin{pmatrix}3\\ 2\\ 1\end{pmatrix}\Rightarrow\begin{pmatrix}\boldsymbol{x}_1\\ \boldsymbol{x}_2\\ \boldsymbol{x}_3\end{pmatrix}=\begin{pmatrix}3\\ 2\\ 1\end{pmatrix}\Rightarrow\begin{cases}\boldsymbol{x}_1=3;\\ \boldsymbol{x}_2=2;\\ \boldsymbol{x}_1=1.\end{cases}$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Matritsalar ustida talab etilgan amallarni bajaring:

$$A=\begin{pmatrix}2 & 5\\ 3 & -4\end{pmatrix};\quad B=\begin{pmatrix}-3 & -2\\ -1 & 5\end{pmatrix};\quad C=\begin{pmatrix}-1 & 0 & -2\\ 3 & 2 & 1\\ 4 & -3 & -4\end{pmatrix};\quad D=\begin{pmatrix}-2 & 5 & 3\\ -4 & -3 & 2\\ 0 & 1 & 6\end{pmatrix};$$

$$E=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix};\quad F=\begin{pmatrix}x_1 & y_1\\ x_2 & y_2\\ x_3 & y_3\end{pmatrix}.$$

261. $2A=?$ **262.** A+B=?**263**. A-B=? **264.** $A\cdot B$=? **265.** $B\cdot A$=?

266. $-\frac{1}{2}A=?$ **267.** $\frac{1}{3}\cdot \boldsymbol{B}=?$ **268.** C+D=? **269.** C-D=?

270. $C\cdot D$=? **271**. $D\cdot C$=? **272.** $2\cdot C$=? **273**. $-2\cdot D$=?

274. $C\cdot E$=? **275**. $C\cdot F$=? **276.** $D\cdot E$=? **277**. $D\cdot F$=?

Quyidagi matritsalarni transponirlang:

278. $A=\begin{vmatrix}2 & 4\\ 7 & 8\\ 9 & 1\\ 0 & 3\end{vmatrix}$; **279**. $B=\begin{Vmatrix}1 & -4 & 5 & 7\\ 8 & 3 & -4 & 2\end{Vmatrix}$; **280.** $C=\begin{Vmatrix}0 & 3 & 1 & 4\\ 2 & -1 & 0 & 7\\ 5 & 6 & -9 & 8\end{Vmatrix}$;

281. $D=\begin{Vmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\end{Vmatrix}$; **282.** $E=\begin{Vmatrix}1 & 4 & 5\\ 9 & 2 & -6\\ -7 & 8 & 3\end{Vmatrix}$; **283**. $X=\begin{Vmatrix}x_1\\ x_2\\ x_3\end{Vmatrix}$; **284**. $Y=\begin{Vmatrix}y_1 & y_2 & y_3\end{Vmatrix}$.

Quyidagi matritsalarning teskari matritsalarini toping va tirkalgan matritsalarini yozing:

285. $A=\begin{pmatrix}3 & 5\\ 7 & -9\end{pmatrix}$; **286**. $B=\begin{pmatrix}3 & -2 & -4\\ 2 & 0 & 1\\ 0 & 1 & 5\end{pmatrix}$; **287**. $C=\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & 4\\ 1 & 3 & 9\end{pmatrix}$;

288. $D=\begin{pmatrix}1 & 2 & 1\\ 4 & 3 & -2\\ -5 & -4 & -1\end{pmatrix}$; **289.** $E=\begin{pmatrix}1 & 2 & 3 & 4\\ -2 & 1 & -2 & 3\\ 0 & 1 & 0 & 1\\ -1 & -2 & -3 & 0\end{pmatrix}$.

Quyidagi matritsalar uchun ko'rsatilgan amallarni bajaring:

$A=\begin{pmatrix}3 & -2 & -4\\ 2 & 0 & 1\\ 0 & 1 & 5\end{pmatrix}$; $B=\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & 4\\ 1 & 3 & 9\end{pmatrix}$; $C=\begin{pmatrix}1 & 2 & 1\\ 4 & 3 & -2\\ -5 & -4 & -1\end{pmatrix}$; $E=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$.

290. $\boldsymbol{A\cdot A^{-1}=?}$ **291.** $\boldsymbol{B\cdot B^{-1}=?}$ **292.** $\boldsymbol{C\cdot C^{-1}=?}$ **293.** $A\cdot E=?$

Quyidagi matritsalarning rangini toping:

294. $A=\begin{pmatrix}3 & 5\\ 7 & -9\end{pmatrix}$; **295.** $B=\begin{pmatrix}3 & 5\\ 30 & 50\end{pmatrix}$; **296**. $C=\begin{pmatrix}3 & 5\\ -1{,}5 & -2{,}5\end{pmatrix}$; **297.** $D=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$;

298. $E=\begin{pmatrix}3 & -2 & -4\\ 2 & 0 & 1\\ 0 & 1 & 5\end{pmatrix}$; **299**. $F=\begin{pmatrix}3 & -2 & -4\\ -3 & 2 & 4\\ 0 & 1 & 5\end{pmatrix}$; 300**.** $K=\begin{pmatrix}2 & 1 & -5\\ 3 & -5 & 2\\ 5 & -6 & 3\end{pmatrix}$;

301. $M=\begin{pmatrix}3 & 1 & -5\\ 1 & -5 & 2\\ 6 & -6 & 3\end{pmatrix}$; **302**. $N=\begin{pmatrix}2 & 3 & -5\\ 3 & 1 & 2\\ 5 & 6 & 3\end{pmatrix}$; **303**. $L=\begin{pmatrix}2 & 1 & 3\\ 3 & -5 & 1\\ 5 & -6 & 6\end{pmatrix}$.

Quyidagi chiziqli tenglamalr sistemasini matritsa usulida yeching:

304. $\begin{cases}\boldsymbol{2x+3y+1=0;}\\ \boldsymbol{5x+4y-1=0.}\end{cases}$ 305. $\begin{cases}\boldsymbol{2x-3y-5=0;}\\ \boldsymbol{4x-5y-7=0.}\end{cases}$ 306. $\begin{cases}\boldsymbol{4x+9y=-7;}\\ \boldsymbol{x+3y=5.}\end{cases}$

Javobi: *x=1, y=-1.* Javobi: *x=-2, y=-3.* Javobi: *x=-22, y=9.*

307. $\begin{cases}2x-3y=22;\\ -3x-4y=1.\end{cases}$ 308. $\begin{cases}2x-11y+3z=-2;\\ 3x+13y+5z=-4;\\ 4x-17y-2z=12.\end{cases}$ 309. $\begin{cases}3x-5y+7z=9;\\ 2x-4y-5z=6;\\ -5x+2y-3z=-15.\end{cases}$

Javobi: *x=5, y= -4.* Javobi: *x=2, y=0, z=-2.* Javobi: *x=3, y=0, z=0.*

310. $\begin{cases} 3x+4y+7z=-11; \\ -2x-5y-15z=4; \\ 4x-3y+5z=6. \end{cases}$ 311. $\begin{cases} -5x+7y-4z+5=0; \\ 3x-8y+3z+2=0; \\ 2x+5y+5z-3=0. \end{cases}$

312. $\begin{cases} 2x-3y+z=-5; \\ x+2y-4z=-9; \\ 5x-4y+6z=5. \end{cases}$

Javobi: *x=-2, y=-3, z=1.* Javobi: *x=4, y=1, z=-2.* Javobi: *x=-1, y=2, z=3.*

313. $\begin{cases} 3x_1+4x_2+2x_3+x_4=16; \\ x_1+7x_2+x_3+x_4=23; \\ 2x_1+x_2+3x_3+5x_4=10; \\ 4x_1-3x_2+4x_3+6x_4=1; \end{cases}$

Javobi: $x_1=1$, $x_2=3$, $x_3=0$, $x_4=1$.

9- MAVZU. KOMPLEKS SONLAR

$z=x+iy$ ko'rinishdagi ifoda kompleks son deyiladi , bunda x va y - haqiqiy sonlar. x- kompleks sonning haqiqiy qismi, iy – mavhum qismi. $\sqrt{-1}=i$ yoki $i^2=-1$.

$z_1=x_1+iy_1$ va $z_2=x_2+iy_2$ kompleks sonlar ustida amallar:

$$z_1+z_2=(x_1+iy_1)+(x_2+iy_2)=(x_1+x_2)+i(y_1+y_2),$$

$$z_1-z_2=(x_1+iy_1)-(x_2+iy_2)=(x_1-x_2)+i(y_1-y_2),$$

$$z_1\cdot z_2=(x_1+iy_1)\cdot(x_2+iy_2)=(x_1x_2-y_1y_2)+i(x_1y_2+x_2y_1),$$

$$\frac{z_1}{z_2}=\frac{z_1\cdot\overline{z_2}}{z_2\cdot\overline{z_2}}=\frac{(x_1+iy_1)\cdot(x_2-iy_2)}{(x_2+iy_2)\cdot(x_2-iy_2)}=\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}+\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2}i.$$

Bunda $\bar{z}_2$ - maxrajning qo'shmasi.

Kompleks sonlarni darajaga ko'tarish ikki hadni darajaga ko'tarish kabi bajariladi, bunda i sonning darajalari quyidagi formulalar bo'yicha aniqlanadi:

$$i^2=-1,\quad i^3=-i, i^4=1 \qquad i^{4k}=1,\quad i^{4k+1}=i,\quad i^{4k+2}=-1,\quad i^{4k+3}=-i.$$

Misol 314. Ushbu $z_1=3-i,\ z_2=-2+3i, z_3=4+3i$ kompleks sonlar berilgan bo'lsin. $z=\dfrac{z_1-z_2\cdot z_3}{z_1^3+z_3}$ ni hisoblang.

Yechilishi. Bu misolni yechish jarayonida kompleks sonlarni ko'paytirish, ayirish, darajaga ko'tarish, qo'shish va bo'lish amallarini bajarish o'rganiladi:

$$z_2\cdot z_3=(-2+3i)(4+3i)=(-8-9)+i(-6+12)=-17+6i;$$

$$z_1-z_2\cdot z_3=(3-i)-(-17+6i)=(3+17)+i(-1-6)=20-7i;$$

$$z_1^3=(3-i)^3=27-27i+9i^2-i^3=(27-9)+i(-27+1)=18-26i;$$

$$z_1^3+z_3=(18-26i)+(4+3i)=(18+4)+i(-26+3)=22-23i.$$

Demak,

$$z=\frac{20-7i}{22-23i}=\frac{(20-7i)(22+23i)}{(22-23i)(22+23i)}=\frac{(440+161)+i(460-154)}{22^2+23^2}=\frac{601}{1013}+\frac{306i}{1013}=$$

$$=\frac{601}{1013}+\frac{306}{1013}i.$$

Misol 315. $z=-\sqrt{3}+i$ kompleks sonning r moduli va φ argumentini toping.

Yechilishi. $x=-\sqrt{3},\ y=1$ bo'lgani uchun

$$|z|=r\Rightarrow r=\sqrt{x^2+y^2}=\sqrt{(-\sqrt{3})^2+1^2}=2.\qquad tg\varphi=\frac{y}{x}=-\frac{1}{\sqrt{3}}\Rightarrow$$

$$\Rightarrow\varphi=arctg(-\frac{1}{\sqrt{3}})+\pi k\Rightarrow\varphi=-arctg\frac{1}{\sqrt{3}}+\pi k\Rightarrow\begin{cases}k=1\\ \varphi=-\frac{\pi}{6}+\pi=\frac{5\pi}{6}.\end{cases}$$

$z=x+iy$ - Kompleks sonning algebraik shakli .

$z=r(\cos\varphi+i\sin\varphi)$- kompleks sonning trigonometrik shakli . $\cos\varphi+i\sin\varphi=e^{i\varphi}$ - Eylir formulasi

$z=re^{i\varphi}$ – kompleks sonning ko'rsatkichli shakli.

Kompleks sonlarni ko'paytirish, bo'lish, darajaga ko'tarish, ulardan ildiz chiqarishda, kompleks sonning trigonometrik va ko'rsatkichli shakllaridan foydalaniladi:

Agar $z_1=r_1(\cos\varphi_1+i\sin\varphi_1),$
$z_2=r_2(\cos\varphi_2+i\sin\varphi_2)$

bo'lsa, ushbu formulalar o'rinli:

$$z_1\cdot z_2=r_1\cdot r_2(\cos(\varphi_1+\varphi_2)+i\sin(\varphi_1+\varphi_2))=r_1\cdot r_2e^{i(\varphi_1+\varphi_2)},$$

$$\frac{z_1}{z_2}=\frac{r_1}{r_2}(\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2))=\frac{r_1}{r_2}e^{i(\varphi_1-\varphi_2)},$$

$$z^n=r^n(\cos n\varphi+i\sin n\varphi)=r^n\cdot e^{in\varphi}.$$

Oxirgi formula Muavr formulasi deyiladi.

Trigonometrik yoki ko'rsatkichli shakldagi kompleks sondan n darajali ildiz chiqarish uchun ushbu formuladan foydalaniladi:

$$w_k=\sqrt[n]{z}=\sqrt[n]{r(\cos\varphi+i\sin\varphi)}=\sqrt[n]{r}(\cos\frac{\varphi+2\pi k}{n}+i\sin\frac{\varphi+2\pi k}{n})=\sqrt[n]{r}e^{\frac{i(\varphi+2\pi k)}{n}}.$$

Bunda $0\le k\le n-1.$

Misol 316. $(-\sqrt{3}+i)^6$ ni hisoblang**.**

Yechilshi.

$$n=6;\quad x=-\sqrt{3};\quad y=1;\quad r=\sqrt{x^2+y^2}=\sqrt{(-\sqrt{3})^2+1^2}=2;\quad tg\varphi=\frac{y}{x}=-\frac{1}{\sqrt{3}}\Rightarrow\varphi=\frac{5\pi}{6}$$

bo'ladi.

Muavr formulasidan foydalanib quyidagi yechim olinadi:

$$z^6=2^6(\cos\frac{5\pi}{6}\cdot6+i\sin\frac{5\pi}{6}\cdot6)=2^6e^{5\pi i}=64(\cos5\pi+i\sin5\pi)=-64.$$

Misol 317. $z=\sqrt[3]{-1}$ ni toping.

Yechilishi. $z = \sqrt[3]{-1} \Rightarrow z = -1. \quad x = -1; \quad y = 0$

$r = \sqrt{(-1)^2 + 0^2} = 1; \; tg\varphi = \frac{y}{x} = 0 \Rightarrow \varphi = \pi k; k = 1 \Rightarrow \varphi = \pi.$ u holda

$z = 1 \cdot (\cos\pi + i\sin\pi)$

n- darajali ildiz chiqarish formulasiga asosan

$$w_k = \sqrt[3]{\cos\pi + i\sin\pi} = \cos\frac{\pi + 2\pi k}{3} + i\sin\frac{\pi + 2\pi k}{3} = e^{\frac{i(\pi + 2\pi k)}{3}}, \; bunda \; k = 0,1,2.$$

k ga ketma-ket 0,1,2 qiymatlar berib, ildizning uchta qiymati topiladi:

$$w_0 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = e^{\frac{i\pi}{3}} = \frac{1}{2} + i\frac{\sqrt{3}}{2},$$

$$w_1 = \cos\pi + i\sin\pi = e^{i\pi} = -1,$$

$$w_2 = \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} = e^{\frac{5\pi i}{3}} = \frac{1}{2} - i\frac{\sqrt{3}}{2},$$

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318. Agar $z_1 = 1 - i, z_2 = 3 + 4i, z_3 = -1 + 3i$ *bo'lsa*, $z = \frac{z_1 + 3z_2}{z_1 z_2 - z_3^2}$ ning qiymatini hisoblang. **Javobi:** $\frac{227}{274} + \frac{99}{274}i.$

$z_1 = 3 - 2i, z_2 = 4 + i, z_3 = -2 + i$ *kompleks sonlar berilgan*

319. $z = \frac{z_1(z_2 + z_3)}{z_3^2 + z_1}$ *ni hisoblang*

Javobi: $-\frac{45}{41} - \frac{87}{41}i.$

320. Agar $z_1 = i - 1, z_2 = -2 + i, z_3 = 3 - 4i$ *bo'lsa*, $z = \frac{z_1(z_2 + 3z_3^2)}{z_1 - z_2}$

ni toping

Javobi: $-\frac{64}{5} - \frac{38}{5}i.$

Quyidagi kompleks sonlarni trigonometrik va ko'rsatkichli shakllarda ifodalang:

321. $z_1 = 3 - 3i;$ *Javobi*: $z_1 = 3\sqrt{2}(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})) = 3\sqrt{2}e^{-\frac{\pi}{4}i};$

322. $z_2 = -1 - i;$ *Javobi*: $z_2 = \sqrt{2}(\cos(-\frac{3\pi}{4}) + i\sin(-\frac{3\pi}{4})) = \sqrt{2}e^{-\frac{3\pi}{4}i};$

323. $z_3 = -i;$ *Javobi*: $z_3 = \cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2}) = e^{-\frac{\pi}{2}i};$

324. $z_4 = -2;$ *Javobi*: $z_4 = 2(\cos\pi + i\sin\pi) = 2e^{\pi i}.$

Quyidagilarni hisoblang:

325.

315. $\sqrt[6]{-1}$;

$Javobi: k=0, \quad w_0=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6};$

$k=1, \quad w_1=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2};$

$k=2, \quad w_2=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6};$

$k=3, \quad w_3=\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6};$

$k=4, \quad w_4=\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2};$

$k=5, \quad w_5=\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}.$

326. $\sqrt[3]{i}$;

$Javobi: \ k=0 \quad w_0=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6};$

$k=1, \quad w_1=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6};$

$k=2, \quad w_2=\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}.$

10-мavzu. Algebraning asosiy teoremasi. Kubik tenglamalar. Kardano formulasi

Misol 327. $\boldsymbol{P}_5(\boldsymbol{x})=2\boldsymbol{x}^5-3\boldsymbol{x}^4+5\boldsymbol{x}^3-7\boldsymbol{x}^2+2\boldsymbol{x}-7$ beshinchi darajali ko'phadni $\boldsymbol{Q}_2(\boldsymbol{x})=\boldsymbol{x}^2-1$ ikkinchi darajali ro'hadga bo'ling.

Yechilishi.

$$\begin{array}{r|l}
-\ 2x^5-3x^4+5x^3-7x^2+2x-7 & x^2-1 \\
\underline{2x^5-2x^3\qquad\qquad\qquad\qquad} & 2x^3-3x^2+7x-10 \\
-\ -3x^4+7x^3-7x^2+2x-7 & \\
\underline{-3x^4+3x^2\qquad\qquad\qquad} & \\
-\ 7x^3-10x^2+2x-7 & \\
\underline{7x^3-7x\qquad\qquad} & \\
-\ -10x^2+9x-7 & \\
\underline{-10x^2+10} & \\
9x-17 &
\end{array}$$

Demak, $2\boldsymbol{x}^5-3\boldsymbol{x}^4+5\boldsymbol{x}^3-7\boldsymbol{x}^2+2\boldsymbol{x}-7=(\boldsymbol{x}^2-1)\cdot(2\boldsymbol{x}^3-3\boldsymbol{x}^2+7\boldsymbol{x}-10)+(9\boldsymbol{x}-17)$

$$x^3+ax^2+bx+c=0 \qquad (1)$$

ko'rinishdagi tenglamaga kubik tenglama deyiladi.

Viyet teoremasi. Agar x_1, x_2, x_3 lar (1) tenglamaning ildizlari bo'lsa, uni quyidagi ko'rinishda yozish mumkin:

$$(x-x_1)(x-x_2)(x-x_3)=0$$

Bundan, $\quad a=-(x_1+x_2+x_3),$

$b=x_1x_2+x_2x_3+x_1x_3,$

$c=-x_1x_2x_3.$

$x^3+ax^2+bx+c=0$ tenglamani $x=z-\frac{a}{3}$ almashtirish yordamida $z^3+pz+q=0$ ko'rinishga keltiriladi. $z^3+pz+q=0$ tenglama ***Kardano formulasi*** yordamida quyidagicha yechiladi:

$$z=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}=u+v$$

1. Agar $\Delta=\frac{q^2}{4}+\frac{p^3}{27}>0$ bo'lsa, u holda $\boldsymbol{z}_1=\boldsymbol{u}_1+\boldsymbol{v}_1, \quad \boldsymbol{z}_{2,3}=-\frac{\boldsymbol{u}_1+\boldsymbol{v}_1}{2}\pm\frac{\sqrt{3}(\boldsymbol{u}_1-\boldsymbol{v}_1)}{2}\boldsymbol{i}$.

2. Agar $\Delta = \frac{q^2}{4} + \frac{p^3}{27} = 0$ bo'lsa, u holda $z_1 = \frac{3q}{p}, \quad z_2 = z_3 = -\frac{z_1}{2}$.

3. Agar $\Delta = \frac{q^2}{4} + \frac{p^3}{27} < 0$ bo'lsa, u holda $z_1 = 2\sqrt{\frac{-p}{3}}\cos\frac{\varphi}{3}$,

$z_{2,3} = 2\sqrt{\frac{-p}{3}}\cos\left(\frac{\varphi}{3} \pm 120^0\right)$, bu yerda $\cos\varphi = -\frac{q}{2} : \sqrt{\frac{-p^3}{27}}$.

Misol 328. $x^3 - 4x^2 + x + 6 = 0$ tenglamaning butun ildizlari orasaidan tanlab, chap tomonini *(x-x₁)* ko'paytuvchiga ajratib qolgan ildizlarini toping va

$x_1 + x_2 + x_3 = -a, \quad x_1x_2 + x_2x_3 + x_1x_3 = b, \quad x_1x_2x_3 = -c$ orqali tekshiring.

Yechilishi: 6 sonining ko'paytuvchilari 1,2,3,6 ,-1,-2,-3,-6.

Berilgan to'rt had *x-1, x-2, x-3, x-6* yoki *x+1, x+2, x+3, x+6* larga bo'linadi:

$x^3 - 4x^2 + x + 6 = (x-2)(x^2 - 2x - 3) = (x-2)(x-3)(x+1).$

Demak, $x_1 = -1, \ x_2 = 2, \ x_3 = 3$.

Tekshirish:

$x_1 + x_2 + x_3 = -1 + 2 + 3 = 4 = -a;$

$x_1x_2 + x_2x_3 + x_1x_3 = -1*2 + 2*3 + (-1)*3 = 1 = b;$

$x_1 * x_2 * x_3 = -1*2*3 = -6 = -c.$

Misol 329. $z^3 - 6z - 9 = 0.$ tenglamani Kardano formulasi yordamida yeching.

Yechilishi.

$$p = -6, \ q = -9. \qquad \Delta = \frac{q^2}{4} + \frac{p^3}{27} = \frac{81}{4} - \frac{216}{27} = \frac{2187 - 864}{108} = \frac{49}{4} > 0;$$

$$u = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} = \sqrt[3]{\frac{9}{2} + \frac{7}{2}} = \sqrt[3]{8} = 2;$$

$$v = \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} = \sqrt[3]{\frac{9}{2} - \frac{7}{2}} = \sqrt[3]{1} = 1;$$

$$z_1 = u + v = 2 + 1 = 3.$$

$$z_2 = -\frac{u+v}{2} + \frac{u-v}{2} i\sqrt{3} = -\frac{3}{2} + \frac{\sqrt{3}}{2} i.$$

$$z_3 = -\frac{u+v}{2} - \frac{u-v}{2} i\sqrt{3} = -\frac{3}{2} - \frac{\sqrt{3}}{2} i.$$

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330. $P_4(x) = 2x^4 - 5x^3 + 2x$ ko'phadni $Q_2(x) = x^2 - 1$ ko'hadga bo'ling.

Javobi: $2x^4 - 5x^3 + 2x = (x^2 - 1)\cdot(2x^2 - 5x + 2) + (-3x + 2).$

331. $P_4(x) = x^4 - 3x^3 + 3x^2 + 2x - 1$ ko'phadni $Q_2(x) = x^2 - 1$ ko'hadga bo'ling.

Javobi: $x^4 - 3x^3 + 3x^2 + 2x - 1 = (x^2 - 1)\cdot(x^2 - 3x + 4) + (5x + 3).$

332. $P_5(x) = 2x^5 - 3x^4 - 5x^3 - 3x^2 + 5$ ko'phadni $Q_2(x) = 2x^2 - 1$ ko'hadga bo'ling.

Javobi: $2x^5 - 3x^4 - 5x^3 - 3x^2 + 5 = (2x^2 - 1)\cdot\left(x^3 - \frac{3}{2}x^2 - 2x - \frac{9}{4}\right) + \left(-2x + \frac{29}{4}\right).$

Quyidagi tenglamalar ozod hadlarining tub ko'paytuvchilari orasidan tenglamaning bitta ildizi x_1 ni topib, so'ngra tenlamaning chap tomonini $(x-x_1)$ ga bo'lib, qolgan ildizlari topilsin:

333. $x^3 - 4x^2 + x + 6 = 0.$ *Javobi* $: -1; 2; 3.$

334. $x^3 - 4x^2 - 4x - 5 = 0.$ *Javobi* $: 5; \frac{-1 + i\sqrt{3}}{2}; \frac{-1 - i\sqrt{3}}{2}$

335. $x^3 - 5x^2 - 2x + 24 = 0.$ *Javobi* $: 3; 4; -2.$

336. $x^4 + x^3 + 2x - 4 = 0.$ *Javobi* $: 1; -2; i\sqrt{2}; -i\sqrt{2}.$

337. $9x^3 + 18x^2 - x - 2 = 0.$ *Javobi* $: -2; \frac{1}{3}; -\frac{1}{3}.$

338. $4x^3 - 4x^2 + x - 1 = 0.$ *Javobi* $: 1; \frac{i}{2}; -\frac{i}{2}.$

Quyidagi tenglamalarni Kardano formulasi bo'yicha yeching:

339. $z^3 - 6z - 9 = 0.$ *Javobi* $: \Delta = \frac{49}{4} > 0,\ z_1 = 3,\ z_{2,3} = \frac{-3 \pm i\sqrt{3}}{2}.$

340. $z^3 - 12z - 16 = 0.$ *Javobi* $: \Delta = 0,\ z_1 = 4,\ z_2 = z_3 = -2.$

341. $z^3 - 12z - 8 = 0.$ *Javobi* $: \Delta < 0,\ z_1 = 4\cos 40^o,\ z_{2,3} = 4\cos\left(20^o \pm 120^o\right).$

342. $z^3 + 18z - 19 = 0.$ *Javobi* $: \Delta = \frac{1225}{4} > 0,\ z_1 = 1,\ z_{2,3} = \frac{-1 \pm 5i\sqrt{3}}{2}.$

343. $z^3 - 6z - 4 = 0.$ *Javobi* $: \Delta = -4 < 0,\ z_1 = 2\sqrt{2}\cos 15^o = 1 + \sqrt{3},\ z_2 = -2,\ z_3 = 1 - \sqrt{3}.$

344. $z^3 - 9z + 9 = 0.$ *Javobi* $: \Delta = -\frac{27}{4} < 0,\ z_1 = 2\sqrt{3}\cos 50^o,\ z_{2,3} = 2\sqrt{3}\cos(50^o \pm 120^o).$

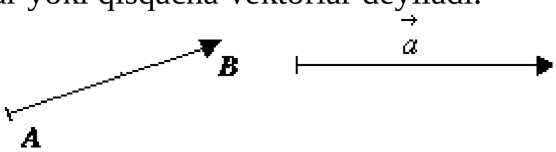
Quyidagi tenglamalarni $z^3+pz+q=0$ **ko'rinishga keltiring va Kardano usulidan foydalanib ildizlarini toping:**

345. $x^3 + 9x^2 + 18x + 9 = 0.$ *Javobi* $: \Delta = -\frac{27}{4} < 0,\ z_1 = 2\sqrt{3}\cos 50^o - 3,\ z_{2,3} = 2\sqrt{3}\cos(50^o \pm 120^o) - 3.$

346. $x^3 + 6x^2 + 9x + 4 = 0.$ *Javobi* $: \Delta = 0,\ x_1 = -4,\ x_2 = x_3 = -1.$

11-MAVZU. VEKTORLAR

Ta'rif: Uzunliklari teng va yo'nalishlari bir xil bo'lgan barcha yo'nalgan kesmalar to'plamiga ozod vektorlar yoki qisqacha vektorlar deyiladi.



A nuqta vektorning boshi .
B nuqta vektorning oxiri.

Vektorlar $\vec{AB}$ yoki $\vec{a}$ ko'rinishda yoziladi.

Vektorning uzunligi $\left|\vec{AB}\right|$ yoki $\left|\vec{a}\right|$ ko'rinishda yoziladi.

Uzunligi nolga teng bo'lgan vektorni nol vektor deyiladi va $\vec{0}$ ko'rinishda yoziladi. Nol vektorning yo'nalishi aniqlanmagan hisoblanadi.

Uzunligi birga teng bo'lgan vektorlarni birlik vektorlar, ortlar, koordinata vektorlari yoki bazis vektorlar deyiladi.

Har qanday vektor o'zining uzunligiga teskari songa ko'paytirilsa birlik vektor hosil bo'ladi:

$$\vec{a}_0 = \frac{1}{|\vec{a}|} * \vec{a}. \qquad \textbf{(1)}$$

$\vec{a}_0$- birlik vektor, ya'ni $|\vec{a}_0| = 1.$

Bitta to'g'ri chiziqqa parallel joylashgan vektorlarni kollinear vektorlar deyiladi:

$$\vec{a} = \lambda \vec{b} \qquad \textbf{(2)}$$

(2) tenglik ikki vektorning kollinearlik sharti.

Bitta tekislikka parallel joylashgan vektorlarni komplanar vektorlar deyiladi.

Ikkita vektorning bir xil yo'nalishli ekanligi $\vec{a}\uparrow\uparrow\vec{b}$, qarama-qarshi yo'nalishli ekanligi $\vec{a}\uparrow\downarrow\vec{b}$ ko'rinishda belgilanadi.

Ikki vektorning tenglik sharti:

$$\vec{a} = \vec{b} \Leftrightarrow \begin{cases} |\vec{a}| = |\vec{b}|; \\ \vec{a}\uparrow\uparrow\vec{b}. \end{cases} \qquad \textbf{(3)}$$

Shuningdek

$$\vec{AB} = -\vec{BA} \qquad \textbf{(4)}$$

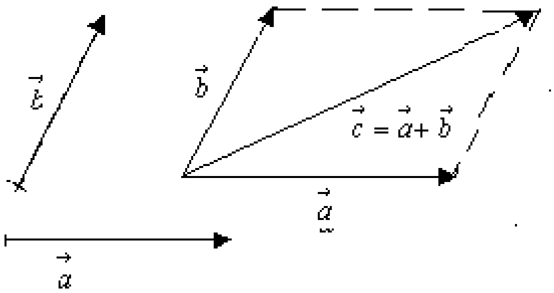
bo'ladi.

Vektorlarni oldingi vaziyatiga nisbatan yo'nalishi va uzunligini o'zgartirmasdan parallel ko'chirish mumkin.

VEKTORLAR USTIDA CHIZIQLI AMALLAR

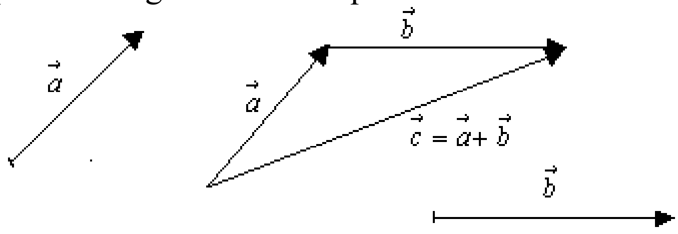
Vektorlar ustida bo'lish amali mavjud emas.

Vektorlarni qo'shishning parallelogramm qoidasi



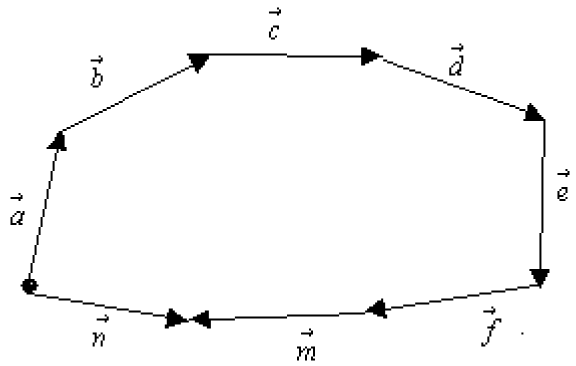
Ikkita vektor bitta nuqtaga ko'chirilib, ulardan parallelogramm yasaladi. Papallelogrammning diagonali ikki vektorning yig'indi vektori bo'ladi va $\vec{c} = \vec{a} + \vec{b}$ ko'rinishda yoziladi.

Vektorlarni qo'shishning uchburchak qoidasi:



$\vec{a}$ vektorning oxiridan $\vec{b}$ vektor qo'yiladi. Qoplovchi vektor, yig'indi vektor bo'ladi va $\vec{c}=\vec{a}+\vec{b}$ ko'rinishda yoziladi.

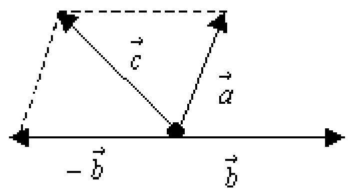
Vektorlarni qo'shishning ko'pburchak qoidasi:



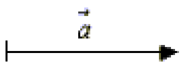
$\vec{n}=\vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f}+\vec{m}.$

$\vec{n}$ - qoplovchi vektor.

Vektorlarni ayirish. $\vec{a}$ vektordan $\vec{b}$ vektorni ayirish uchun $\vec{a}$ vektorga $\vec{b}$ vektorning qarama-qarshi $-\vec{b}$ vektorini qo'shish kerak.

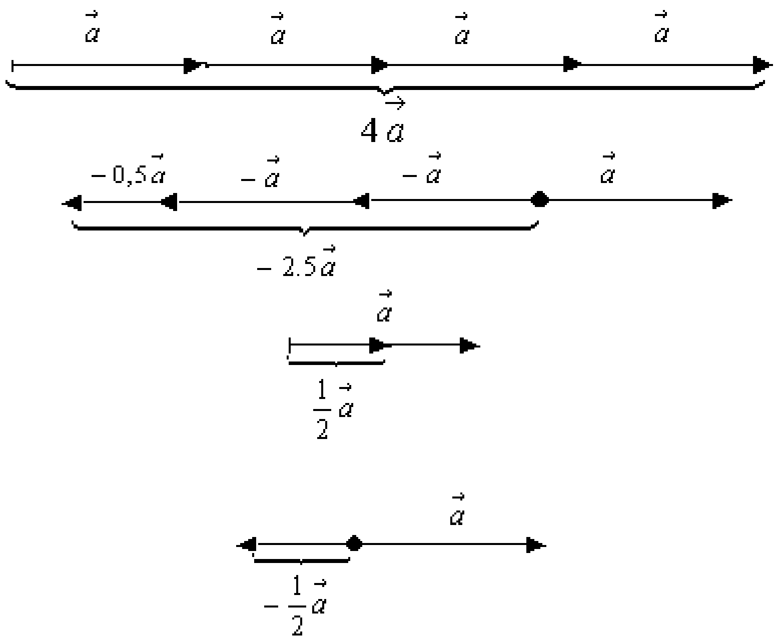


$$\vec{c}=\vec{a}+(-\vec{b})=\vec{a}-\vec{b}.$$

Misol 347. 

vektor berilgan, $4\vec{a}$, $-2{,}5\vec{a}$, $\frac{1}{2}\vec{a}$, $-\frac{1}{2}\vec{a}$, vektorlarni yasang.

Yechilishi.



IKKI VEKTORNING SKALYAR KO'PAYTMASI

$$\vec{a}*\vec{b}=\left|\vec{a}\right|*\left|\vec{b}\right|*\cos(\vec{a}\,\hat{,}\,\vec{b}); \qquad (5)$$

$$\vec{a}*\vec{a}=\left|\vec{a}\right|*\left|\vec{a}\right|*\cos 0^0;$$

$$\vec{a}^{\,2}=\left|\vec{a}\right|^2. \qquad (6)$$

Ikki vektor orasidagi burchak

$$\cos(\vec{a}\,\hat{,}\,\vec{b})=\frac{\vec{a}*\vec{b}}{\left|\vec{a}\right|*\left|\vec{b}\right|}. \qquad (7)$$

Ikkita perpendikulyar vektorlarning skalyar ko'paytmasi nolga teng bo'ladi:

$$\vec{a}\perp\vec{b}\Leftrightarrow\vec{a}*\vec{b}=0. \qquad (8)$$

Misol 348. $\left|\vec{a}\right|=4,$ $\left|\vec{b}\right|=3,$ $(\vec{a}\,\hat{,}\,\vec{b})=60^0$. λ ning qanday qiymatida $(\vec{a}+\lambda\vec{b})\perp\vec{a}$ bo'ladi?

Yechilishi. (8) ga asosan $(\vec{a}+\lambda\vec{b})\perp\vec{a}\Leftrightarrow\vec{a}^{\,2}+\lambda\vec{a}\vec{b}=0;$ (6) va (5) ga asosan

$$\left|\vec{a}\right|^2+\lambda\left|\vec{a}\right|*\left|\vec{b}\right|*\cos(\vec{a}\,\hat{,}\,\vec{b})=0;$$

$$4^2+\lambda*4*3*\cos 60^0=0;$$

$$16+\lambda*12*\frac{1}{2}=0;$$

$$6\lambda=-16\Leftrightarrow\lambda=-\frac{16}{6}=-\frac{8}{3}=-2\frac{2}{3}\Rightarrow\lambda=-2\frac{2}{3}.$$

Misol 349. Agar $\vec{m}$ va $\vec{n}$ o'zaro perpendikulyar birlik vektorlar bo'lsa, $\vec{a}=2\vec{m}+\vec{n}$ vektorning uzunligini toping.

Yechilishi. $\vec{m}\perp\vec{n}\Leftrightarrow\vec{m}*\vec{n}=0;$ $\quad\left|\vec{m}\right|=\left|\vec{n}\right|=1;$ $\quad\left|\vec{a}\right|=?$

Vektorlarga doir ayrim masalalar tenglikning ikkala tomoni kvadratga ko'tarish orqali yechiladi:

$$\vec{a}^{\,2}=(2\vec{m}+\vec{n})^2=(2\vec{m})^2+4\vec{m}\vec{n}+\vec{n}^{\,2}=4\vec{m}^{\,2}+4*0+\vec{n}^{\,2}\Rightarrow$$

$$\Rightarrow\left|\vec{a}\right|^2=4*\left|\vec{m}\right|^2+\left|\vec{n}\right|^2\Rightarrow\quad\left|\vec{a}\right|^2=4*1^2+1^2=5\quad\Rightarrow\quad\left|\vec{a}\right|=\sqrt{5}.$$

Misol 350. $\vec{a}$ va $\vec{b}$ nokollinear vektorlar berilgan. $\left|\vec{a}\right|=\left|\vec{b}\right|=4$ bo'lsa, $(\vec{a}+\vec{b})$ bilan $(\vec{a}-\vec{b})$ vektorlar qanday burchak tashkil etadi?

Yechilishi. $(\vec{a}-\vec{b})*(\vec{a}+\vec{b})=\left|\vec{a}\right|^2-\left|\vec{b}\right|^2=4^2-4^2=16-16=0\Rightarrow(\vec{a}-\vec{b})\perp(\vec{a}+\vec{b}),$

ya'ni 90^0 li burchak tashkil etadi.

Misol 351. Agar $\vec{a}-2\vec{b}$ va $4\vec{b}+5\vec{a}$ vektorlar perpendikulyar bo'lsa, $\vec{a}$ va $\vec{b}$ birlik vektorlar orasidagi burchakni toping.

Yechilishi.

$$\left|\vec{a}\right|=\left|\vec{b}\right|=1; \quad (\vec{a}-2\vec{b})\perp(4\vec{b}+5\vec{a}); \quad (\vec{a}-2\vec{b})*(4\vec{b}+5\vec{a})=0;$$

$$4\vec{a}\vec{b}+5\vec{a}^2-8\vec{b}^2-10\vec{a}\vec{b}=0; \quad \Rightarrow \quad -6\vec{a}\vec{b}+5\left|\vec{a}\right|^2-8\left|\vec{b}\right|^2=0; \Rightarrow$$

$$\Rightarrow -6\left|\vec{a}\right|\left|\vec{b}\right|\cos(\vec{a}\,\hat{,}\,\vec{b})+5*1^2-8*1^2=0; \Rightarrow -6\left|\vec{a}\right|\left|\vec{b}\right|\cos(\vec{a}\,\hat{,}\,\vec{b})+5-8=0; \Rightarrow$$

$$\Rightarrow -6*\cos(\vec{a}\,\hat{,}\,\vec{b})-3=0; \quad \Rightarrow 6*\cos(\vec{a}\,\hat{,}\,\vec{b})=-3; \quad \Rightarrow \cos(\vec{a}\,\hat{,}\,\vec{b})=\frac{-3}{6};$$

$$\Rightarrow \cos(\vec{a}\,\hat{,}\,\vec{b})=-\frac{1}{2}; \quad (\vec{a}\,\hat{,}\,\vec{b})=120^0.$$

KOORDINATALARI BILAN BERILGAN VEKTORLAR

Boshi $A(x_1;y_1)$ oxiri $B(x_2;y_2)$ nuqtada bo'lgan $\vec{AB}$ vektorni koordinatalari bilan yozish uchun ikkinchi o'rinda turgan nuqtaning koordinatalaridan birinchi o'rinda turgan nuqtaning mos koordinatalarini ayirish kerak:

$$\vec{AB}=\{x_2-x_1\ ; y_2-y_1\}. \qquad \textbf{(9)}$$

Misol 352. A(-2;3) va B(1;5) nuqtalardan foydalanib, $\vec{AB}$ vektorni koordinatalari bilan yozing.

Yechilishi. $\vec{AB}=\{1-(-2);5-3\}=\{3;2\}.$

Koordinatalari bilan berilgan vektor ortlar bo'yicha quyidagicha yoyiladi:

$$\vec{a}=\{x;\ y;\ z\}=x*\vec{i}+y*\vec{j}+z*\vec{k} \qquad \textbf{(10)}$$

Misol 353. $\vec{a}=\{2;-3;7\}$ vektorni ortlar bo'yicha yoying.

Yechilishi. $\vec{a}=\{2;-3;7\}=2*\vec{i}-3*\vec{j}+7*\vec{k}$.

Koordinatalari bilan berilgan vektorning uzunligini topish uchun har bir koordinata alohida-alohida kvadratga ko'tarilib qo'shiladi va arifmetik ildizga olinadi:

$$\vec{AB}=\{x_2-x_1;y_2-y_1;z_2-z_1\}\Rightarrow\left|\vec{AB}\right|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

yoki

$$\vec{a}=\{x;y;z\}\Rightarrow\left|\vec{a}\right|=\sqrt{x^2+y^2+z^2}\ . \qquad \textbf{(11)}$$

Misol 354. $\vec{a}=\{2;-3;-4\}$ vektorning uzunligini toping va birlik vektor ko'rinishida yozing.

$$\left|\vec{a}\right|=\sqrt{2^2+(-3)^2+(-4)^2}=\sqrt{4+9+16}=\sqrt{29};$$

Yechilishi. $\vec{a}_0=\frac{1}{\left|\vec{a}\right|}*\vec{a}=\frac{1}{\sqrt{29}}\ \{2;-3;-4\}=\left\{\frac{2}{\sqrt{29}},-\frac{3}{\sqrt{29}},-\frac{4}{\sqrt{29}}\right\}.$

Koordinatalari bilan berilgan ikki vektorni qo'shish (ayirish) uchun ularning mos koordinatalari qo'shiladi (ayriladi):

$\vec{a}=\{x_1;y_1;z_1\}$ va $\vec{b}=\{x_2;y_2;z_2\}$.

$$\vec{a}\pm\vec{b}=\{x_1\pm x_2;y_1\pm y_2;z_1\pm z_2\}. \qquad \textbf{(12)}$$

Misol 355. $\vec{a}=\{2;\ 3;\ 4\},\vec{b}=\{-2;5;-3\}$ vektorlar berilgan . $\vec{a}+\vec{b}=?$ $\quad\vec{a}-\vec{b}=?$

Yechilishi.

$$\vec{a}+\vec{b}=\{2;3;4\}+\{-2;\ 5;-3\}=\{2+(-2);\ 3+5;\ 4+(-3)\}=\{2-2;\ 8;\ 4-3\}=\{0;\ 8;\ 1\}.$$

$$\vec{a}-\vec{b}=\{2;3;4\}-\{-2;\ 5;-3\}=\{2-(-2);\ 3-5;\ 4-(-3)\}=\{2+2;\ -2;\ 4+3\}=\{4;-2;\ 7\}.$$

Koordinatalari bilan berilgan ikki vektorni skalyar ko'paytirish uchun ularning mos koordinatalari ko'paytirilib qo'shiladi:

$$\vec{a}=\{x_1;y_1;z_1\} \quad \text{va} \quad \vec{b}=\{x_2;y_2;z_2\}.$$

$$\vec{a}*\vec{b}=\{x_1;y_1;z_1\}*\{x_2;y_2;z_2\}=x_1*x_2+y_1*y_2+z_1*z_2. \qquad \textbf{(13)}$$

Misol 356. $\vec{a}=\{2;\ 3;\ 4\}$ va $\vec{b}=\{-2;5;-3\}$ vektorlarni skalyar ko'paytiring.

Yechilshi. $\vec{a}*\vec{b}=\{2;\ 3;\ 4\}*\{-2;\ 5;\ -3\}=2*(-2)+3*5+4*(-3)=-4+15-12=-1.$

Sonni koordinatalari bilan berilgan vektorga ko'paytirish uchun, bu son har bir koordinataga alohida-alohida ko'paytiriladi:

$\vec{a}=\{x;y;z\};\quad \lambda\vec{a}=\lambda*\{x;\ y;\ z\}=\{\lambda x;\ \lambda y;\ \lambda z\}.$

Misol 357. $\vec{a}=\{-3;4;-2\}$ vektorni 3 ga ko'paytiring va uzunligini toping.

Yechilishi.

$$3*\vec{a}=3*\{-3;4;-2\}=\{-9;12;-6\}.$$

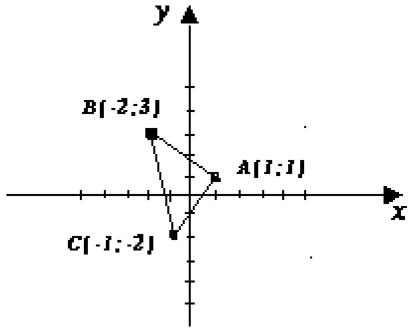
$$\left|3\vec{a}\right|=\sqrt{(-9)^2+12^2+(-6)^2}=\sqrt{81+144+36}=\sqrt{261}\ .$$

Koordinatalari bilan berilgan ikki vektor orasidagi burchakni topish formulasi:

$$\cos(\vec{a}\,\hat{,}\,\vec{b})=\frac{\vec{a}*\vec{b}}{\left|\vec{a}\right|*\left|\vec{b}\right|}=\frac{x_1x_2+y_1y_2+z_1z_2}{\sqrt{x_1^2+y_1^2+z_1^2}*\sqrt{x_2^2++y_2^2+z_2^2}} \qquad \textbf{(14).}$$

Misol 358. Uchlari A(1;1), B(-2;3) va C(-1;-2) nuqtalarda bo'lgan uchburchakning burchaklarini toping.

Yechilishi. $\angle A$ ni topish uchun $\vec{AB}$ va $\vec{AC}$ vektorlar hosil qilinadi.



$$\vec{AB} = \{-2-1; 3-1\} = \{-3; 2\}; \qquad \vec{AC} = \{-1-1; -2-1\} = \{-2; -3\};$$

$$\vec{AB} * \vec{AC} = \{-3; 2\} * \{-2; -3\} = -3*(-2) + 2*(-3) = 6 - 6 = 0;$$

$$\left|\vec{AB}\right| = \sqrt{(-3)^2 + 2^2} = \sqrt{9+4} = \sqrt{13}; \qquad \left|\vec{AC}\right| = \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13};$$

Bu ma’lumotlar (14) ga qo’yiladi:

$$\cos(\vec{AB} \hat{,} \vec{AC}) = \frac{\vec{AB} * \vec{AC}}{\left|\vec{AB}\right| * \left|\vec{AC}\right|} = \frac{0}{\sqrt{13} * \sqrt{13}} = 0.$$

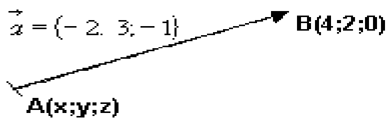
Bundan $(\vec{AB} \hat{,} \vec{AC}) = \angle A = 90^0$.

$\left|\vec{AB}\right| = \left|\vec{AC}\right| = \sqrt{13}$ ekanligidan $\angle B = \angle C = 45^0$ bo’ladi.

Eslatma: $\angle B$ ni $\vec{BA}$ va $\vec{BC}$, $\angle C$ ni $\vec{CA}$ va $\vec{CB}$ vektorlar orasidagi burchak sifatida ham topish mumkin.

Misol 359. B(4;2;0) nuqta $\vec{a} = \{-2; 3; -1\}$ vektorning oxiri bo’lsa, bu vektor boshining koordinatalarini toping.

Yechilishi. $\vec{AB} = \{4-x; 2-y; 0-z\}$;



$\vec{a} = \{-2; 3; -1\}$

$$\vec{AB} = \vec{a} \Leftrightarrow \{4-x; 2-y; -z\} = \{-2; 3; -1\} \Rightarrow \begin{cases} 4-x = -2 \\ 2-y = 3 \\ -z = -1 \end{cases} \Rightarrow \begin{cases} x = 6 \\ y = -1 \\ z = 1 \end{cases} \Rightarrow A(x; y; z) = A(6; -1; 1).$$

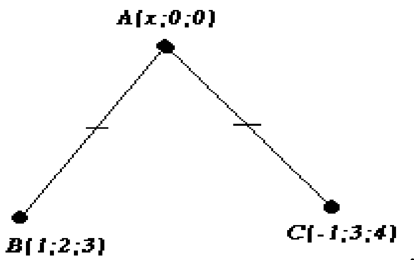
Misol 360. **A(x;0;0)** nuqta **B(1;2;3)** va **C(-1;3;4)** nuqtalardan teng uzoqlikdaligi ma’lum bo’lsa x ni toping.

Yechilishi.

$\vec{AB} = \{1 - x; 2; 3\};$

$\left|\vec{AB}\right| = \sqrt{(1-x)^2 + 2^2 + 3^2} =$

$= \sqrt{1 - 2x + x^2 + 4 + 9} = \sqrt{x^2 - 2x + 14};$



$\vec{AC} = \{-1 - x; 3; 4\};$

$$\left|\vec{AC}\right| = \sqrt{(-1-x)^2 + 3^2 + 4^2} = \sqrt{1 + 2x + x^2 + 9 + 16} = \sqrt{x^2 + 2x + 26};$$

$$\left|\vec{AB}\right| = \left|\vec{AC}\right| \Leftrightarrow \sqrt{x^2 - 2x + 14} = \sqrt{x^2 + 2x + 26} \Leftrightarrow x^2 - 2x + 14 = x^2 + 2x + 26 \Rightarrow$$

$$\Rightarrow 4x = -12 \Rightarrow x = -3 \Rightarrow A(x;0;0) = A(-3;0;0).$$

Misol 361. $\vec{a} = \{0;1\}$ va $\vec{b} = \{2;1\}$ vektorlar berilgan. x ning qanday qiymatlarida $\vec{b} + x\vec{a}$ vektor $\vec{b}$ vektorga perpendikulyar bo'ladi?

Yechilishi.

$$(\vec{b} + x\vec{a}) \perp \vec{b} \quad \Leftrightarrow \quad (\vec{b} + x\vec{a}) * \vec{b} = 0 \quad \Leftrightarrow \quad \vec{b}^2 + x\vec{a} * \vec{b} = 0 \quad \Rightarrow \quad \left|\vec{b}\right|^2 + x\vec{a} * \vec{b} = 0;$$

$$\left|\vec{b}\right| = \sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5};$$

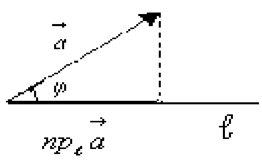
$$\vec{a} * \vec{b} = \{0;1\} * \{2;1\} = 0 * 2 + 1 * 1 = 1;$$

U holda

$$\left|\vec{b}\right|^2 + x * \vec{a} * \vec{b} = 0 \Rightarrow \quad (\sqrt{5})^2 + x * 1 = 0 \quad \Rightarrow \quad 5 + x = 0 \quad \Rightarrow \quad x = -5.$$

VEKTORNING O'QDAGI PROEKSIYASI

Vektorning o'qdagi proeksiyasi, shu vektor uzunligining vektor bilan o'q orasidagi burchak kosinusi ko'paytmasiga teng.



$$np_\ell \vec{a} = \left|\vec{a}\right| * \cos\varphi. \qquad \textbf{(15)}$$

Ikki vektorning skalyar ko'paytmasi shu vektorlardan birining uzunligi bilan shu vektor yo'nalishiga tushirilgan ikkinchi vektor proeksiyasining ko'paytmasiga teng:

$$\vec{a} * \vec{b} = \left|\vec{a}\right| * np_{\vec{a}} \vec{b} = \left|\vec{b}\right| np_{\vec{b}} \vec{a}. \qquad \textbf{(16)}$$

Misol 362. $\vec{a}=\{3;-1;5\}$ vektorning $\vec{b}=\{1;2;2\}$ vektor yo'nalishiga tushirilgan proeksiyasi topilsin.

Yechilishi. (16) dan

$$np_{\vec{b}}\,\vec{a}=\frac{\vec{a}*\vec{b}}{|\vec{b}|}=\frac{3*1+(-1)*2+5*2}{\sqrt{1^2+2^2+2^2}}=\frac{3-2+10}{3}=\frac{11}{3}=3\frac{2}{3}$$

bo'ladi.

VEKTORNING YO'NALTIRUVCHI KOSINUSLARI

$\vec{a}=\{x;y;z\}$ vektorning yo'naltiruvchi kosinuslari:

$$\cos\alpha=\frac{x}{\sqrt{x^2+y^2+z^2}};$$
$$\cos\beta=\frac{y}{\sqrt{x^2+y^2+z^2}}; \qquad \textbf{(17)}$$
$$\cos\gamma=\frac{z}{\sqrt{x^2+y^2+z^2}}.$$

$\vec{a}$ vektorning $\vec{a}_0$ birlik vektori yo'naltiruvchi kosinuslar yordamida topiladi:

$$\vec{a}_0=\{\cos\alpha;\cos\beta;\cos\gamma\}=\cos\alpha*\vec{i}+\cos\beta*\vec{j}+\cos\gamma*\vec{k} \qquad \textbf{(18)}$$

Misol 363. $\vec{a}=\{1;2;2\}$ vektorning birlik vektori topilsin.

Yechilishi.

$$\cos\alpha=\frac{1}{\sqrt{1^2+2^2+2^2}}=\frac{1}{\sqrt{9}}=\frac{1}{3};$$
$$\cos\beta=\frac{2}{3}; \qquad \cos\gamma=\frac{2}{3}.$$

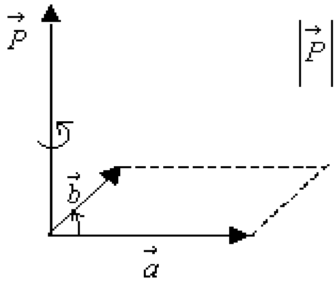
U holda

$$\vec{a}_0=\{\cos\alpha;\cos\beta;\cos\gamma\}=\{\frac{1}{3};\frac{2}{3};\frac{2}{3}\}=\frac{1}{3}\vec{i}+\frac{2}{3}\vec{j}+\frac{2}{3}\vec{k}.$$

Haqiqatan,

$$|\vec{a}_0|=\sqrt{\left(\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^2}=\sqrt{\frac{1}{9}+\frac{4}{9}+\frac{4}{9}}=\sqrt{\frac{9}{9}}=1.$$

IKKI VEKTORNI VEKTOR KO'PAYTIRISH



$$\vec{P} \perp \vec{a};\ \vec{P} \perp \vec{b};\ \vec{P} = \left[\vec{a} * \vec{b}\right];$$

$$\left|\vec{P}\right| = \left|\vec{a}\right| * \left|\vec{b}\right| * \sin(\vec{a}\,\hat{,}\,\vec{b}). \qquad \textbf{(19)}$$

Ikkita vektor bir-biriga vektor ko'paytirilsa, natija yana vektor chiqadi: $\left[\vec{a} * \vec{b}\right] = \vec{P}$. $\vec{a}$ va $\vec{b}$ vektorlarni vektor ko'paytirishdan hosil bo'lgan $\vec{P}$ vektorning uzunligi, $\vec{a}$ va $\vec{b}$ vektorlardan yasalgan parallelogrammning yuzi necha kvadrat birlik bo'lsa, shuncha uzunlik birlikka teng bo'ladi. Masalan, parallelogrammning yuzi 5 sm^2 bo'lsa, vektorning uzunligi 5 sm bo'ladi.

$$S_{\lozenge} = \left|\vec{P}\right| = \left\|\left[\vec{a} * \vec{b}\right]\right\|. \qquad \textbf{(20)}$$

$$S_{\Delta} = \frac{1}{2}\left\|\left[\vec{a} * \vec{b}\right]\right\|. \qquad \textbf{(21)}$$

Ikkita vektor koordinatalari bilan $\vec{a} = \{x_1; y_1; z_1\}$ va $\vec{b} = \{x_2; y_2; z_2\}$ ko'rinishda berilgan bo'lsa, ularning $\left[\vec{a} * \vec{b}\right]$ vektor ko'paytmasi quyidagicha topiladi:

$$\vec{P} = \left[\vec{a} * \vec{b}\right] = \left\{\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix};\ \begin{vmatrix} z_1 & x_1 \\ z_2 & x_2 \end{vmatrix};\ \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}\right\}. \qquad \textbf{(22)}$$

Misol 364. $\vec{a} = \{2; 0; 0\}$ va $\vec{b} = \{3;4;0\}$ vektorlar berilgan :

1) $\vec{a}$ va $\vec{b}$ vektorlarning vektor ko'paytmasini toping;

2) hosil bo'lgan vektorning uzunligini toping;

3) $\vec{a}$ va $\vec{b}$ vektorlarning hosil bo'lgan vektorga perpendikulyarligini ko'rsating;

4) $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogrammning yuzuni toping.

Yechilishi.

$$x_1 = 2; \quad y_1 = 0; \quad z_1 = 0;$$
$$x_2 = 3; \quad y_2 = 4; \quad z_2 = 0.$$

1) $\vec{P} = \left[\vec{a} * \vec{b}\right] = \left\{ \begin{vmatrix} 0 & 0 \\ 4 & 0 \end{vmatrix}; \begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix}; \begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix} \right\} = \{0;\ 0;\ 8\};$

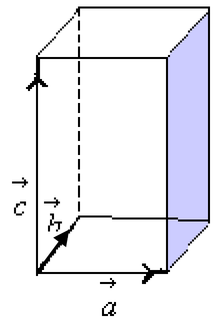
2) $|\vec{\boldsymbol{P}}| = \sqrt{0^2 + 0^2 + 8^2} = \sqrt{64} = 8$ ***uzunlik birlik.***

3) $\vec{\boldsymbol{a}} * \vec{\boldsymbol{p}} = \{2;\ 0;0\} * \{0;\ 0;\ 8\} = 2 * 0 + 0 * 0 + 0 * 8 = 0 \Leftrightarrow \vec{\boldsymbol{a}} \perp \vec{\boldsymbol{p}};$

$\vec{b} * \vec{P} = \{3;4;0\} * \{0;0;8\} = 3 * 0 + 4 * 0 + 0 * 8 = 0 \Leftrightarrow \vec{b} \perp \vec{P};$

4) $S_{\Diamond} = \left|\vec{P}\right| = 8$ *kv. birlik*.

UCH VEKTORNING ARALASH KO'PAYTMASI



Ikkita vektorning vektor ko'paytmasi uchinchi vektorga skalyar ko'paytirilsa, uch vektorning aralash ko'paytmasi hosil bo'ladi:

$$\left[\vec{a} * \vec{b}\right] * \vec{c} \quad yoki \quad (\vec{a} * \vec{b} * \vec{c}). \qquad \textbf{(23)}$$

Uch vektorni aralash ko'paytirishdan hosil bo'lgan son, bu vektorlarga qurulgan parallelepipedning hajmini ifodalaydi.

Agar uchta vektor koordinatalari bilan $\vec{a} = \{x_1; y_1; z_1\}$, $\vec{b} = \{x_2; y_2; z_2\}$, $\vec{c} = \{x_3; y_3; z_3\}$, ko'rinishda berilgan bo'lsa, ularning aralash ko'paytmasi

$$(\vec{a}, \vec{b}, \vec{c}) = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \qquad \textbf{(24)}$$

formula yordamida topiladi.

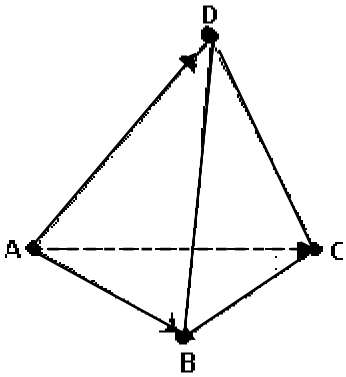
Misol 365. $\vec{a} = \{2;-3;1\}$, $\vec{b} = \{1;2;-4\}$, $\vec{c} = \{5;-4;6\}$ vektorlarga qurilgan parallelepipedning hajmini toping.

Yechilishi. (24) ga asosan

$$V = \left[\vec{a}; \vec{b}\right] * \vec{c} = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix} = 56 \ kub\ birlik.$$

Tetraedr uchlarining koordinatalari bilan berilgan:

$A(x_1; y_1; z_1)$,
$B(x_2; y_2; z_2)$,
$C(x_3, y_3; z_3)$,
$D(x_4; y_4; z_4)$.



U holda

$$\vec{AB} = \{x_2 - x_1;\ y_2 - y_1;\ z_2 - z_1\};$$
$$\vec{AC} = \{x_3 - x_1;\ y_3 - y_1;\ z_3 - z_1\};$$
$$\vec{AD} = \{x_4 - x_1;\ y_4 - y_1;\ z_4 - z_1\}.$$

Tetraedrning hajmi bir nuqtadan chiqqan uchta vektorga qurulgan parallelepiped hajmining $\frac{1}{6}$ qismiga teng bo'lgani uchun (24) ga asosan

$$V_{tet} = \frac{1}{6} * \mathrm{mod}\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} \qquad \textbf{(25)}$$

bo'ladi.

Bu hajmni quyidagi formula yordamida ham topish mumkin:

$$V_{tet} = \frac{1}{6}\mathrm{mod}\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix} \qquad \textbf{(26)}$$

Misol 366. $\vec{AB} = \{2; 0; 0\}$, $\vec{AC} = \{3; 4; 0\}$, $\vec{AD} = \{3; 4; 2\}$ vektorlarga qurulgan tetraedr berilgan. Quyidagilarni toping:

1) tetraedrning hajmini;

2) ABC yoqning yuzuni;

3) D uchudan tushirilgan balandlikni;

4) AB va AC qirralar orasidagi burchak kosinusini;

5) ABC va ADC yoqlar orasidagi burchak kosinusini.

Yechilishi.

$$1)V_{tet}=\frac{1}{6}\mathrm{mod}\begin{vmatrix}2 & 0 & 0\\ 3 & 4 & 0\\ 3 & 4 & 2\end{vmatrix}=\frac{1}{6}*16=\frac{8}{3}=2\frac{2}{3}\quad kub\,birlik.$$

$$2)\ S_{\Delta}=\frac{1}{2}\left\|\left[\vec{AB}*\vec{AC}\right]\right\|=\frac{1}{2}\sqrt{\begin{vmatrix}0 & 0\\ 4 & 0\end{vmatrix}^2+\begin{vmatrix}0 & 2\\ 0 & 3\end{vmatrix}^2+\begin{vmatrix}2 & 0\\ 3 & 4\end{vmatrix}^2}=\frac{1}{2}\sqrt{8^2}=4\ \ kv.birlik.$$

$$3)\ V_{tet}=\frac{1}{3}S_{asos}*H\Rightarrow\frac{8}{3}=\frac{1}{3}*4*H\Rightarrow H=2\ \ uz.birlik.$$

$$4)\ cos(\vec{AB}\,\hat{,}\,\vec{AC})=\frac{\vec{AB}*\vec{AC}}{\left|\vec{AB}\right|*\left|\vec{AC}\right|}=\frac{3}{5}.$$

5) ABC va ADC yoqlar orasidagi burchak, ularning normal vektorlari orasidagi burchakka teng bo'ladi.

ABC yoqqa perpendikulyar vektor

$$\vec{P_1}=\left[\vec{AB}*\vec{AC}\right]=\left\{\begin{vmatrix}0 & 0\\ 4 & 0\end{vmatrix};\begin{vmatrix}0 & 2\\ 0 & 3\end{vmatrix};\begin{vmatrix}2 & 0\\ 3 & 4\end{vmatrix}\right\}=\{0;0;8\}.$$

ADC yoqqa perpendikulyar vektor

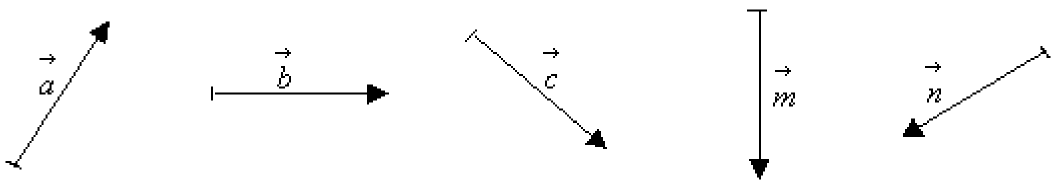
$$\vec{P_2}=\left[\vec{AC}*\vec{AD}\right]=\left\{\begin{vmatrix}4 & 0\\ 4 & 2\end{vmatrix};\begin{vmatrix}0 & 3\\ 2 & 3\end{vmatrix};\begin{vmatrix}3 & 4\\ 3 & 4\end{vmatrix}\right\}=\{8;-6;0\}.$$

U holda

$$\cos(\vec{P_1}\,\hat{,}\,\vec{P_2})=\frac{\vec{P_1}*\vec{P_2}}{\left|\vec{P_1}\right|*\left|\vec{P_2}\right|}=\frac{0*8+0*(-6)+8*0}{\sqrt{8^2}*\sqrt{8^2+(-6)^2}}=0\Rightarrow(\vec{P_1}\perp\vec{P_2}).$$

Demak, yoqlar perpendikulyar ekan.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR
Quyidagi vektorlar ustida talab etilgan amallarni bajaring



367. $\vec{a}+\vec{b}=?$ 368. $\vec{m}+\vec{n}=?$ 369. $\vec{a}+\vec{b}+\vec{c}+\vec{m}+\vec{n}=?$

370. $\vec{a}-\vec{b}=?$ 371. $\vec{b}-\vec{a}=?$ 372. $\vec{m}-\vec{n}=?$

373. $\vec{n}-\vec{m}=?$ 374. $3\vec{c}=?$ 375. $-4\vec{c}=?$

376. $\frac{1}{2}\vec{c}=?$ 377. $-\frac{1}{2}\vec{c}=?$ 378. $\vec{a}*\vec{b}=?$

379. $|\vec{a}|=3,\ |\vec{b}|=4,\ (\vec{a}\,\hat{,}\,\vec{b})=60^0$. λ ning qanday qiymatida $(\vec{a}-\lambda\vec{b})\perp\vec{a}$ bo'ladi ?

Javobi: 1,5.

380. $\vec{a}$ va $\vec{b}$ nokollinear vektorlar berilgan. $|\vec{a}|=|\vec{b}|=3$ bo'lsa, $(\vec{a}+\vec{b})$ bilan $(\vec{a}-\vec{b})$ vektorlar orasidagi burchakni toping.

Javobi: 90^0.

381. Agar $\vec{c}-2\vec{b}$ va $4\vec{b}+5\vec{c}$ vektorlar perpendikulyar bo'lsa, $\vec{b}$ va $\vec{c}$ birlik vektorlar orasidagi burchakni toping.

Javobi: 120^0.

382. Boshi va oxiri quyidagi nuqtalarda bo'lgan vektorlarni koordinatalari bilan yozing, yasang, ortlar bo'yicha yoying va uzunliklarini toping:

356. A(2;4;7) va B(1;2;4); 357. C(-1;2;3) va D(1;3;5);
358. E(3;5;7) va F(-2;-1;2); 359. M(2;-3;4) va N(-2;-3;1).

383. $\vec{b}=\{0;-2\}$ va $\vec{c}=\{-3;4\}$ vektorlar berilgan. $\vec{a}=3\vec{b}-2\vec{c}$ vektorning koordinatalarini toping.

Javobi: $\{6;-14\}$.

384. $\vec{a}=\{2;-3\}$ va $\vec{b}=\{-2;-3\}$ vektorlar berilgan. $\vec{m}=\vec{a}-2\vec{b}$ vektorning koordinatalarini toping.

Javobi: {6;3}.

385. $\vec{a}=\{4;1\}$ va $\vec{b}=\{-2;2\}$ vektorlar berilgan. Agar $\vec{a}=\vec{c}+3\vec{b}$ bo'lsa, $\vec{c}$ vektorning koordinatalarini toping.

Javobi: {10; -5}.

386. $\vec{a}=\{1;\frac{4}{3}\}$ vektor berilgan $3\vec{a}$ vektorning modulini toping.

Javobi: 5.

387. Agar $\vec{a}=\{6;\ 2;\ 1\}$ va $\vec{b}=\{0;-1;\ 2\}$ bo'lsa, $\vec{c}=2\vec{a}-\vec{b}$ vektorning uzunligini toping.

Javobi: 13.

Quyidagi vektorlarning skalyar ko'paytmasini toping:

388. $\vec{a}=\{2;-3;4\}$ va $\vec{b}=\{-2;-3;1\}$; Javobi: 9.

389. $\vec{m}=\{-1;5;3\}$ va $\vec{n}=\{2;-2;4\}$; Javobi: 0.

390. $\vec{e}=\{0;-4;2\}$ va $\vec{k}=\{-2;2;3\}$; Javobi: -2.

Quyidagi vektorlar orasidagi burchakni toping:

391. $\vec{a}=\{2;5\}$ va $\vec{b}=\{-7;-3\}$; Javobi: 135^0.

392. $\vec{c}=\{1;0\}$ va $\vec{d}=\{1;-1\}$; Javobi: 45^0.

393. $\vec{m}=\{5;-3\}$ va $\vec{n}=\{4;1\}$; Javobi: 45^0.

394. Agar M(1;1), N(2;3) va K(-1;2) bo'lsa, MNK uchburchakni yasang va eng katta burchagini toping.

Javobi: 90^0.

395. N(2;0;4) nuqta $\vec{c}=\{1;-2;3\}$ vektorning oxiri bo’lsa, bu vektor boshining koordinatalarini toping.
Javobi: (1;2;1)

396. A(0;y;0) nuqta M(1;2;3) va N(-1;3;4) nuqtalardan teng uzoqlikda bo’lsa, y ni toping.
Javobi: 6.

397. ***x*** ning qanday qiymatlarida $\vec{a}=\{2;x;x\}$ va $\vec{b}=\{2;5;x\}$ vektorlar o’zaro perpendikulyar bo’ladi? Javobi: -1;-4.

398. ***m*** ning qanday qiymatida $\vec{a}=\{m;-2;4\}$ va $\vec{b}=\{m;4m;4\}$ vektorlar perpendikulyar bo’ladi. *Javobi*: 4.

399. ***x*** ning qanday qiymatlarida $\vec{a}=\{8;4;5x\}$ va $\vec{b}=\{2x;x;1\}$ vektorlar o’zaro perpendikulyar bo’ladi? Javobi: 0.

Yo’naltiruvchi kosinuslardan foydalanib, quyidagi vektorlarning birlik vektorlarini toping:

400. $\vec{a}=\{2;-3;4\}$; Javobi: $\{\frac{2}{\sqrt{29}};-\frac{3}{\sqrt{29}};\frac{4}{\sqrt{29}}\}$;

401. $\vec{b}=\{-2;-3;1\}$; Javobi: $\{-\frac{2}{\sqrt{14}};-\frac{3}{\sqrt{14}};\frac{1}{\sqrt{14}}\}$;

402. $\vec{m}=\{-1;5;3\}$; Javobi: $\{-\frac{1}{\sqrt{35}};\frac{5}{\sqrt{35}};\frac{3}{\sqrt{35}}\}$;

403. $\vec{n}=\{2;-2;4\}$; Javobi: $\{\frac{1}{\sqrt{6}};-\frac{1}{\sqrt{6}};\frac{2}{\sqrt{6}}\}$; 404. $\vec{e}=\{0;-4;2\}$; Javobi: $\{0;-\frac{2}{\sqrt{5}};\frac{1}{\sqrt{5}}\}$;

405. $\vec{k}=\{-2;0;4\}$; Javobi: $\{-\frac{3}{5};0;\frac{4}{5}\}$;

Quyidagi vektorlarning talab etilgan proeksiyalarini toping:

$$\vec{a}=\{3;0;4\},\ \vec{b}=\{0;4;3\},\ \vec{m}=\{-1;5;3\},\ \vec{n}=\{2;-2;4\}.$$

406. $np_{\vec{a}}\vec{b}=?$ Javobi: 2,4.

407. $np_{\vec{b}}\vec{a}=?$ Javobi: 2,4.

408. $np_{\vec{m}}\vec{n}=?$ Javobi: 0.

409. $np_{\vec{n}}\vec{m}=?$ Javobi: 0.

12-MAVZU. TEKISLIK

Tekislik matematikaning ta’riflanmaydigan asosiy tushunchalaridan biri.

1. $\Pi: Ax+By+Cz+D=0$ - tekislikning umumiy tenglamasi .

$\vec{n}=\{A;B;C\}$ - tekislikning normal vektori.

$x,\ y,\ z$ - tekislikda yotgan ixtiyoriy nuqtaning koordinatalari.

$\mu=\frac{1}{\pm\sqrt{A^2+B^2+C^2}}$ - normallovchi.

Normallovchining ishorasi ozod son D ning ishorasiga qarama-qarshi olinadi.

$$\frac{A}{\pm\sqrt{A^2+B^2+C^2}}*x+\frac{B}{\pm\sqrt{A^2+B^2+C^2}}*y+\frac{C}{\pm\sqrt{A^2+B^2+C^2}}*z+\frac{D}{\pm\sqrt{A^2+B^2+C^2}}=0$$

- tekislikning normal tenglamasi.

Misol 410.

2x-9y+6z-22=0 tekislikning normal vektorini, normallovchisini yozing va normal shaklga keltiring.

Yechilishi. ***A=2, B = - 9, C = 6, D = - 22 < 0;*** $\vec{n}=\{A;B;C\}=\{2;-9;6\};$

$$\mu=\frac{1}{\pm\sqrt{A^2+B^2+C^2}}=\frac{1}{\sqrt{2^2+(-9)^2+6^2}}=\frac{1}{\sqrt{4+81+36}}=\frac{1}{\sqrt{121}}=\frac{1}{11};$$

$$\frac{2}{11}x-\frac{9}{11}y+\frac{6}{11}z-2=0.$$

2. Berilgan nuqtaning koordinatalari tekislik tenglamasiga qo'yilganda tenglik buzilmasdan saqlansa, nuqta tekislikda yotadi va aksincha.

Misol 412. A(3;-5;-2) nuqtaning 22x+14y-20z-36=0 tekislikda yotishi yoki yotmasligini tekshiring.

Yechilishi. ***x=3, y=-5, z=-2.***

22*3+14*(-5)-20*(-2)-36=0; $\Rightarrow$ ***66-70+40-36=0;*** $\Rightarrow$ ***0=0*** bo'ladi.

Demak, A(3;-5;-2) nuqta 22x+14y-20z-36=0 tekislikda yotadi.

3. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ - tekislikning koordinata o'qlaridan kesgan kesmalari bo'yicha tenglamasi.

Misol 413.

2x-3y-z+12=0 tekislikning koordinata o'qlaridan kesgan kesmalari bo'yicha tenglamasini tuzing va tekislikni yasang.

Yechilishi.

Tenglamadagi ikkita koordinata nolga tenglanib, tekislikning uchinchi koordinata o'qini kesishishidan hosil bo'lgan kesmaning uzunligi aniqlanadi.

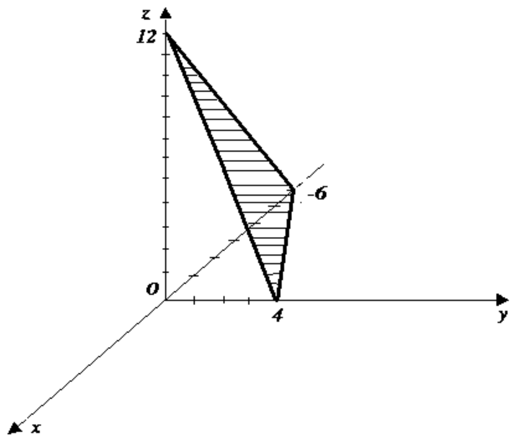
1) $y=z=0 \Rightarrow 2*x-3*0-0+12=0 \Rightarrow 2x=-12 \Rightarrow x=\frac{-12}{2} \Rightarrow x=-6 \Rightarrow a=-6;$

2) $x=z=0 \Rightarrow -3y+12=0 \Rightarrow 3y=12 \Rightarrow y=4 \Rightarrow b=4;$

3) $x=y=0 \Rightarrow -z+12=0 \Rightarrow z=12 \Rightarrow c=12.$

Bulardan $\frac{x}{-6}+\frac{y}{4}+\frac{z}{12}=1$ hosil bo'ladi.

Buni 2x-3y-z=-12 tenglamaning ikkala tamonini –12 ga bo'lish orqali ham hosil qilish mumkin.



4. $M_0(x_0,y_0,z_0)$ nuqtadan Π tekislikkacha bo'lgan masofani topish formulasi:

$$\rho(M_0;\Pi)=\left|\frac{Ax_0+By_0+Cz_0+D}{\pm\sqrt{A^2+B^2+C^2}}\right|.$$

Misol 414. M (4;1;2) nuqtadan П: 2x-9y+6z-22=0 tekislikkacha bo'lgan masofani toping.

Yechilishi. A=2; B=-9; C=6; D=-22<0; $x_0 = 4;\ y_0 = 1;\ z_0 = 2.$

$$\rho(M;\Pi) = \left|\frac{2\cdot 4 - 9\cdot 1 + 6\cdot 2 - 22}{\sqrt{2^2 + (-9)^2 + 6^2}}\right| = \left|\frac{8 - 9 + 12 - 22}{\sqrt{4 + 81 + 36}}\right| = \left|\frac{20 - 31}{\sqrt{121}}\right| = \left|\frac{-11}{11}\right| = 1\,uz.birlik.$$

5. $\Pi_1 : A_1x + B_1y + C_1z + D_1 = 0;\ \vec{n}_1 = \{A_1; B_1; C_1\};$

$$\Pi_2 : A_2x + B_2y + C_2z + D_2 = 0;\ \vec{n}_2 = \{A_2; B_2; C_2\}.$$

$\dfrac{A_1}{A_2} = \dfrac{B_1}{B_2} = \dfrac{C_1}{C_2}$ - ikkita Π_1 va Π_2 tekisliklarning parallellik sharti.

$A_1A_2 + B_1B_2 + C_1C_2 = 0$ - ikkita Π_1 va Π_2 tekisliklarning perpendikulyarlik sharti.

Ikkita Π_1 va Π_2 tekisliklar orasidagi burchak, ularning normal vektorlari orasidagi burchakka teng:

$$\cos(\Pi_1 \hat{;} \Pi_2) = \cos(\vec{n}_1 \hat{;} \vec{n}_2) = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \cdot |\vec{n}_2|} = \pm\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \cdot \sqrt{A_2^2 + B_2^2 + C_2^2}} = \cos\alpha.$$

Misol 415. 4x-5y+3z-1=0 va x-4y-z+9=0 tekisliklar orasidagi burchakni toping.

Yechilishi.

$$A_1 = 4;\ B_1 = -5;\ C_1 = 3;\ D_1 = -1 < 0;\qquad \vec{n}_1 = \{4;\ -5;\ 3\};$$

$$A_2 = 1;\ B_2 = -4;\ C_2 = -1;\ D_2 = 9 > 0;\qquad \vec{n}_2 = \{1;\ -4;\ -1\};$$

$$\cos\alpha = \pm\frac{4\cdot 1 + (-5)\cdot(-4) + 3\cdot(-1)}{\sqrt{4^2 + (-5)^2 + 3^2}\cdot\sqrt{1^2 + (-4)^2 + (-1)^2}} = \pm\frac{4 + 20 - 3}{\sqrt{16 + 25 + 9}\cdot\sqrt{1 + 16 + 1}} =$$

$$= \pm\frac{21}{\sqrt{50}\cdot\sqrt{18}} = \pm\frac{21}{\sqrt{50\cdot 18}} = \pm\frac{21}{\sqrt{25\cdot 2\cdot 9\cdot 2}} = \pm\frac{21}{5\cdot 2\cdot 3} = \pm\frac{7}{10}.\qquad \left\{\begin{array}{l}\alpha = \pi - ars\cos\frac{7}{10};\\ \alpha = ars\cos\frac{7}{10}.\end{array}\right.$$

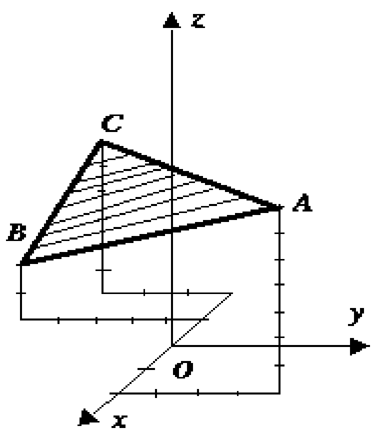
6. Uchta $A(x_1; y_1; z_1),\ B(x_2; y_2; z_2), C(x_3; y_3; z_3)$ nuqtalardan o'tuvchi tekislik tenglamasi quyidagi ko'rinishda bo'ladi:

$$\begin{vmatrix} x & y & z & 1\\ x_1 & y_1 & z_1 & 1\\ x_2 & y_2 & z_2 & 1\\ x_3 & y_3 & z_3 & 1\end{vmatrix} = 0.$$

Misol 416.

Uchta A(2;4;7), B(-1;-5;2), C(-2;-3;6) nuqtalardan o'tuvchi tekislikni yasang va tenglamasini tuzing.

Yechilishi.



$$\begin{vmatrix} x & y & z & 1 \\ 2 & 4 & 7 & 1 \\ -1 & -5 & 2 & 1 \\ -2 & -3 & 6 & 1 \end{vmatrix} = 0 \quad .$$

Bu determinantni minorlarga yoyish orqali yechamiz.

$$(-1)^{1+1}\cdot x\cdot\begin{vmatrix} 4 & 7 & 1 \\ -5 & 2 & 1 \\ -3 & 6 & 1 \end{vmatrix} + (-1)^{1+2}\cdot y\cdot\begin{vmatrix} 2 & 7 & 1 \\ -1 & 2 & 1 \\ -2 & 6 & 1 \end{vmatrix} + (-1)^{1+3}\cdot z\cdot\begin{vmatrix} 2 & 4 & 1 \\ -1 & -5 & 1 \\ -2 & -3 & 1 \end{vmatrix} +$$

$$+(-1)^{1+4}\cdot\begin{vmatrix} 2 & 4 & 7 \\ -1 & -5 & 2 \\ -2 & -3 & 6 \end{vmatrix} = 0 \ ;$$

$x \cdot (8 - 21 - 30 + 6 + 35 - 24) - y \cdot (4 - 14 - 6 + 4 + 7 - 12) +$
$+z \cdot (-10 - 8 + 3 - 10 + 4 + 6) - (-60 - 16 + 21 - 70 + 24 + 12) = 0; \Rightarrow$
$\Rightarrow -26x + 17y - 15z + 89 = 0; \Rightarrow 26x - 17y + 15z - 89 = 0.$

Uchta tekislikning kesishish nuqtasini topish uchun ularning tenglamalarini sistema qilib birgalikda yechish kerak.

Uchlari to'rtta $A(x_1;y_1;z_1),\ B(x_2;y_2;z_2), C(x_3;y_3;z_3), D(x_4;y_4;z_4)$ nuqtalarda bo'lgan tetraedrning hajmi

$$V = \frac{1}{6}\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}$$

formula yordamida topiladi.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi tekisliklarning tenglamalarini normal shaklga keltiring:

417. ***10x+2y-11z+60=0;*** **418.** ***6x-6y-7z+33=0;*** **419**. ***2x+3y-7z+1=0;***
420. ***2x-3y-z-12=0***; **421.** ***x-y+z-1=0***.
422. ***4x-y+3z+1=0*** tekislikning ***A(-1;6;3), B(3;-2;-5), C(0;4;1), D(2;0;5), E(2;7;0), F(0;1;0)*** nuqtalardan o'tishi yoki o'tmasligini tekshiring.

Quyidagi tekisliklarning koordinata o'qlaridan kesgan kesmalari bo'yicha tenglamalarini tuzing va ularni yasang:

423. ***5x+y-3z-15=0;*** **424**. ***x-y+z-1=0;*** **425**. ***x-4z+6=0***;
426. ***5x-2y+z=0;*** **427**. ***x-7=0;*** **428**. y+***5=0;***
429. ***z-4=0.***

Quyidagi nuqtalardan tekislikkacha bo'lgan masofalarni aniqlang:

430. ***O(0;0;0)*** nuqtadan ***2x-9y+6z-22=0*** tekislikkacha. *Javobi*: 2 uzunlik birlik.
431. ***A(3;1;-1)*** nuqtadan ***22x+4y-20z-45=0*** tekislikkacha. *Javobi*: 1,5 uzunlik birlik
432. ***B(4;3;-2)*** nuqtadan ***3x-y+5z+1=0*** tekislikkacha. *Javobi*: 0 (nuqta tekislikda yotadi).

433. $C(2;0;-\frac{1}{2})$ nuqtadan ***4x-4y+2z+17=0*** tekislikkacha. *Javobi*: 4 uzunlik birlik.

434. ***D(2;0;0)*** nuqtadan ***2x-6y+3z-42=0*** tekislikkacha.*Javobi*: $5\frac{3}{7}$ uzunlik birlik.

435. ***E(5; 1 ; -1)*** nuqtadan ***x-2y-2z+4=0*** tekislikkacha. *Javobi*: 3 uzunlik birlik.

Quyidagi tekisliklar orasidagi burchakni toping:

436. ***4x-5y+3z-1=0*** va ***x+4y-z+9=0;*** Javobi: $\alpha = \pi - \arccos\frac{19}{30}$..

437. ***3x-y+2z+15=0*** va ***5x+9y-3z-1=0;*** Javobi: $\alpha = 90^0$.

438. ***6x+2y-4z+17=0*** va ***9x+3y-6z-1=0;*** Javobi: $\alpha = \arccos\frac{1}{7}$.

439. ***x-2y+2z-8=0*** va ***x+z-6=0;*** Javobi: $\alpha = 45^0$.

440. ***x+2z-6=0*** va ***x+2y-4=0;*** Javobi: $\alpha = \arccos\frac{1}{5}$.

Quyidagi uchta nuqtadan o'tuvchi tekislikni yasang va tenglamasini tuzing:

441. ***O(0;0;0), A(2; 3;-3), B(-1;2;3);*** Javobi: ***15x-3y+7z=0.***
442. ***C(1;-3;-2), D(2;3;-4), E(-1;-3;3);*** Javobi: ***30x-y+12z-24=0***.
443. ***M(-3;2;-4), N(1;-2;3), K(0;3;-2);*** Javobi: ***-15x+13y+16z-7=0***.

Quyidagi uchta tekislikning kesishish nuqtasini toping:

444. ***5x+8y-z-7=0; x+2y+3z-1=0; 2x-3y+2z-9=0;*** *Javobi*: (3;-1;0).
445. ***x-4y-2z+3=0; 3x+y+z-5=0; -3x+12y+6z-7=0.*** *Javobi*: tekisliklar kesishmaydi.
446. ***2x-y+5z-4=0; 5x+2y-13z+23=0; 3x-z+5=0.*** *Javobi*: tekisliklar kesishmaydi.

Uchlari quyidagi to'rtta nuqtada bo'lgan tetraedrni yasang va hajmini toping:

447. ***A(0;0;2), B(3;0;5), C(1;1;0), D(4;1;2).*** *Javobi:* $V = \frac{1}{6}(kub.birlik)$.

448. ***S(0;6;4), A(3;5;3), B(-2;11;-5), C(1;-1;4).*** *Javobi*: $V = 31{,}5\,(kub.birlik)$.

449. ***M(0;0;2), N(4;0;5), K(5;3;0), F(-1;4;-2).*** *Javobi*: $V = 8\frac{5}{6}(kub.birlik)$.

13-MAVZU. FAZODA TO'G'RI CHIZIQ

1. Fazodagi to'g'ri chiziqni ikkita tekislikning kesishish chizig'i deb qarash mumkin:

$$\begin{cases} A_1x + B_1y + C_1z + D_1 = 0; \\ A_2x + B_2y + C_2z + D_2 = 0. \end{cases} \quad (1)$$

Sistemadagi *x* va y lar z bilan bo'glab yoziladi.

Misol 450.

$\begin{cases} 6x + 2y - z - 9 = 0, \\ 3x + 2y + 2z - 12 = 0 \end{cases}$ to'g'ri chiziqning *(x0z)* va *(y0z)* tekisliklardagi proeksiyalari tenglamalari tuzulsin.

Yechilishi: Izlanayotgan tenglamalarni hosil qilish uchun berilgan sistemadagi *x* va *y* ni *z* bilan bog'lab yozish kifoya:

1) $$\begin{array}{r} -\ 6x+2y-z-9=0; \\ 3x+2y+2z-12=0. \\ \hline \end{array}$$

$$6x+2y-z-9-3x-2y-2z+12=0;$$

$$3x-3z+3=0;$$

$$3x=3z-3 \Rightarrow x=z-1;$$

2) $$\begin{array}{r} -\ 6x+2y-z-9=0; \\ 6x+4y+4z-24=0. \\ \hline \end{array}$$

$$6x+2y-z-9-6x-4y-4z+24=0;$$

$$-2y-5z+15=0;$$

$$2y=-5z+15; \quad y=\frac{-5z+15}{2}.$$

x=z-1 ni sistema ko'rinishida berilgan to'g'ri chiziqning *(x0z)* tekislikdagi, $y=\frac{-5z+15}{2}$ ni esa *(y0z)* tekislikdagi izi (proeksiyasi) tenglamasi deyiladi.

2. Fazodagi to'g'ri chiziqning kanonik tenglamasi:

$$\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n} \qquad \textbf{(2)}$$

Bu to'g'ri chiziq (x_0, y_0, z_0) nuqtadan o'tadi va uning yo'naltiruvchi vektori $\vec{u}=\{l;m;n\}$ dan iborat.

Misol 451.

$$\begin{cases} 6x+2y-z-9=0; \\ 3x+2y+2z-12=0 \end{cases}$$

to'g'ri chiziq tenglamasini kanonik shaklda yozing.

Yechilishi. Sistemadan x va y topiladi:

$x=z-1$ va $y=\frac{-5z+15}{2}$; bu tenglamalardan z topiladi:

1) $z=x+1$;

2) $2y=-5z+15 \Leftrightarrow 5z=-2y+15 \Leftrightarrow z=\frac{-2y+15}{5} \Leftrightarrow z=\frac{y-\frac{15}{2}}{-\frac{5}{2}}$;

1) va 2) dan foydalanib to'g'ri chiziqning kanonik tenglamasi quyidagicha yoziladi:

$$\frac{x+1}{1}=\frac{y-\frac{15}{2}}{-\frac{5}{2}}=\frac{z}{1}.$$

Bunda

$$x_0=-1; \quad y_0=\frac{15}{2}; \quad z_0=0; l=1; \quad m=-\frac{5}{2}; \quad n=1.$$

Misol 452. *2x-5y+z-3=0; 3x-2y+3z-6=0* tenglamalari bilan berilgan to'g'ri chiziqni kanonik ko'rinishga keltiring.

Yechilishi. Berilgan tenglamalardan navbat bilan x va y ni yo'qotamiz:

1) $\begin{cases} 2x-5y+z-3=0 \\ 3x-2y+3z-6=0 \end{cases}\begin{vmatrix} 3 \\ 2 \end{vmatrix} \Leftrightarrow - \begin{array}{l} 6x-15y+3z-9=0 \\ \\ 6x-4y+6z-12=0 \end{array}$

$$\overline{6x-15y+3z-9-6x+4y-6z+12=0}$$
$-11y-3z+3=0;$

2) $\begin{cases} 2x-5y+z-3=0 \\ 3x-2y+3z-6=0 \end{cases}\begin{vmatrix} 2 \\ 5 \end{vmatrix} \Leftrightarrow \quad - \begin{array}{l} 4x-10y+2z-6=0 \\ \underline{15x-10y+15z-30=0} \end{array}$

$4x-10y+2z-6-15x+10y-15z+30=0;$

$-11x-13z+24=0.$

Hosil bo'lgan $-11y-3z+3=0$ va $-11x-13z+24=0$ tenglamalardan z ni topiladi:

$$3z=-11y+3 \Leftrightarrow z=\frac{-11y+3}{3};$$

$$13z=-11x+24 \Leftrightarrow z=\frac{-11x+24}{13}.$$

Bu tenglamalarning ikkala tomoni x va y oldidagi koeffitsent -11 ga bo'linadi:

$\frac{z}{-11}=\frac{y-\frac{3}{11}}{3}=\frac{x-\frac{24}{11}}{13}$ yoki $\frac{x-\frac{24}{11}}{13}=\frac{y-\frac{3}{11}}{3}=\frac{z}{-11}$ kanonik tenglama hosil bo'ladi.

Bu to'g'ri chiziq $\left(\frac{24}{11},\frac{3}{11},0\right)$ nuqtadan o'tadi va uning yo'naltiruvchi vektori $\vec{u}=\{13;3;-11\}$ dan iborat.

3. Ikki nuqtadan o'tuvchi to'g'ri chiziqning tenglamasi:

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1} \qquad (3)$$

Misol 453. *A(1; 2; 3)* va *B(-1; -2; -3)* nuqtalardan o'tuvchi to'g'ri chiziqning tenglamasini tuzing.

Yechilishi.

$x_1=1; \quad y_1=2; \quad z_1=3;$

$x_2=-1; \quad y_2=-2; \quad z_2=-3.$

U holda

$\frac{x-1}{-1-1}=\frac{y-2}{-2-2}=\frac{z-3}{-3-3}$ yoki $\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$ yoki

$\frac{x-1}{2}=\frac{y-2}{4}=\frac{z-3}{6}$

bo'ladi.

4. Ikki to'g'ri chiziq orasidagi burchak

$u_1: \frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1};$ va $\boldsymbol{u_2: \frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2};}$

$\vec{u_1}=\{l_1; \quad m_1; \quad n_1\};$ $\quad \boldsymbol{\vec{u_2}=\{l_2; \quad m_2; \quad n_2\}.}$

$$\cos\left(\widehat{u_1; u_2}\right) = \cos\left(\vec{u}_1 \wedge \vec{u}_2\right) = \frac{\vec{u}_1 \cdot \vec{u}_2}{\left|\vec{u}_1\right| \cdot \left|\vec{u}_2\right|} = \frac{l_1 \cdot l_2 + m_1 \cdot m_2 + n_1 \cdot n_2}{\sqrt{l_1^2 + m_1^2 + n_1^2} \cdot \sqrt{l_2^2 + m_2^2 + n_2^2}} = \cos\alpha \qquad (4)$$

Misol 454. $\frac{x-1}{1} = \frac{y}{-2} = \frac{z+4}{3}$ va $\frac{x+6}{3} = \frac{y-2}{2} = \frac{z-3}{-1}$ to'g'ri chiziqlar orasidagi burchakni toping.

Yechilishi.

$$\boldsymbol{l_1 = 1;\ m_1 = -2;\ n_1 = 3;}$$

$$\boldsymbol{l_2 = 3;\ m_2 = 2;\ n_2 = -1;}$$

$$\cos\alpha = \frac{1\cdot 3 + (-2)\cdot 2 + 3\cdot(-1)}{\sqrt{1^2 + (-2)^2 + 3^2}\cdot\sqrt{3^2 + 2^2 + (-1)^2}} = \frac{3-4-3}{\sqrt{1+4+9}\cdot\sqrt{9+4+1}} = \frac{-4}{\sqrt{14}\cdot\sqrt{14}} = -\frac{4}{14} = -\frac{2}{7};$$

$$\alpha = arc\cos\left(-\frac{2}{7}\right) = \pi - arc\cos\frac{2}{7}.$$

5. Ikkita u_1 va u_2 to'g'ri chiziqlarning parallellik sharti:

$$\boldsymbol{u_1 \| u_2 \Leftrightarrow \vec{u}_1 \| \vec{u}_2 \Leftrightarrow \frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}.} \qquad (5)$$

Misol 455. $\frac{x-3}{4} = \frac{y-2}{3} = \frac{z-5}{2}$ va $\frac{x+2}{8} = \frac{y-4}{6} = \frac{z-3}{4}$ to'g'ri chiziqlarning parallelligini ko'rsating.

Yechilishi.

$$\vec{u}_1 = \{l_1; m_1; n_1\} = \{4;3;2\}; \qquad \vec{u}_2 = \{l_2; m_2; n_2\} = \{8;6;4\}.$$

$$\frac{4}{8} = \frac{3}{6} = \frac{2}{4} \Leftrightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2}.$$

6. Ikkita u_1 va u_2 to'g'ri chiziqlarning perpendikulyarlik sharti:

$$\boldsymbol{u_1 \perp u_2 \Leftrightarrow \vec{u}_1 \perp \vec{u}_2 \Leftrightarrow \vec{u}_1 * \vec{u}_2 = 0 \Leftrightarrow l_1 l_2 + m_1 m_2 + n_1 n_2 = 0.} \qquad (6)$$

Misol 456. $\frac{x-2}{1} = \frac{y-2}{2} = \frac{z-1}{-2}$ va $\frac{x-15}{4} = \frac{y-9}{1} = \frac{z-11}{3}$ to'g'ri chiziqlarning perpendikulyarligini ko'rsating.

Yechilishi.

$$\vec{u}_1 * \vec{u}_2 = 0 \Leftrightarrow 1*4 + 2*1 + (-2)*3 = 4 + 2 - 6 = 0 \Rightarrow u_1 \perp u_2.$$

7**. Ikki to'g'ri chiziqning kesishish sharti:**

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0 \qquad (7)$$

Misol 457. $\frac{x-1}{2}=\frac{y+2}{1}=\frac{z-2}{-4}$ va $\frac{x-9}{6}=\frac{y-2}{3}=\frac{z+1}{1}$ to'g'ri chiziqlarning kesishishini tekshiring va kesishish huqtasini toping.

Yechilishi.

$$\begin{vmatrix} \mathbf{9-1} & \mathbf{2-(-2)} & \mathbf{1-(-2)} \\ \mathbf{2} & \mathbf{1} & \mathbf{-4} \\ \mathbf{6} & \mathbf{3} & \mathbf{1} \end{vmatrix} = \begin{vmatrix} \mathbf{8} & \mathbf{4} & \mathbf{3} \\ \mathbf{2} & \mathbf{1} & \mathbf{-4} \\ \mathbf{6} & \mathbf{3} & \mathbf{1} \end{vmatrix} = \mathbf{8\cdot 1\cdot 1+4\cdot(-4)\cdot 6+2\cdot 3\cdot 3-3\cdot 1\cdot 6-}$$

$$\mathbf{-4\cdot 2\cdot 1-(-4)\cdot 3\cdot 8=8-96+18-18-8+96=0.}$$

Demak, to'g'ri chiziqlar kesishadi.

Berilgan tenglamalar quyidagicha yoziladi:

$$\begin{cases} y+2=\dfrac{x-1}{2} \Leftrightarrow y=\dfrac{x}{2}-\dfrac{1}{2}-2 \Rightarrow y=\dfrac{x}{2}-\dfrac{5}{2}; \\ y+2=\dfrac{z-2}{-4} \Leftrightarrow y=-\dfrac{z}{4}+\dfrac{1}{2}-2 \Rightarrow y=-\dfrac{z}{2}-\dfrac{3}{2}; \\ z+1=\dfrac{x-9}{6} \Leftrightarrow z=\dfrac{x}{6}-\dfrac{3}{2}-1 \Rightarrow z=\dfrac{x}{6}-\dfrac{5}{2}; \\ z+1=\dfrac{y-2}{3} \Leftrightarrow z=\dfrac{y}{3}-\dfrac{2}{3}-1 \Rightarrow z=\dfrac{y}{3}-\dfrac{5}{3}. \end{cases}$$

Dastlabki uchta tenglama sistema qilinib birgalikda yechiladi:

$$\begin{cases} \boldsymbol{y}=\dfrac{\boldsymbol{x}}{2}-\dfrac{5}{2} \\ \boldsymbol{y}=-\dfrac{\boldsymbol{z}}{4}-\dfrac{3}{2} \\ \boldsymbol{z}=\dfrac{\boldsymbol{x}}{6}-\dfrac{5}{2} \end{cases} \Rightarrow \begin{cases} \boldsymbol{y}=\boldsymbol{y} \\ \dfrac{\boldsymbol{x}}{2}-\dfrac{5}{2}=-\dfrac{\boldsymbol{z}}{4}-\dfrac{3}{2} \quad /2 \\ \boldsymbol{z}=\dfrac{\boldsymbol{x}}{6}-\dfrac{5}{2} \end{cases} \Rightarrow \begin{cases} \boldsymbol{x}-5=-\dfrac{\boldsymbol{z}}{2}-3 \\ \boldsymbol{z}=\dfrac{\boldsymbol{x}}{6}-\dfrac{5}{2} \end{cases} \Rightarrow \begin{cases} 2\boldsymbol{x}-10=-\boldsymbol{z}-6 \\ \boldsymbol{z}=\dfrac{\boldsymbol{x}}{6}-\dfrac{5}{2} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} z=-2x+10-6 \\ z=\dfrac{x}{6}-\dfrac{5}{2} \end{cases} \Rightarrow \begin{cases} z=-2x+4 \\ -2x+4=\dfrac{x}{6}-\dfrac{5}{2} \end{cases} \Rightarrow \begin{cases} z=-2x+4 \\ -12x+24=x-15 \end{cases} \Rightarrow \begin{cases} z=-2x+4 \\ 13x=39 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} z=-2 \\ x=3 \end{cases} \Rightarrow y=\frac{x}{2}-\frac{5}{2}=\frac{3}{2}-\frac{5}{2}=\frac{3-5}{2}=-1$$

Demak, x=3, y=-1, z=-2. Bular to'rtinchi $\boldsymbol{z=\frac{y}{3}-\frac{5}{3}}$ tenglamaga qo'yilib tekshiriladi:

$$-2=\frac{-1}{3}-\frac{5}{3}=\frac{-1-5}{3}=\frac{-6}{3}=-2 \Rightarrow -2=-2.$$

Demak, *A(x; y; z)=A(3; -1; -2).*

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi to'g'ri chiziqlarning

1) *(x0z)* va *(y0z)* tekisliklardagi proeksiyalari tenglamalarini tuzing;
2) kanonik shaklga keltiring;
3) to'g'ri chiziq o'tadigan nuqtani toping;

4) to’g’ri chiziqning yo’naltiruvchi vektorini yozing.

458. $\begin{cases} 5x+8y-3z+9=0; \\ 2x-4y+z-1=0. \end{cases}$ $\quad Javobi: \frac{x+\frac{7}{9}}{\frac{1}{9}}=\frac{y+\frac{23}{36}}{\frac{11}{36}}=\frac{z}{1}.$

459. $\begin{cases} 3x+2y-z+5=0; \\ x-y-z+1=0. \end{cases}$ $\quad Javobi: \frac{x+7}{-3}=\frac{y-\frac{2}{5}}{5}=\frac{z}{-5}.$

460. $\begin{cases} x-4y+2z-5=0; \\ 3x+y-z+2=0 \quad . \end{cases}$ $\quad Javobi: \frac{x+\frac{3}{13}}{2}=\frac{y+\frac{17}{13}}{7}=\frac{z}{13}$

461. $\begin{cases} x-2y+z-5=0; \\ 3x+2y+z+3=0. \end{cases}$ $\quad Javobi: \frac{x-3}{-2}=\frac{y+\frac{15}{4}}{1}=\frac{z}{4}.$

Quyidagi ikki nuqtadan o’tuvchi to’g’ri chiziq tenglamasini tuzing:

462. A(3; 4;5) va B(-3; -4; -5); Javobi: $\frac{x-3}{6}=\frac{y-4}{8}=\frac{z-5}{10}$.

463. C(2; 3; 4) va D(-2; -3; -4); Javobi: $\frac{x-2}{4}=\frac{y-3}{6}=\frac{z-4}{8}$.

464. M(1; 4; 0) va N(4; 2; 3); Javobi: $\frac{x-1}{3}=\frac{y-4}{-2}=\frac{z}{3}.$

465. E(0; -2; 1) va F(1; 3; -2); Javobi: $\frac{x}{1}=\frac{y+2}{6}=\frac{z-1}{-3}.$

466. K(2; -1;0) va L(7; 1; 3). Javobi:. $\frac{x-2}{5}=\frac{y+1}{2}=\frac{z}{3}$

Quyidagi ikki to’g’ri chiziq orasidagi burchakni toping:

467. $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z+2}{5}$ va $\frac{x-1}{4}=\frac{y}{7}=\frac{z-2}{3};$ $\quad \boldsymbol{Javobi: arc\cos\frac{8}{\sqrt{555}}}$.

468. $\frac{x+2}{3}=\frac{y-5}{4}=\frac{z}{1}$ va $\frac{x-4}{5}=\frac{y+3}{2}=\frac{z}{1};$ $\quad \boldsymbol{Javobi: arc\cos\frac{12}{\sqrt{195}}}$.

469. $\frac{x-3}{5}=\frac{y+1}{2}=\frac{z-2}{4}$ va $\frac{x-5}{3}=\frac{y-1}{1}=\frac{z-6}{-2};$ $\quad \boldsymbol{Javobi: arc\cos\frac{3}{\sqrt{70}}}$.

470. $\frac{x-1}{3}=\frac{y+2}{6}=\frac{z-5}{2}$ va $\frac{x}{2}=\frac{y-3}{9}=\frac{z+1}{6};$ $\quad \boldsymbol{Javobi: arc\cos\frac{72}{71}}$.

471. $\frac{x-1}{2}=\frac{y-7}{1}=\frac{z-5}{4}$ va $\frac{x-6}{3}=\frac{y+1}{-2}=\frac{z}{1}.$ $\quad \boldsymbol{Javobi: arc\cos\frac{8}{7\sqrt{6}}}$.

Quyidagi ikki to’g’ri chiziqning parallelligini ko’rsating:

472. $\frac{x-9}{3}=\frac{y+1}{4}=\frac{z}{2}$ va $\frac{x-7}{3}=\frac{y-1}{4}=\frac{z-3}{2};$

473. $\frac{x+3}{4}=\frac{y-6}{-3}=\frac{z-3}{2}$ va $\frac{x-4}{8}=\frac{y+1}{-6}=\frac{z+7}{1}$;

474. $\frac{x}{7}=\frac{y+2}{3}=\frac{z-1}{5}$ va $\frac{x-1}{7}=\frac{y-3}{3}=\frac{z+2}{5}$.

Quyidagi ikki to'g'ri chiziqning kesishishini tekshiring va kesishish nuqtasini toping:

475. $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z+2}{5}$ va $\frac{x-1}{4}=\frac{y}{7}=\frac{z-2}{3}$; Javobi: Kesishmaydi.

476. $\frac{x+2}{3}=\frac{y-5}{4}=\frac{z}{1}$ va $\frac{x-4}{5}=\frac{y+3}{2}=\frac{z}{1}$; Javobi: Kesishmaydi.

477. $\frac{x-3}{5}=\frac{y+1}{2}=\frac{z-2}{4}$ va $\frac{x-5}{3}=\frac{y-1}{1}=\frac{z-6}{2}$; Javobi: A(23; 18; 7).

478. $\frac{x-1}{3}=\frac{y+2}{6}=\frac{z-5}{2}$ va $\frac{x}{2}=\frac{y-3}{9}=\frac{z+1}{6}$; Javobi: Kesishmaydi.

479. $\frac{x-1}{2}=\frac{y-7}{1}=\frac{z-5}{4}$ va $\frac{x-6}{3}=\frac{y+1}{-2}=\frac{z}{1}$. Javobi: A(-3; 5; -3).

14-MAVZU. IKKINCHI TARTIBLI SIRTLAR

$$S: a_{II}x^2+a_{22}y^2+a_{33}z^2+2a_{12}xy+2a_{13}xz+2a_{23}yz+2a_{14}x+2a_{24}y+2a_{34}z+a_{44}=0$$ **(1)**

\- ikkinchi tartibli sirtlarning umumiy tenglamasi.

$$\left(a_{II}x_0+a_{12}y_0+a_{13}z_0+a_{14}\right)\left(x-x_0\right)+\left(a_{21}x_0+a_{22}y_0+a_{23}z_0+a_{24}\right)\left(y-y_0\right)+$$
$$+\left(a_{31}x_0+a_{32}y_0+a_{33}z_0+a_{34}\right)\left(z-z_0\right)=0$$ **(2)**

\- sirtning $M_0(x_0;y_0;z_0)$ nuqtasiga o'tkazilgan urinma tekislik tenglamasi.

$(x-a)^2+(y-b)^2+(z-c)^2=R^2$ **(3)** **y**oki $x^2+y^2+z^2-2ax-2by-2cz+a^2+b^2+c^2-R^2=0$ **(4)**

-markazi $(a;b;c;)$ nuqtada va radiusi R bo'lgan sfera

$F\left(x-\frac{1}{n}\cdot z; y-\frac{m}{n}\cdot z\right)=0$ **(5)** - silindrik sirt tenglamasi.

$F\left(x_0+\frac{x-x_0}{z_0-z}\cdot z_0, y_0+\frac{y-y_0}{z_0-z}\cdot z_0\right)=0$ **(6)** - konus tenglamasi.

$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ **(7)** – ellipsoid.

Misol 480. $x^2-xy+yz-5z=0$ sirt bilan $\frac{x-10}{7}=\frac{y-5}{3}=\frac{z}{-1}$ to'g'ri chiziqning kesishish nuqtalarini toping.

Yechilishi. To'g'ri chiziq parametrik shaklda yoziladi: $x=7t+10, y=3t+5, z=-t$. Bular berilgan sirt tenglamasiga qo'yiladi:

$$(7t+10)^2-(7t+10)\cdot(3t+5)+(3t+5)\cdot(-t)-5\cdot(-t)=0\Rightarrow$$

$$\Rightarrow 49t^2+140t+100-(21t^2+35t+30t+50)-3t^2-5t+5t=0\Rightarrow$$

$$\Rightarrow 49t^2+140t+100-21t^2-65t-50-3t^2=0\Rightarrow 25t^2+75t+50=0\Rightarrow t^2+3t+2=0\Rightarrow$$

$$\Rightarrow\begin{cases} t_1=-1\Rightarrow x=7\cdot(-1)+10=3;\ y=3\cdot(-1)+5=2;\ z=1;\\ t_2=-2\Rightarrow x=7(-2)+10=-4;\ y=3(-2)+5=-1;\ z=2\end{cases}\Rightarrow\begin{cases} A(x;y;z)=A(3;2;1);\\ B(x;y;z)=B(-4;-1;2);\end{cases}$$

Misol 481. $x^2-y^2-2x+z-3=0$ sirtning $(1;1;5)$ nuqtasidagi urinma tekislik tenglamasini tuzing.

Yechilishi. Berilgan sirt tenglamasi **(1)** bilan taqqoslanadi:

$$a_{11}=1, a_{22}-1, a_{33}=0, a_{12}=0, a_{13}=0, a_{23}=0, 2a_{14}=-2\Rightarrow a_{14}=-1, a_{24}=0,\ \ 2a_{34}=1\Rightarrow a_{34}=\frac{1}{2}, a_{44}=-3.$$

$x_0=1,\qquad y_0=1,\qquad z_0=5.$ Bular **(2)** ga qo'yiladi

$$[1\cdot1+0\cdot1+0\cdot5+(-1)](x-1)+[0\cdot1+(-1)\cdot1+0\cdot5+0](y-1)+\left[0\cdot1+0\cdot1+0\cdot5+\frac{1}{2}\right](z-5)=0\Rightarrow$$

$$\Rightarrow-(y-1)+\frac{1}{2}(z-5)=0\Rightarrow-y+1+\frac{1}{2}z-\frac{5}{2}=0\Rightarrow-2y+z-3=0\Rightarrow 2y-z+3=0$$

Misol 482. $x^2+3y^2-4xz-2yz+z-6=0$ sirtning $z=0$ tekislik bilan kesimini toping.

Yechilishi. $z=0$ sirt tenglamasiga qo'yiladi:

$$x^2+3y^2-6=0\Rightarrow x^2+3y=6\Rightarrow\frac{x^2}{6}+\frac{y^2}{2}=1.$$

Bu yarim o'qlari $a=\sqrt{6}$ va $b=\sqrt{2}$ bo'lgan ellips.

Misol 483. $\mathbf{2x^2+2y^2+2z^2-2x+4y+2z+1=0}$ sferaning markazi va radiusi topilsin.

Yechilishi. Berilgan tenglamadan

$$x^2+y^2+z^2-x+2y+z+\frac{1}{2}=0\Rightarrow x^2-x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+y^2+2y+1^2-1^2+z^2+z+\left(\frac{1}{2}\right)^2-$$

$$-\left(\frac{1}{2}\right)^2+\frac{1}{2}=0\Rightarrow\left(x-\frac{1}{2}\right)^2+(y+1)^2+\left(z+\frac{1}{2}\right)^2-\frac{1}{4}-1-\frac{1}{4}+\frac{1}{2}=0\Rightarrow\left(x-\frac{1}{2}\right)^2+(y+1)^2+\left(z+\frac{1}{2}\right)^2=1$$

Demak, $\left(\frac{1}{2};-1;-\frac{1}{2}\right)$ va R=1.

Misol 484. $(x-2)^2+y^2+z^2=4$ sferaning $M(3;\sqrt{2};1)$ nuqtasiga o'tkazilgan urunma tekislik tenglamasi topilsin.

Yechilishi. (2) ga koeffetsiyentlar qo'yilib yechilsa ham bo'ladi. Boshqacha yo'l tutamiz.

Tenglamadan sfera markazi $A(2;0;0)$ va R=2.

Sferaning M nuqtasiga o'tkazilgan urunma tekislik, sfera radiusiga perpendikulyarligidan $\overrightarrow{AM}$ vektor tekislikning normal vektori bo'ladi.

$$\overrightarrow{AM}=\{3-2;\sqrt{2}-0;1-0\}=\{1;\sqrt{2};1\}.$$

Bular $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$ tekislik tenglamasiga qo'yiladi.

$$1\cdot(x-3)+\sqrt{2}(y-\sqrt{2})+1(z-1)=0\Rightarrow x-3+\sqrt{2}\cdot y-2+z-1=0\Rightarrow x+\sqrt{2}\cdot y+z-6=0$$

Misol 485. Asosi (yo'naltiruvchisi) $(x0y)$ tekislikda $x^2+2xy-3y^2-x=0$ tenglama bilan aniqlanuvchi, yasovchilari $\{1;0;1\}$ vektorga parallel bo'lgan silindrik sirt tenglamasini toping.

Yechilishi. $\boldsymbol{L: F(x;y) = x^2+2xy-3y^2-x=0;}$

$\vec{\boldsymbol{u}} = \{1;0;1\} \Rightarrow \boldsymbol{l}=1, \boldsymbol{m}=0, \boldsymbol{n}=1.$

U holda sirt tenglamasi: $F(x-z;y) = (x-z)^2+2(x-z)\cdot y+3y^2-(x-z)=0$

Misol 486. Asosi $(x0y)$ tekislikdagi $x^2-2y^2=1$ giperboladan iborat, uchi (-1;2;1) nuqtada bolgan konus tenglamasini tuzing.

Yechilishi. $\boldsymbol{L: F(x;y) = x^2-2y^2-1=0;}$ $x_0=-1; y_0=2; z_0=1$ **(5)** ga asosan berilgan tenglamadagi x ni $-1+\frac{x-(-1)}{1-z}\cdot 1=\frac{x+1}{1-z}-1=\frac{x+1-1+z}{1-z}=\frac{x+z}{1-z}$ bilan,

y ni $2+\frac{y-2}{1-z}\cdot 1=\frac{2-2z+y-2}{1-z}=\frac{y-2z}{1-z}$ bilan almashtiriladi:

$$\left(\frac{x+z}{1-z}\right)^2-2\cdot\left(\frac{y-2z}{1-z}\right)^2-1=0 \Rightarrow x^2-2y^2-8z^2+8yz+2xz+2z-1=0$$

Misol 487. O'qlari koordinata o'qlarida joylashgan hamda M(2;0;1) nuqtadan o'tib, $(x0y)$ tekislik bilan $\frac{x^2}{8}+\frac{y^2}{1}=1$ ellips bo'yicha kesishuvchi ellipsoid tenglamasini tuzing.

Yechilishi. (7) ellipsoid z=0, ya'ni $(x0y)$ tekislik bilan kesilsa $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellips hosil bo'ladi.

U holda $a^2=8, b^2=1.$ $M(2;0;1)\in\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \Rightarrow \frac{2^2}{8}+\frac{0^2}{1}+\frac{1^2}{c^2}=1 \Rightarrow c^2=2.$

Izlangan tenglama $\frac{x^2}{8}+\frac{y^2}{1}+\frac{z^2}{2}=1.$

Misol 488. Fazodagi shunday nuqtalarning geometrik o'rni topilsinki, ularning har biridan $M_I(0;0;3)$ va $M_2(0;0;-3)$ nuqtalargacha bo'lgan masofalar ayirmasining moduli 4ga teng bo'lsin.

Yechilishi. $M(x;y;z)$ izlanayotgan nuqta bo'lsin $\overrightarrow{MM}_1=\{x;y;z-3\};$

$\overrightarrow{MM}_2=\{x;y;z+3\};$

$$\left|\sqrt{x^2+y^2+(z-3)^2}-\sqrt{x^2+y^2+(z+3)^2}\right|=4 \Rightarrow \sqrt{x^2+y^2+(z-3)^2}-\sqrt{x^2+y^2+(z+3)^2}=\pm 4 \Rightarrow$$

$$\Rightarrow\left(\sqrt{x^2+y^2+(z-3)^2}\right)^2=\left(\pm 4+\sqrt{x^2+y^2+(z+3)^2}\right)^2 \Rightarrow x^2+y^2+z^2-6z+9=16\pm$$

$$\pm 8\sqrt{x^2+y^2+(z+3)^2}+x^2+y^2+z^2+6z+9 \Rightarrow \left(\pm 8\sqrt{x^2+y^2+(z+3)^2}\right)^2=(16+12z)^2 \Rightarrow$$

$$\Rightarrow 64\left(x^2+y^2+z^2+6z+9\right)=256+384z+144z^2 \Rightarrow 64(x^2+y^2+z^2+6z+9)=64(4+6z+2.25\cdot z^2) \Rightarrow$$

$$x^2+y^2+z^2+6z+9-2.25z^2-6z-4=0 \Rightarrow x^2+y^2-1.25z^2=-5 \Rightarrow -\frac{x^2}{5}-\frac{y^2}{5}+\frac{z^2}{4}=1.$$

Bu ikki pallali giperboloid.

Misol 489. $x^2+y^2+4z^2-2xy-8z+5=0$ sirtning geometrik ma'nosini aniqlang.

Yechilishi. $x^2-2xy+y^2+4(z^2-2z+1)+1=0 \Rightarrow (x+y)^2+4(z-1)^2 \neq -1.$ **Ø**

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyida tenglamalari bilan berilgan sferaning markazi va radiusini toping:

490. $x^2+y^2+z^2-x+2y+1=0$ J: $\left(\frac{1}{2};-1;0\right), R=\frac{1}{2}.$

491. $(x+1)^2+(y+2)^2+z^2=25$ J: (-1;-2;0), R=5.

492. $x^2+y^2+z^2-4x+6y+2z-2=0$ J: $(2;-3;-1), R=4.$

493. $2x^2+2y^2+2z^2+4y-3z+2=0$ J: $\left(0;-1;\frac{3}{4}\right), R=\frac{3}{4}.$

494. $x^2+y^2+z^2=2x$ J: $(1;0;0), R=1.$

495. $x^2+y^2+z^2=4z-3$ J: $(0;0;2), R=1.$

$M(1;-1;3)$ **nuqtaning quyidagi sferalarga nisbatan joylashishini aniqlang:**

496. $(x-1)^2+(y+2)^2+z^2=19$

497. $x^2+y^2+z^2-x+y=0$

498. $x^2+y^2+z^2-4x+y-2z=0$

Quyidagi sirtlarning geometrik ma'nosini aniqlang:

499. $x^2-xy-xz+yz=0$

500. $x^2+z^2-4x-4z+4=0$

501. $x^2+2y^2+z^2-2xy-2yz=0$

502. $x^2+y^2-z^2-2y+2z=0$

503. $x^2+2y^2+2z^2-4y+4z+4=0$

504. $4x^2+y^2-z^2-24x-4y+2z+35=0$

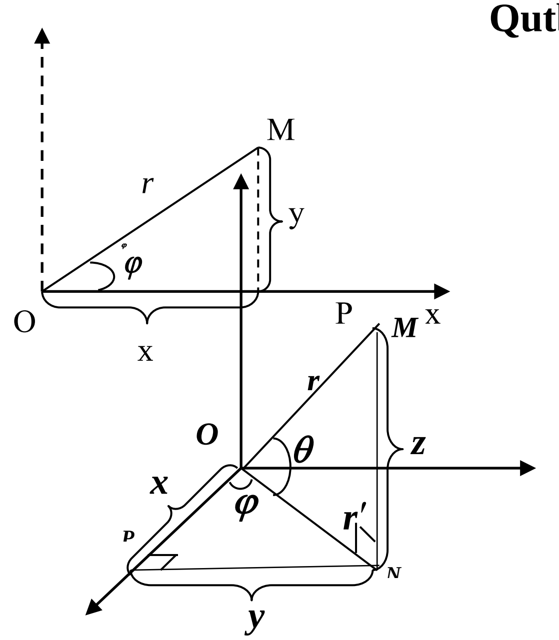
505. $x^2+y^2-z^2-2x-2y+2z+2=0$

506. $x^2+y^2-6x+6y-4z+18=0$

507. $9x^2-z^2-18x-18y-6z=0$

15-mavzu. Fazoda silindrik va sferik koordinatalar sistemasi

Qutb koordinatalar



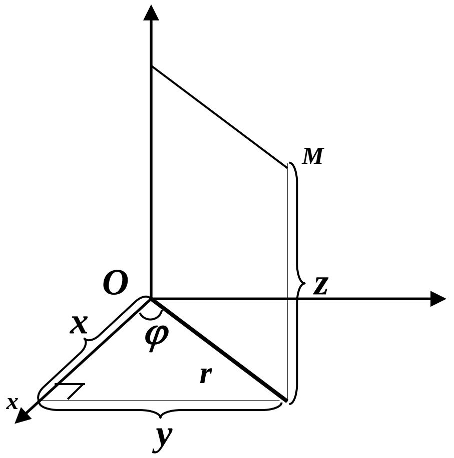
$$\begin{cases} x = r\cdot \cos\varphi; \\ y = r\cdot \sin\varphi. \end{cases}$$

$$r=\sqrt{x^2+y^2};$$

$$tg\varphi=\frac{y}{x} \Rightarrow \varphi = arctg\frac{y}{x}.$$

Silindrik koordinatalar

$$\begin{cases} x = r\cos\varphi; \\ y = r\sin\varphi; \\ z = h. \end{cases}$$



Sferik koordinatalar

$$\begin{cases} x = r\cos\theta\cos\varphi; \\ y = r\cos\theta\sin\varphi; \\ z = r\sin\theta. \end{cases}$$

Misol 508. Dekart koordinatalar sistemasida berilgan $A(1;\sqrt{3};4)$ nuqtani silindrik koordinatalarda ifodalang.

Yechilishi. $x = 1,\quad y = \sqrt{3},\quad z = 4$ dan foydalanib silindrik koordinatalar topiladi:

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (\sqrt{3})^2} = 2;$$

$$\varphi = arctg\frac{y}{x} = arctg\frac{\sqrt{3}}{1} = arctg\sqrt{3} = \frac{\pi}{3}.$$

Demak, $A(r;\varphi;h) = A\left(2;\frac{\pi}{3};4\right)$.

Misol 509. Silindrik koordinatalari bilan berilgan $M\left(2;\frac{\pi}{6};7\right)$ nuqtani Dekart koordinatalar sistemasida yozing.

Yechilishi. $r = 2,\quad \varphi = \frac{\pi}{6},\quad h = 7.$

$$\begin{cases} x = r\cos\varphi, \\ y = r\sin\varphi, \\ z = h \end{cases} \Rightarrow \begin{cases} x = 2\cdot\cos\frac{\pi}{6} = 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3}, \\ y = 2\cdot\sin\frac{\pi}{6} = 2\cdot\frac{1}{2} = 1, \\ z = 7. \end{cases}$$

Demak, $M(x;y;z) = M(\sqrt{3};1;7)$.

Misol 510. Dekart koordinatalar sistemasida berilgan $M(1;\sqrt{3};4)$ nuqtani sferik koordinatalarda ifodalang.

Yechilishi. $x=1,\ y=\sqrt{3},\ z=4.$

r' qutb radiusi va φ qutb burchagi aniqlanadi:

$$r' = \sqrt{x^2 + y^2} = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2;$$

$$\varphi = arctg\,\frac{y}{x} = arctg\sqrt{3} = \frac{\pi}{3}.$$

r' va z dan foydalanib r va θ topiladi:

$$\begin{cases} \dfrac{r'}{r} = \cos\theta \\ \dfrac{z}{r} = \sin\theta \end{cases} \Rightarrow \begin{cases} r' = r\cos\theta \\ z = r\sin\theta \end{cases} \Rightarrow \begin{cases} 2 = r\cos\theta \\ 4 = r\sin\theta \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} r = \dfrac{2}{\cos\theta} \\ 4 = \dfrac{2}{\cos\theta}\cdot\sin\theta \Rightarrow tg\theta = 2 \Rightarrow \theta = arctg2. \end{cases} \Rightarrow \begin{cases} r = \dfrac{2}{\cos arctg2} = \dfrac{2}{\dfrac{1}{\sqrt{1+2^2}}} = 2\sqrt{5}; \\ \theta = arctg2. \end{cases}$$

Demak, $M(r;\varphi;\theta) = M\left(2\sqrt{5};\frac{\pi}{3};arctg2\right)$.

Misol 511. Sferik koordinatalari bilan berilgan $M\left(4;\frac{\pi}{6};\frac{\pi}{3}\right)$ nuqtani Dekart koordinatalarida yozing.

Yechilishi. $r = 4,\quad \varphi = \frac{\pi}{6},\quad \theta = \frac{\pi}{3}.$

$$\begin{cases} x = r\cos\theta\cos\varphi, \\ y = r\cos\theta\sin\varphi, \\ z = r\sin\theta. \end{cases} \Rightarrow \begin{cases} x = 4\cdot\cos\frac{\pi}{3}\cdot\cos\frac{\pi}{6} = 4\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \sqrt{3}, \\ y = 4\cdot\cos\frac{\pi}{3}\cdot\sin\frac{\pi}{6} = 4\cdot\frac{1}{2}\cdot\frac{1}{2} = 1, \\ z = 4\sin\frac{\pi}{3} = 4\cdot\frac{\sqrt{3}}{2} = 2\sqrt{3}. \end{cases}$$

Demak, $M(x;y;z) = M\left(\sqrt{3};1;2\sqrt{3}\right)$.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi Dekart koordinatalari bilan berilgan nuqtalarni silindrik koodinatalar sistemasida ifodalang:

512. $A\left(\sqrt{3};1;5\right)$. *Javobi:* $A\left(2;\frac{\pi}{6};5\right)$.

513. $B\left(\sqrt{2};\sqrt{2};6\right)$. *Javobi:* $B\left(2;\frac{\pi}{4};6\right)$

514. $C\left(1;\sqrt{3};8\right)$. *Javobi:* $C\left(2;\frac{\pi}{3};8\right)$.

515. *D(3; 0; 10.)* *Javobi:* *D(3;0;10).*

516. *E(1; 2; 3).* *Javobi:* $E\left(\sqrt{5};arctg2;3\right)$.

Quyidagi silindrik koordinatalari bilan berilgan nuqtalarni Dekart koordinatalar sistemasida yozing:

517. $A\left(2;\frac{\pi}{6};5\right)$.	*Javobi:* $A\left(\sqrt{3};1;5\right)$.
518. $B\left(2;\frac{\pi}{4};6\right)$.	*Javobi:* $B\left(\sqrt{2};\sqrt{2};6\right)$.
519. $C\left(2;\frac{\pi}{3};8\right)$.	*Javobi:* $C\left(1;\sqrt{3};8\right)$
520. *D(3; 0; 10.)*	*Javobi:* ***D**(3;0;10).*
521. $E\left(\sqrt{5};arctg2;3\right)$.	*Javobi:* ***E**(1; 2; 3).*

Quyidagi Dekart koordinatalari bilan berilgan nuqtalarni sferik koordinatalar sistemasida yozing:

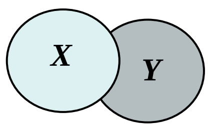
522. $A\left(\sqrt{3};1;4\right)$.	*Javobi:* $A\left(2\sqrt{5};\frac{\pi}{6};arctg2\right)$.
523. $B\left(\sqrt{2};\sqrt{2};6\right)$.	*Javobi:* $B\left(2\sqrt{10};\frac{\pi}{4};arctg3\right)$.
524. $C\left(1;\sqrt{3};8\right)$.	*Javobi:* $C\left(2\sqrt{17};\frac{\pi}{3};arctg4\right)$.
525. ***D**(3;0;10).*	*Javobi:* $D\left(2\sqrt{26};0;arctg5\right)$.
526. ***E**(1; 2; 3).*	*Javobi:* $E\left(\sqrt{14};arctg2;\ arctg\frac{3}{\sqrt{5}}\right)$.

16-MAVZU. TO'PLAM

To'plam matematikaning ta'riflanmaydigan asosiy tushunchalaridan biri.

To'plamlarni bosh harflar *A, B, C, X, Y, …*, uning elementlarini kichik harflar *a, b, c, x, y, …* bilan belgilash qabul qilingan. To'plamlar chekli va cheksiz bo'ladi.

Agar *a* element *A* to'plamga qarashli bo'sa $a \in A$, qarashli bo'lmasa $a \notin A$ yoki $a \overline{\in} A$ ko'rinishda belgilanadi. Bo'sh to'plam, ya'ni hech qanday elementga ega bo'lmagan to'plam **Ø** kabi belgilanadi.

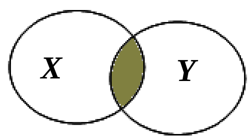


Z = X∪Y

Ta'rif. *X* va *Y* to'plamlarning *yig'indisi* deb shunday *Z* to'plamga aytiladiki, bu to'plamning istalgan elementi *X* yoki *Y* to'plamga tegishli bo'ladi va $Z=X\cup Y$ ko'rinishda yoziladi.

Misol 527. $A=\{1,2,3,4\}$; $B=\{1,2,3,5,7\}$. $C=A\cup B$ to'plamni toping.

Yechilishi. $C=A\cup B=\{1,2,3,4\}\cup\{1,2,3,5,7\}=\{1,2,3,4,5,7\}$.



Z = X∩Y

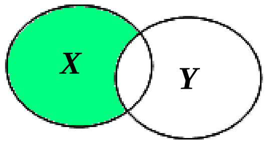
Ta'rif. *X* va *Y* to'plamlarning *kesishmasi (ko'paytmasi)* deb shunday *Z* to'plamga aytiladiki, bu to'plamning istalgan elementi ham *X*, ham *Y* to'plamga tegishli bo'ladi va $Z=X\cap Y$ ko'rinishda yoziladi.

Misol 528. $A=\{1,2,3,4\}$; $B=\{1,2,3,5,7\}$. $C=A\cap B$ to'plamni toping.

Yechilishi. $C = A \cap B = \{1,2,3,4\} \cap \{1,2,3,5,7\} = \{1,2,3\}$.

Ta'rif. Y to'plamning har bir elementi X to'plamning ham elementi bo'lsa, Y to'plamga X to'plamning *qism to'plami* deyiladi va $X \subset Y$ yoki $Y \supset X$ kabi belgilanadi. Teng to'plamlar esa $X=Y$ kabi tasvirlanadi.

Ta'rif. X to'plamdan Y *to'plamning farqi* deb, shunday Z to'plamga aytiladiki, uning har bir elementi X ga tegishli bo'lsa Y ga tegishli bo'lmaydi, ya'ni $Z=X \backslash Y$.



Z = X \ Y

Misol 529. $A = \{1,2,3,4\}$; $B = \{1,2,3,5,7\}$. $C = A \backslash B$ to'plamni toping.

Yechilishi. $C = A \backslash B = \{1,2,3,4\} \backslash \{1,2,3,5,7\} = \{4\}$.

Ta'rif. Haqiqiy sonlarning har qanday to'plamiga *sonli to'plam* deyiladi. Quyida eng ko'p uchraydigan sonli to'plamlarni qarab o'tamiz.

Boshi a va oxiri b bilan chegaralangan *yopiq oraliq* (yoki *kesma)* quyidagicha belgilanadi:

$$[a,b] = \{x \in R,\ a \le x \le b\}$$

Boshi a va oxiri b bilan chegaralangan *ochiq oraliq* quyidagicha belgilanadi:

$$(a,b) = \{x \in R,\ a < x < b\}$$

Boshi a va oxiri b bilan chegaralangan *yarim ochiq oraliq* quyidagicha belgilanadi:

$$(a,b] = \{x \in R,\ a < x \le b\}$$
$$[a,b) = \{x \in R,\ a \le x < b\}$$

Ta'rif. Barcha elementlarini natural sonlar bilan tartiblash (nomerlash) mumkin bo'lgan to'plamga *sanoqli to'plam* deyiladi. Sanoqli to'plam quvvati cheksiz to'plamlar quvvati orasida eng kichigidir. Juft sonlar, toq sonlar, ratsional sonlar kabilar sanoqli to'plamga misol bo'la oladi. Sanoqli bo'lmagan to'plamlar – *sanoqsiz to'plam* deyiladi.

Misol 530. Agar to'plam quyidagi xossalarga ega bo'lsa, uning elementlarini aniqlang:

a) $A = \{x \in N,\ x \le 0\}$; *b)* $B = \{x \in N,\ x \le 3\}$; *c)* $C = \{x \in Z,\ |x| \le 2\}$

Yechilishi. *a)* Ma'lumki, N natural sonlar to'plamida manfiy sonlar va nol mavjud emas. Shuning uchun A to'plamning elementlari yo'q, ya'ni $A = \varnothing$.

b) N natural sonlar to'plamida 3 ga teng va undan kichik bo'lgan sonlar 1, 2 va 3 dan iborat. Demak $B = \{1,2,3\}$

c) Butun sonlar to'plamida berilgan $|x| \le 2$ shartni qanoatlantiruvchi sonlar to'plami $C = \{-2;-1;0;1;2\}$

Misol 531. $X = \{1;2\}$ to'plamning barcha qism to'plamlaridan iborat to'plamni tuzing.

Yechilishi. Qism to'plamlarning ta'rifiga asosan quyidagilarni yozish mumkin:

$$\varnothing \in X,\ \{1\} \in X,\ \{2\} \in X,\quad \{1,2\} \in X.$$

Demak,

$$Y = [\varnothing, \{1\}, \{2\}, \{1,2\}]$$

Misol 532. Agar $X = \{2,3,4,6,7\}$ va $Y = \{3,6\}$ berilgan bo'lsa, bu to'plamlarning birlashmasi, ayirmasi va kesishmasini toping.

Yechilishi. Ikki to'plamning birlashmasi ta'rifiga asosan:

$$X \cup Y = \{2,3,4,6,7\}.$$

X va Y to’plamlarning kesishmasi

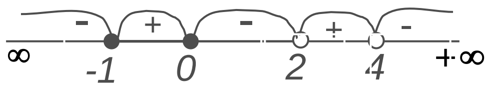
$$X\cap Y=\{3,6\}.$$

X va Y to’plamlarning ayirmasi

$$X\setminus Y=\{2,4,7\}.$$

Misol 533. $y=\sqrt{\dfrac{x\cdot(x+1)}{(x-2)\cdot(4-x)}}$ funksiyaning aniqlanish sohasini toping.

Yechilishi. $\dfrac{x\cdot(x+1)}{(x-2)\cdot(4-x)}\ge 0\Rightarrow\begin{cases}x(x+1)=0\\(x-2)\cdot(4-x)\ne 0\end{cases}\Rightarrow\begin{cases}x=0,\\x=-1,\\x\ne 2,\\x\ne 4.\end{cases}$

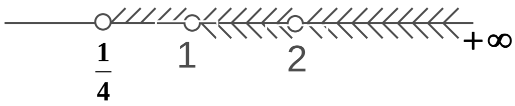


$x=-2\Rightarrow\dfrac{x\cdot(x+1)}{(x-2)\cdot(4-x)}=\dfrac{-2\cdot(-2-1)}{(-2-2)\cdot(4+2)}=\dfrac{6}{4\cdot 6}=-\dfrac{1}{4}<0$ bulardan

D(y)=[-1;0]$\cup$(2;4).

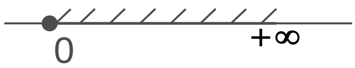
Misol 534. y=log$_{x-1}$(x-$\frac{1}{4}$) *funksiyaning aniqlanish sohasini toping.*

Yechilishi. $\begin{cases}x-1>0\\x-1\ne 1\\x-\dfrac{1}{4}>0\end{cases}\Rightarrow\begin{cases}x>1\\x\ne 2\\x>\dfrac{1}{4}\end{cases}\Rightarrow$



$\Rightarrow$D(y)=(1;2)$\cup$(2;+∞).

Misol 535. $y=2^{\sqrt{x}}$ *funksiyaning aniqlanish sohasini toping.*



Yechilishi. 2>0, 2≠1, x≥0 D(y)=[0;+∞).

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

536. Natural sonlar (*N*) va butun sonlar (*Z*) to’plami birlashmasini toping.

537. Agar ratsional sonlar to’plami *Q* va haqiqiy sonlar to’plami *R* berilgan bo’lsa, ularning kesishmasini toping

538. Ratsional va irratsional sonlar to’plamlarining birlashmasi va kesishmasini toping.

539. Agar *X* – to’g’ri to’rtburchaklar, *Y* – romblar to’plami bo’lsa, ularning kesishmasini toping.

540. Toq va juft sonlar to’plamlarining birlashmasi va kesishmasini toping.

541. Agar $A=\{1,3,6\}$ berilgan bo’lsa, barcha qism to’plamlarini aniqlang.

542. N natural sonlar to’plami, A toq sonlar to’plami va B juft sonlar to’plami berilgan bo’lsa, $N\cup A,\ N\cap A,\ N\cup B,\ N\cap B,\ A\cup B,\ A\cap B$ larni toping.

543. A – juft sonlar to’plami, B – toq sonlar to’plami va C – tub sonlar top’lami berilgan bo’lsa, $A\cup B,\ A\cap B,\ A\cup C$ larni toping.

544. Agar $X=\{0,2,3,5\},\quad Y-\{1,2,4,5,6\}$ bo’lsa, $X\cup Y,\ X\cap Y$, $X\backslash Y$ va $Y\backslash X$ ni toping.

545. Agar *N* – natural sonlar to'plami va *B* – toq sonlar to'plami bo'lsa, $N\backslash B$ ni toping.

Quyidagi funksiyalarning berilgan nuqtalardagi qiymatlari to'plamini toping:

546. $y=\dfrac{x^2-1}{3x+2}, \quad x=-1,\ x=0,\ x=\dfrac{3}{2}, \quad x=3.$

547. $y=\sin^2 3x, \quad x=0, \quad x=\dfrac{\pi}{6}, \quad x=\dfrac{\pi}{2}, \quad x=\dfrac{3\pi}{2}.$

Quyidagi funksiyalarning aniqlanish sohasini toping:

548. $y=\sqrt{9-x^2};$

549. $y=\dfrac{\sqrt{(x-2)(5-x)}}{x};$

550. $y=\dfrac{4}{1+\sqrt{x-4}};$

551. $y=\sqrt{\dfrac{(x-2)(4-x)}{(x+3)(x-5)}};$

552. $f(x)=\dfrac{x}{\sqrt{x^2-5x+6}};$

553 $g(x)=\sqrt{\sin 2x}.$

554. $y=\log_{x+1}(x-1)$

555. $y=5^{x^2-1}$

556. $y=\log_{x^2-1}\left(x^2-7x+2\right)$

557. $f(x)=e^{tgx}$

Quyidagi funksiyalarning qiymatlar sohasini toping:

558. $y=x^2-3x+4;$

559. $y=\sqrt{9-x^2};$

17-MAVZU. SONLAR KETMA – KETLIGI VA FUNKSIYA LIMITI

LIMITLAR HAQIDA TEOREMALAR.

1.$\lim c=c; \qquad c=const.$

2.$\lim(x\pm y\pm z)=\lim x\pm\lim y\pm\lim z;$

3.$\lim(x\cdot y\cdot z)=\lim x\cdot\lim y\cdot\lim z;$

4.$\lim\dfrac{x}{y}=\dfrac{\lim x}{\lim y}, \qquad y\neq 0.$

$\lim\limits_{n\to\infty}\dfrac{a}{n^k}=0$ tenglik o'rinli.

Misol 560.

$$\lim_{n\to\infty}\frac{6n^3+n^2-2n+15}{2n^3+5n-7}=\lim_{n\to\infty}\frac{\dfrac{6n^3+n^2-2n+15}{n^3}}{\dfrac{2n^3+5n-7}{n^3}}=\lim_{n\to\infty}\frac{\dfrac{6n^3}{n^3}+\dfrac{n^2}{n^3}-\dfrac{2n}{n^3}+\dfrac{15}{n^3}}{\dfrac{2n^3}{n^3}+\dfrac{5n}{n^3}-\dfrac{7}{n^3}}=$$

$$=\lim_{n\to\infty}\frac{6+\dfrac{1}{n}-\dfrac{2}{n^2}+\dfrac{15}{n^3}}{2+\dfrac{5}{n^2}-\dfrac{7}{n^3}}=\frac{\lim\limits_{n\to\infty}(6+\dfrac{1}{n}-\dfrac{2}{n^2}+\dfrac{15}{n^3})}{\lim\limits_{n\to\infty}(2+\dfrac{5}{n^2}-\dfrac{7}{n^3})}=\frac{\lim\limits_{n\to\infty}6+\lim\limits_{n\to\infty}\dfrac{1}{n^2}-\lim\limits_{n\to\infty}\dfrac{2}{n^2}+\lim\limits_{n\to\infty}\dfrac{15}{n^3}}{\lim\limits_{n\to\infty}2+\lim\limits_{n\to\infty}\dfrac{5}{n^2}-\lim\limits_{n\to\infty}\dfrac{7}{n^3}}=$$

$$=\frac{6+0-0+0}{2+0-0}=\frac{6}{2}=3.$$

AJOYIB LIMITLAR

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \;\; yoki \;\; \lim_{x\to 0}\frac{x}{\sin x}=1;$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e \;\; yoki \;\; \lim_{\alpha\to 0}(1+\alpha)^{\frac{1}{\alpha}}=e,$$

$$2<e<3, \quad e=2{,}71.$$

Misol 561.

$$\lim_{x\to 0}\frac{\sin 3x}{x}=\lim_{x\to 0}\frac{3\cdot\sin 3x}{3\cdot x}=3\cdot\lim_{x\to 0}\frac{\sin 3x}{3x}=3\cdot 1=3.$$

Misol 562.

$$\lim_{x\to 0}\frac{\sin 5x}{\sin 7x}=\lim_{x\to 0}\frac{\sin 5x}{x}\cdot\frac{x}{\sin 7x}=\lim_{x\to 0}\frac{\sin 5x}{x}\cdot\lim_{x\to 0}\frac{x}{\sin 7x}=$$

$$=\lim_{x\to 0}\frac{5\cdot\sin 5x}{5\cdot x}\cdot\lim_{x\to 0}\frac{7\cdot x}{7\cdot\sin 7x}=5\cdot\lim_{x\to 0}\frac{\sin 5x}{5\cdot x}\cdot\frac{1}{7}\cdot\lim_{x\to 0}\frac{7\cdot x}{\sin 7x}=5\cdot 1\cdot\frac{1}{7}\cdot 1=\frac{5}{7}.$$

Misol 563. $\lim_{n\to\infty}\left(1-\frac{2}{n}\right)^n.$

Yechilishi.

$$\alpha=-\frac{2}{n}\Leftrightarrow n=-\frac{2}{\alpha}\Leftrightarrow n=-\frac{1}{\alpha}\cdot 2; \qquad n\to\infty\Rightarrow\alpha\to 0;$$

$$\lim_{n\to\infty}\left(1-\frac{2}{n}\right)^n=\lim_{\alpha\to 0}(1+\alpha)^{-\frac{1}{\alpha}\cdot 2}=\frac{1}{\left[\lim_{\alpha\to 0}(1+\alpha)^{-\frac{1}{\alpha}}\right]^2}=\frac{1}{e^2}=e^{-2}.$$

Misol 564. $\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^n.$

Yechilishi. $\alpha=\frac{3}{n}\Leftrightarrow n=\frac{3}{\alpha}\Leftrightarrow n=\frac{1}{\alpha}\cdot 3; \quad n\to\infty\Rightarrow\alpha\to 0;$

$$\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^n=\lim_{\alpha\to 0}(1+\alpha)^{\frac{1}{\alpha}\cdot 3}=\left[\lim_{\alpha\to 0}(1+\alpha)^{\frac{1}{\alpha}}\right]^3=e^3.$$

LOPITAL QOIDASI

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}, \qquad g(x)\neq 0.$$

Misol 565.

$$\lim_{x\to 1}\frac{2x^3+3x^2-x-4}{x^3-2x^2+1}=\frac{2\cdot 1^3+3\cdot 1^2-1-4}{1^3-2\cdot 1^2+1}=\frac{0}{0}-\text{ aniqmaslik }.$$

Yechilishi.

$$\lim_{x\to 1}\frac{2x^3+3x^2-x-4}{x^3-2x^2+1}=\lim_{x\to 1}\frac{6x^2+6x-1}{3x^2-4x}=\frac{6\cdot 1^2+6\cdot 1-1}{3\cdot 1^2-4\cdot 1}=\frac{11}{-1}=-11$$

Misol 566. $\lim\limits_{x\to\infty}\dfrac{6x^3+x^2-2x+15}{2x^3+2x-7}=\dfrac{\infty}{\infty}$ – aniqmaslik .

Yechilishi.

$$\lim_{x\to\infty}\frac{6x^3+x^2-2x+15}{2x^3+5x-7}=\lim_{x\to\infty}\frac{18x^2+2x-2}{6x^2+5}=\lim_{x\to\infty}\frac{36x+2}{12x}=\lim_{x\to\infty}\frac{36}{12}=3$$

Misol 567. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{x}=\dfrac{\sin 3*0}{0}=\dfrac{0}{0}$ – ***aniqmaslik***

Yechilishi: $\lim\limits_{x\to 0}\dfrac{\sin 3x}{x}=\lim\limits_{x\to 0}\dfrac{3*\cos 3x}{1}=\lim\limits_{x\to 0}\dfrac{3*\cos 3*0}{1}=3*1=3.$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

568. 550. $\lim\limits_{x\to 1}\dfrac{4x^5+9x+7}{3x^6+x^5+1}$; ***javobi*** : 4

569. 551. $\lim\limits_{x\to 5}\dfrac{2x^2-5x-25}{x^2-25}$; ***javobi*** : $\dfrac{3}{4}$

570. 552. $\lim\limits_{x\to 2}\dfrac{x^2-5x+6}{x^2-7x+10}$; ***javobi*** : $\dfrac{1}{3}$

571. 553. $\lim\limits_{x\to 0}\dfrac{\sin 4x}{x}$; ***javobi*** : 4

572. 554. $\lim\limits_{x\to 0}\dfrac{\sin\frac{x}{3}}{x}$; ***javobi*** : $\dfrac{1}{3}$

573. 555. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 5x}$; ***javobi*** : $\dfrac{2}{5}$

574. 556. $\lim\limits_{n\to\infty}\left(1-\dfrac{5}{n}\right)^n$; ***javobi*** : e^{-5}

575. 557. $\lim\limits_{n\to\infty}\left(1+\dfrac{5}{n}\right)^n$; ***javobi*** : e^{5}

576. 558. $\lim\limits_{x\to\infty}(1+2x)^{\frac{1}{x}}$; ***javobi*** : e^{-5}

577. 559. $\lim\limits_{x\to 0}\left(1-\dfrac{x}{2}\right)^{\frac{1}{x}}$. ***javobi*** : $\dfrac{1}{\sqrt{e}}$.

578. 560. $\lim\limits_{n\to\infty}\dfrac{2x^2+x+5}{x^3+2x+1}$; ***javobi*** : 2

579. 561. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x}-\sqrt{x^2-7x}\right)$; ***javobi*** : 4

580. 562. $\lim\limits_{x\to\infty}\left(\sqrt{9x^2+1}-3x\right)$; ***javobi*** : 0

581. 563. $\lim\limits_{n\to\infty}\dfrac{3n^2+2}{5n^2-2n-1}$; ***javobi*** : $\dfrac{3}{5}$

18-MAVZU. H O S I L A

DIFERENSIALLASH QOIDALARI VA ASOSIY ELEMENTAR FUNKSIYALARNING HOSILALARI

$$y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}.$$

Misol 582. Hosilaning ta'rifidan foydalanib ***y=x²*** funksiya hosilasini toping.

Yechilishi.

1) $y+\Delta y=\left(x+\Delta x\right)^2$;

2) $\Delta y=f\left(x+\Delta x\right)-f\left(x\right)=\left(x+\Delta x\right)^2-x^2=x^2+2\Delta x\cdot x+\left(\Delta x\right)^2-x^2=\Delta x\left(2x+\Delta x\right)$;

3) $\dfrac{\Delta y}{\Delta x}=\dfrac{\Delta x\left(2x+\Delta x\right)}{\Delta x}=2x+\Delta x$;

3) $y'=\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\left(2x+\Delta x\right)=2x$. *Demak*, $y'=2x$.

Diferensiallash qoidalari

I. $\left[f(x)\pm g(x)\right]'=f'(x)\pm g'(x)$; III. $\left[f(x)\cdot g(x)\right]'=f'(x)\cdot g(x)+f(x)\cdot g'(x)$;

II. $\left[c\cdot f(x)\right]'=c\cdot f'(x)$; IV. $\left[\dfrac{f(x)}{g(x)}\right]'=\dfrac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g^2(x)}$, $g(x)\neq 0$

ASOSIY ELEMENTAR FUNKSIYALARNING HOSILALARI:

1. $(c)'=0$.
2. $(x)'=1$.
3. $(x^m)'=m\cdot x^{m-1}$.
4. $(\sqrt{x})'=\dfrac{1}{2\sqrt{x}}$.
5. $(\sin x)'=\cos x$
.6. $(\cos x)'=-\sin x$.

7. $(tgx)'=\dfrac{1}{\cos^2 x}$.

8 $(ctgx)'=-\dfrac{1}{\sin^2 x}$.

9. $(\log_a x)'=\dfrac{1}{x\ln a}$.

10. $(\ln x)'=\dfrac{1}{x}$

.11. $(a^x)'=a^x\ln a$.

12. $(e^x)'=e^x$.

13. $(\arcsin x)'=-\dfrac{1}{\sqrt{1-x^2}}$.

14. $(\arccos x)'=-\dfrac{1}{\sqrt{1-x^2}}$

15. $(arctgx)'=\dfrac{1}{1+x^2}$.

16. $(arcctgx)'=-\dfrac{1}{1+x^2}$.

MURAKKAB FUNKSIYANING HOSILASI

Agar ***y=f(u)*** bo'lib ***u= φ(x)*** bo'lsa, ya'ni ***y*** funksiya ***x*** argument bilan oraliqda turgan ***u*** argument orqali bog'langan bo'lsa, ***y*** ni ***x*** ning ***MURAKKAB FUNKSIYASI*** deyiladi.

Murakkab funksiyaning hosilasi, y ning u oraliq argument bo'yicha hosilasi $\dfrac{dy}{du}$ bilan oraliq argument ***u*** ning ***x*** bo'yicha $\dfrac{du}{dx}$ hosilaning ko'paytmasiga teng, ya'ni $\dfrac{dy}{dx}=\dfrac{dy}{du}\cdot\dfrac{du}{dx}$ yoki $y'_x=y'_u\cdot u'_x$.

Agar ***y=f(u)*** bo'lib, ***u=φ(x)*** bo'lsa, hosilalar jadvali quyidagi ko'rinishni oladi:

1. $(u^m)'=m\cdot u^{m-1}\cdot u'$;
2. $\left(\sqrt{u}\right)'=\dfrac{1}{2\sqrt{u}}\cdot u'$;
3. $(a^u)'=a^u\ln a\cdot u'$;;
4. $(e^u)'=e^u.u'$;;
5. $(\log_a u)'=\dfrac{1}{u\cdot\ln a}\cdot u'$;

6. $(\ln u)'=\dfrac{1}{u}\cdot u'$;;
7. $(\sin u)'=\cos u\cdot u'$;
8. $(\cos u)'=-\sin u\cdot u'$;
9. $(tgu)'=\dfrac{1}{\cos^2 u}\cdot u'$;
10. $(ctgu)'=-\dfrac{1}{\sin^2 u}\cdot u'$;

11. $(\arcsin u)'=\dfrac{1}{\sqrt{1-u^2}}.u'$;
12. $(\arccos u)'=-\dfrac{1}{\sqrt{1-u^2}}.u'$;
13. $(arctgu)'=\dfrac{1}{1+u^2}\cdot u'$;
14. $(arcctgu)'=-\dfrac{1}{1+u^2}\cdot u'$.

Misol 583. $y=\log_3 5x+\cos\left(x^3+2x\right)$

Yechilishi. $y'=\left[\log_3 5x+\cos\left(x^3+2x\right)\right]'=\left(\log_3 5x\right)'+\left[\cos\left(x^3+2x\right)\right]'=$

$$=\frac{1}{5x\cdot\ln 3}\cdot\left(5x\right)'+\left(-\sin\left(x^3+2x\right)\right)\cdot\left(x^3+2x\right)'=\frac{1}{5x\cdot\ln 3}\cdot 5-\sin\left(x^3+2x\right)\cdot\left(3x^2+2\right)=$$

$$=\frac{1}{x\ln 3}-\left(3x^2+2\right)\cdot\sin\left(x^3+2x\right)\quad\Rightarrow\quad y'=\frac{1}{x\ln 3}-\left(3x^2+2\right)\cdot\sin\left(x^3+2x\right).$$

OSHKORMAS FUNKSIYANING HOSILASI

F(x;y)=0 (1) tenglama o'zgaruvchilardan birortasiga nisbatan yechilmaganligi tufayli uni ***OSHKORMAS*** funksiya deyiladi.

Agar (1) funksiya ***y*** o'zgaruvchiga nisbatan ***y=f(x)*** ko'rinishda yechilgan bo'lsa, uni (1) ning ***OSHKOR*** ko'rinishi deyiladi.

Misol 584. $x^2+y^2-a^2=0$ (2) oshkormas funksiya. Uni y ga nisbatan yechsak $y^2=a^2-x^2 \Rightarrow y=\pm\sqrt{a^2-x^2}$, ya'ni $y=-\sqrt{a^2-x^2}$ (3) va $y=\sqrt{a^2-x^2}$ (4) larga ega bo'lamiz. (3) va (4) ni (2) ga qo'yilsa uni ayniyatga aylantiradi.

Har qanday oshkor ko'rinishdagi ***y=f(x)*** funksiyani ***y-f(x)=0*** oshkormas funksiya ko'rinishida yozish mumkin.

Endi oshkormas funksiyadan hosila olishni ko'rsatamiz:

Misol 585. $x^2+y^2-a^2=0$

Bunda ***y*** o'zgaruvchi ***x*** ning funksiyasi sifatida qaraladi va tenglikning ikki tomonidan ***x*** bo'yicha hosila olinadi:

$$2x+2y\,y'=0\Rightarrow 2yy'=-2x\Rightarrow y'=-\frac{x}{y}$$

Haqiqatan $x^2+y^2-a^2=0$ funksiya ***y*** ga nisbatan yechilib, undan ***x*** bo'yicha hosila olinganda ham shu natija olinadi:

$$y=\sqrt{a^2-x^2}\Rightarrow y'=\frac{1}{2\sqrt{a^2-x^2}}\cdot(a^2-x^2)'=\frac{-2x}{2\sqrt{a^2-x^2}}=-\frac{x}{\sqrt{a^2-x^2}}=-\frac{x}{y}.$$

TESKARI FUNKSIYA VA UNING HOSILASI

Bizga ***y=f(x)*** funksiya berilgan bo'lib, bu funksiya ***x*** ga nisbatan $x=\varphi(y)$ ko'rinishda yechilgan bo'lsa, ***y=f(x)*** va $x=\varphi(y)$ funksiyalarni o'zaro teskari funksiyalar deyiladi.

y=f(x) va $x=\varphi(y)$ funksiyalarning grafiklari bitta egri chiziqdan iborat bo'ladi, biroq $x=\varphi(y)$ dagi ***x*** ni ***y*** bilan, ***y*** ni ***x*** bilan almashtirilsa, har xil egri chiziqlar hosil bo'ladi.

y=f(x) va $x=\varphi(y)$ lar uchun $y'(x)=\dfrac{1}{\varphi'(y)}$ tenglik to'g'ri bo'ladi.

Misol 586.

1) $y=2x+3\Rightarrow y'_x=2\cdot 1+0\ \Rightarrow y'_x=2.$

2) $2x=y-3\Rightarrow x=\frac{1}{2}y-\frac{3}{2}\ \Rightarrow x'_y=\frac{1}{2}.$ Haqiqatdan $y'_x=\dfrac{1}{x'_y}$ bo'ladi.

TESKARI TRIGONOMETRIK FUNKSIYANING HOSILASI

1) $y=\arcsin x\Rightarrow \sin y=\sin\arcsin x\Rightarrow x=\sin y\Rightarrow -1\le x\le 1;\ -\frac{\pi}{2}\le y\le\frac{\pi}{2}$

Teorema. ***y=arcsinx*** funksiyaning hosilasi $y'=\dfrac{1}{\sqrt{1-x^2}}$ ga teng.

Isboti. $y=\arcsin x\Rightarrow x=\sin y\Rightarrow x'_y=\cos y$.

Teskari funksiyaning hosilasini olish qoidasiga asosan: $y'_x = \frac{1}{x'_y} = \frac{1}{\cos y}$ bo'ladi.

Biroq $\cos^2 y = 1 - \sin^2 y \Rightarrow \cos y = \sqrt{1-\sin^2 y} = \sqrt{1-x^2} \Rightarrow y'_x = \frac{1}{\sqrt{1-x^2}}$ bo'ladi.

Misol 587. $y = \arcsin e^x$

Yechilishi. $y' = \frac{1}{\sqrt{1-(e^x)^2}} \cdot (e^x)' = \frac{e^x}{\sqrt{1-e^{2x}}}$

Teorema. $y=\arccos x$ funksiyaning hosilasi $y' = -\frac{1}{\sqrt{1-x^2}}$ ga teng.

Isboti.

$y=\arccos x \Rightarrow \cos y = \cos \arccos x \Rightarrow x = \cos y \Rightarrow x'_y = -\sin y$

$y'_x = \frac{1}{x'y} = \frac{1}{-\sin y} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1-\cos^2 y}} = -\frac{1}{\sqrt{1-x^2}}$ bo'ladi.

Teorema. $y=\mathrm{arctg}\, x$ funksiyasining hosilasi $y' = \frac{1}{1+x^2}$ bo'ladi.

Isboti. $\mathrm{tg}\, y = \mathrm{tg}\,\mathrm{arctg}\, x \Rightarrow x = \mathrm{tg}\, y$; $x'_y = \frac{1}{\cos^2 y}$;

$y'_x = \frac{1}{x'_y} \quad \frac{1}{\frac{1}{\cos^2 y}} = \cos^2 x;$ $\cos^2 y = \frac{1}{\sec^2 y} = \frac{1}{1+\mathrm{tg}^2 y} = \frac{1}{1+x^2};$ $y'_x = \frac{1}{1+x^2}.$

Teorema. $y=\mathrm{arcctg}\, x$ funksiyaning hosilasi $y' = -\frac{1}{1+x^2}$ ga teng.

Isboti. $y=\mathrm{arcctg}\, x \Rightarrow \mathrm{ctg}\, y = \mathrm{ctg}\,\mathrm{arcctg}\, x \Rightarrow x = \mathrm{ctg}\, y \Rightarrow x'_y = -\frac{1}{\sin^2 y};$

$y'_x = \frac{1}{x'_y} \quad \frac{1}{-\frac{1}{\cos^2 y}} = -\sin^2 y = \frac{1}{\mathrm{cosec}^2 y} = -\frac{1}{1+\mathrm{ctg}^2 y} = -\frac{1}{1+x^2}$ bo'ladi.

YUQORI TARTIBLI HOSILALAR

y=f(x) funksiya ***(a;b)*** oraliqda defferensiallanuvchi bo'lsa, undan olingan birinchi tartibli ***y'=f'(x)*** hosila ***(a;b)*** oraliqda aniqlangan bo'ladi. Agar ***y'=f'(x)*** funksiyaning x_0 nuqtadagi hosilasi mavjud bo'lsa, uni ***y=f(x)*** funksiyaning x_0 nuqtadagi ikkinchi tartibli hosilasi deyiladi va qo'yidagicha yoziladi:

$$y''(x_0) = f''(x_0) = \frac{d^2 f(x_0)}{dx^2};$$

Shuningdek

$$(y'')' = y''' = f'''(x) = \frac{d^3 y}{dx^3}; \quad (y''')' = y^{IV} = f^{IV}(x) = \frac{d^4 y}{dx^4}; \quad \ldots \quad (y)^{n-1} = y^{(n)} = f^{(n)}(x) = \frac{d^n y}{dx^n}.$$

Misol 588. Mahsulot ishlab chiqarish harajati y va mahsulot hajmi x orasida $y = 100x - \frac{1}{30}x^3$ bog'lanish bo'lsin. Ishlab chiqarish hajmi:

1. 5 birlik;
2. 10 birlik bo'lganda limitik harajatni aniqlang.

Yechilishi. $y' = 100 - \frac{1}{10}x^2;$

$$y'(5)=100-\frac{1}{10}\cdot 5^2=100-2{,}5=97{,}5;$$

$$y'(10)=100-\frac{1}{10}\cdot 10^2=100-10=90.$$

Buning iqtisodiy mazmuni quyidagicha: mahsulot ishlab chiqarish hajmi 5 birlik bo'lganda, mahsulot ishlab chiqarish harajati kelgusi mahsulotni ishlab chiqarishga o'tishda 97,5 nitashkil etadi; ishlab chiqarish hajmi 10 birlik bo'lganda esa u 90 ni tashkil etadi.

LEYBNITS FORMULASI:

$$(u\cdot v)^n=u^{(n)}\cdot v+\frac{n}{1!}\cdot u^{(n-1)}\cdot v'+\frac{n\cdot(n-1)}{2!}\cdot u^{(n-2)}\cdot v''+...$$

$$+\frac{n\cdot(n-1)....(n-k+1)}{k!}.u^{(n-k)}.v^{(k)}+..+n\cdot u'\cdot v^{(n-1)}+u\cdot v^{(n)}.$$

Xususan: $(u\cdot v)'=u'v+u\cdot v'$.

Misol 589. $y=x^6+3x^3-5x^2+7$; $y'=6x^5+9x^2-10x$;

$y''=30x^4+18x$; $y'''=120x^3+18$;

$y^{IV}=720x$; $y^{V}=720$; $y^{VI}=0$;

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi funksiyalarning hosilalarini ta'rifdan foydalanib toping:

590. $y=x$. **591.** $y=x^2+x$. **592.** $y=\sqrt{2x}$. **593.** $y=\ln x$.

Quyidagi murakkab funksiyalarning hosilalarini toping:

594. $f(x)=(x^2+1)^2$ Javobi: $4x(x^2+1)$.

595. $y=\sin(x^3-2)$. Javobi: $3x^2\cdot\cos(x^3-2)$.

596. $y=\sin(\sin x)$. Javobi: $\cos(\sin x)\cdot\cos x$.

597. $y=\sin(\cos x)$ Javobi: $-\sin x\cdot\cos(\cos x)$.

598. $y=\cos(x^3-5)$. Javobi: $-3x^2\sin(x^3-5)$.

599. $f(x)=\cos(x^2+3)$. Javobi: $-2x\sin(x^2+3)$.

600. $y=\log_2 4x+\cos(x^2+3x)$. Javobi: $\frac{1}{x\ln 2}-(2x+3)\sin(x^2+3x)$.

601. $f(x)=\ln(x^2-3\sin x)$. Javobi: $\frac{2x-3\cos x}{x^2-3\sin x}$.

602. $f(x)=\ln(x^2+3\sin x)$. Javobi: $\frac{2x+3\cos x}{x^2+3\sin x}$.

603. $f(x)=e^{\sin 2x}$. Javobi: $2\cos 2x\cdot e^{\sin 2x}$.

Quyidagi funksiyalarning berilgan nuqtadagi hosilasining qiymatini toping:

604. $f(x)=(x^2+1)^2$, $f'\left(\frac{1}{2}\right)=?$ Javobi: 2,5.

605. $f(x)=\frac{x^2}{x^2+1}$, $f'(1)=?$ Javobi: 0,5.

606. $f(x)=\frac{1}{3}ctg3x$, $f'\left(\frac{\pi}{18}\right)=?$ Javobi:-4.

607. $f(x)=3\cos 2x-\sin 2x,\quad f'\left(\frac{\pi}{8}\right)=?$ Javobi: $-4\sqrt{2}$.

608. $f(x)=\frac{1}{2}tg2x,\quad f'\left(\frac{\pi}{6}\right)=?$ Javobi: 4.

609. $f(x)=\ln\sin x,\quad f'\left(\frac{\pi}{3}\right)=?$ Javobi:1.

610. $f(x)=\ln\cos x,\quad f'\left(\frac{\pi}{4}\right)=?$ Javobi:-1.

611. $f(x)=-\frac{2}{3}tg3x,\quad f'\left(\frac{\pi}{9}\right)=?$ Javobi: -4.

612. $f(x)=x\cdot 2^{x+1},\quad f'(0)=?$ Javobi: 2.

613. $f(x)=3x\cdot 2^{x},\quad f'(0)=?$ Javobi: 3.

Quyidagi oshkormas funksiyalarning hosilalarini toping:

614. $x^4+y^4-3xy=0.$ Javobi: $\frac{4x^3-3y}{3x-4y^3}$.

615. $x^2+y^2=64.$ Javobi: $-\frac{x}{y}$.

616. $xy-x-1=0.$ Javobi: $\frac{1-y}{x}$.

617. $2y-3x^2+3=0.$ Javobi: 3x.

618. $y^2-4x=0.$ Javobi: $\frac{2}{y}$.

619. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$ Javobi: $-\frac{b^2x}{a^2y}$.

Quyidagi funksiyalarning ko'rsatilgan tartibdagi hosilalarini toping:

620. $y=x^4+3x^2-2x+3,\quad y'''=?$ Javobi: 24x.

621. $y=(2x-1)^4,\quad y''=?$ Javobi: $48(2x-1)^2$.

622. $y=e^{3x}(\cos 2x+\sin 2x),\quad y''=?$ Javobi: $e^{3x}(17\cos 2x-7\sin 2x)$.

623. $y=\frac{1}{x},\quad y^{(n)}=?$ Javobi: $(-1)^n\cdot\frac{n!}{x^{n+1}}$.

624. $y=\ln x,\quad y^{(n)}=?$ Javobi: $(-1)^{n-1}\cdot\frac{1\cdot 2\cdot 3\cdots(n-1)}{x^n}$.

625. $y=\sin x,\quad y^{(n)}=?$ Javobi: $\sin\left(x+n\cdot\frac{\pi}{2}\right)$.

19-MAVZU. FUNKSIYANING DIFFERENSIALI

y = y(x) funksiyaning differensiali ***dy*** ni topish uchun funksiyadan hosila olinib ***dx*** ga ko'paytiriladi.

$$dy=y'\cdot dx$$

Misol 626. $y=2x^3+7x-5$ funksiyaning differensialini toping.

Yechilishi. $y'=6x^2+7 \;\Rightarrow\; dy=y'dx=(6x^2+7)dx=6x^2dx+7dx$.

Misol 627 $y = cos4x$, $dy = ?$

Yechilishi. $y' = -4\sin 4x \Rightarrow dy = y'dx = -4\sin 4x dx$

DIFFERENSIAL YORDAMIDA TAQRIBIY HISOBLASH

Mutaxassis kundalik mehnat faoliyati jarayonida turli xil taqribiy hisoblashlarga duch keladi. Ulardan ko'pchiligi differensialga doir ushbu

$$y(x+\Delta x) = y(x) + y'(x)\cdot \Delta x$$

formula yordamida osongina hal bo'ladi.

Misol 628. $\sqrt{17}$ ni hisoblang.

Yechilishi. Ildiz chiqarishda formuladagi x o'rnida turadigan son ildiz ko'rsatkichga darajaga ko'tariladi. Hosil bo'ladigan farq Δx bilan belgilanadi.

$$\sqrt{17} = \sqrt{16+1} = \sqrt{4^2+1} \Rightarrow \begin{cases} x = 4^2; \\ \Delta x = 1. \end{cases} \quad y(x) = \sqrt{x} \Rightarrow y\left(4^2\right) = \sqrt{4^2} = 4;$$

$$y'(x) = \frac{1}{2\sqrt{x}} \Rightarrow y'(4^2) = \frac{1}{2\sqrt{4^2}} = \frac{1}{8}.$$

Bu topilgan ma'lumotlar formulaga qo'yilsa:

$\sqrt{17} = 4 + \frac{1}{8} * 1 = 4\frac{1}{8} = 4{,}125$ bo'ladi.

Misol 629. $\sqrt[3]{17}$ ni toping.

Yechilishi.

$$\sqrt[3]{17} = \sqrt[3]{2{,}6^3 + (-0{,}576)} \Rightarrow \begin{cases} x = 2{,}6^3; \\ \Delta x = -0{,}576. \end{cases} \quad y(x) = \sqrt[3]{x} \Rightarrow y(2{,}6^3) = \sqrt[3]{2{,}6^3} = 2{,}6;$$

$$y'(x) = \frac{1}{3\sqrt[3]{x^2}} \Rightarrow y'\left(2{,}6^3\right) = \frac{1}{3\sqrt[3]{(2{,}6^3)^2}} = \frac{1}{3\cdot 2{,}6^2} = \frac{1}{3\cdot 6{,}76} = \frac{1}{20{,}28} = 0{,}0493.$$

Formulaga asosan: $\sqrt[3]{17} = 2{,}6 + 0{,}4093\cdot(-0{,}576) = 2{,}6 - 0{,}0284 = 2{,}5715.$

Misol 630. $\sqrt[3]{17}$ ni toping.

Yechilishi.

$$\sqrt[3]{17} = \sqrt[3]{3^3 + (-10)} \Rightarrow \begin{cases} x = 3^3; \\ \Delta x = -10. \end{cases} \quad y(3^3) = \sqrt[3]{3^3} = 3;$$

$$y'\left(3^3\right) = \frac{1}{3\sqrt[3]{(3^3)^2}} = \frac{1}{27}.$$

Formulaga asosan:

$$\sqrt[3]{17} = 3 + \frac{1}{27}\cdot(-10) = 3 - \frac{10}{27} = \frac{71}{27} = 2{,}6296.$$

629 va **630-** misollar natijalarini o'zaro taqqoslash, Δx o'rniga qancha kichik son olinsa natija shuncha aniqroq bo'lishini ko'rsatadi.

Misol 631. $\sqrt[4]{17}$ ni toping.

Yechilishi.

$$\sqrt[4]{17} = \sqrt[4]{2^4+1} \Rightarrow \begin{cases} x = 2^4; \\ \Delta x = 1. \end{cases} \quad y(x) = \sqrt[4]{x} \Rightarrow y(2^4) = \sqrt[4]{2^4} = 2;$$

$$y'(x) = \frac{1}{4\sqrt[4]{x^3}} \Rightarrow y'\left(2^4\right) = \frac{1}{4\sqrt[4]{(2^4)^3}} = \frac{1}{32} = 0{,}03125.$$

Formulaga asosan: $\sqrt[4]{17}=2+0,031125\cdot 1=2,03125.$

Misol 632. $\sqrt[5]{17}$ ni toping.

Yechilishi.

$$\sqrt[5]{17}=\sqrt[5]{1,8^4+(-1,89568)}\Rightarrow\begin{cases}x=1,8^5;\\ \Delta x=-1,89568.\end{cases}\quad y(x)=\sqrt[5]{x}\Rightarrow y(1,8^5)=\sqrt[5]{1,8^5}=1,8;$$

$$y'(x)=\frac{1}{5\sqrt[5]{x^4}}\Rightarrow y'\left(1,8^5\right)=\frac{1}{5\sqrt[5]{(1,8^5)^4}}=\frac{1}{5\cdot 1,8^4}=\frac{1}{5\cdot 10,4976}=\frac{1}{52,488}=0,019.$$

Formulaga asosan: $\sqrt[5]{17}=1,8+0,019\cdot(-1,89568)=1,8-0,036=1,764.$

Misol 633. $\sqrt[6]{4099}$ ni toping.

Yechilishi.

$$\sqrt[6]{4099}=\sqrt[6]{4^6+3}\Rightarrow\begin{cases}x=4^6;\\ \Delta x=3.\end{cases}\quad y(x)=\sqrt[6]{x}\Rightarrow y(4^6)=\sqrt[6]{4^6}=4;$$

$$y'(x)=\frac{1}{6\sqrt[6]{x^5}}\Rightarrow y'\left(4^6\right)=\frac{1}{6\sqrt[6]{(4^6)^5}}=\frac{1}{6\cdot 4^5}=\frac{1}{6\cdot 1026}=\frac{1}{6144}=0,00016.$$

Formulaga asosan: $\sqrt[6]{4099}=4+0,00016\cdot 3=4,0004881.$

Misol 634. $\sqrt{0,994}$ ni toping.

Yechilishi.

$$\sqrt{0,994}=\sqrt{1+(-0,006)}\Rightarrow\begin{cases}x=1;\\ \Delta x=-0,006.\end{cases}\quad y(x)=\sqrt{x}\Rightarrow y(1)=1;$$

$$y'(x)=\frac{1}{2\sqrt{x}}\Rightarrow y'(1)=\frac{1}{2}.$$

Formulaga asosan:

$$\sqrt{0,994}=1+\frac{1}{2}\cdot(-0,006)=1-0,003=0,997.$$

Misol 635 . $\sqrt[7]{0,994}$ ni toping.

Yechilishi.

$$\sqrt[7]{0,994}=\sqrt[7]{1+(-0,006)}\Rightarrow\begin{cases}x=1;\\ \Delta x=-0,006.\end{cases}\quad y(x)=\sqrt[7]{x}\Rightarrow y(1)=1;$$

$$y'(x)=\frac{1}{7\sqrt[7]{x^6}}\Rightarrow y'(1)=\frac{1}{7}.$$

Formulaga asosan: $\sqrt[7]{0,994}=1+\frac{1}{7}\cdot(-0,006)=1-0,0008571=0,9991429.$

Misol 636. $\sqrt[3]{1,15}$ ni toping.

Yechilishi.

$$\sqrt[3]{1,15}=\sqrt[3]{1+0,15}\Rightarrow\begin{cases}x=1;\\ \Delta x=0,15.\end{cases}\quad y(x)=\sqrt[3]{x}\Rightarrow y(1)=1;$$

$$y'(x)=\frac{1}{3\sqrt[3]{x^2}}\Rightarrow y'(1)=\frac{1}{3}.$$

Formulaga asosan: $\sqrt[3]{1,15}=1+\frac{1}{3}\cdot 0,15=1+0.05=1,05.$

Misol 637. $\sqrt[3]{9,2}$ ni toping.

Yechilishi.

$$\sqrt[3]{9,2}=\sqrt[3]{2^3+1,2}\Rightarrow\begin{cases}x=2^3;\\ \Delta x=1,2.\end{cases}\quad y(x)=\sqrt[3]{x}\Rightarrow y(2^3)=\sqrt[3]{2^3}=2;$$

$$y'(x)=\frac{1}{3\sqrt[3]{x^2}}\Rightarrow y'\left(2^3\right)=\frac{1}{3\sqrt[3]{(2^3)^2}}=\frac{1}{12}.$$

Formulaga asosan: $\sqrt[3]{9,2}=2+\frac{1}{12}\cdot 1,2=2+0,1=2,1.$

Misol 638. ***lg30,23*** ni toping.

Maktab o'quv dasturi o'quvchidan qo'yidagilarni og'zaki bilishni talab etadi:

$$lg\,2=0,3010;\quad lg\,3=0,4771;\ lg\,4=0,6021;\ lg\,5=0,6990;\ lg\,e=0,4342.$$

Yechilishi.

$$lg\,30,23=lg(30+0,23)\Rightarrow\begin{cases}x=30;\\ \Delta x=0,23.\end{cases}$$

$$y(x)=lg\,x\Rightarrow y(30)=lg\,30=lg\,3\cdot 10=lg\,3+lg\,10=0,4771+1=1,4771;$$

$$y'(x)=\frac{1}{x\cdot ln10}=\frac{1}{x\frac{lg10}{lg\,e}}=\frac{0,4342}{x};$$

$$y'(30)=\frac{0,4342}{30}=0,0145.$$

Formulaga asosan:

$$lg\,30,23=1,4771+0,0145\cdot 0,23=1,4771+0,0039=1,4804.$$

Misol 639. $sin\,32^0$ ni toping.

Yechilishi.

$$sin32^0=sin(30^0+2^0)\Rightarrow\begin{cases}x=30^0;\\ \Delta x=2^0.\end{cases}\quad 1^0=\frac{\pi}{180^0}\Rightarrow 2^0=\frac{2\pi}{180^0}=\frac{\pi}{90^0}=\frac{3,14}{90^0}=0,035.$$

$$y(x)=sin\,x\Rightarrow y(30)=sin30^0=\frac{1}{2}=0,5.\qquad y'(x)=cosx\Rightarrow y'\left(30^0\right)=cos\,30^0=\frac{\sqrt{3}}{2}=\frac{1,73}{2}=0,865.$$

Formulaga asosan: $sin32^0=0,5+0,865\cdot 0,035=0,5+0,0303=0,5303.$

Misol 640. $cos\,29^0$ ni toping.

Yechilishi.

$$cos\,29^0=cos(30^0-1^0)\Rightarrow\begin{cases}x=30^0;\\ \Delta x=-1^0=-\frac{\pi}{180^0}=-0,0174.\end{cases}\quad y(x)=cos\,x\Rightarrow y\left(30^0\right)=\frac{\sqrt{3}}{2};$$

$$y'(x)=-sin\,x\Rightarrow y'(30^0)=-\frac{1}{2}.$$

Formulaga asosan:

$$\cos 29^0=\frac{\sqrt{3}}{2}+\left(-\frac{1}{2}\right)\cdot(-0,0174)=\frac{1,73}{2}+0,5\cdot 0,0174=0,8660+0,0087=0,8747.$$

Misol 641. $tg\,45^0 12'$ ni toping.

Yechilishi.

$$12'=\frac{1}{5}\cdot 1^0=\frac{1}{5}\cdot\frac{\pi}{180^0}=\frac{3,14}{900}=0,0035;\quad tg\,45^0 12'=tg(45^0+12')\Rightarrow\begin{cases}x=45^0;\\ \Delta x=12'.\end{cases}$$

$$y(x)=tgx\Rightarrow y\left(45^0\right)=1;\qquad y'(x)=\frac{1}{\cos^2 x}\Rightarrow y'(45^0)=\frac{1}{(\cos 45^0)^2}=\frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}=2.$$

Formulaga asosan: $tg\,45^0 12'=1+2\cdot 0,0035=1,007.$

Misol 642. $ctg\,46^0 12'$ ni toping.

Yechilishi.

$$ctg\,46^0 12'=ctg(46^0+12')\Rightarrow ctg(45^0+72')=ctg\left(45^0+\frac{6}{5}\cdot 1^0\right)\Rightarrow\begin{cases}x=45^0;\\ \Delta x=\frac{6}{5}\cdot 1^0=\\ =\frac{6}{5}\cdot\frac{\pi}{180^0}=0,021.\end{cases}$$

$$y(x)=ctgx\Rightarrow y\left(45^0\right)=1;\qquad y'(x)=-\frac{1}{\sin^2 x}\Rightarrow y'(45^0)=-2.$$

Formulaga asosan: $ctg\,46^0 12'=1-2\cdot 0,021=1-0,042=0,958.$

Misol 643. $arctg\,1,007$ ni toping. 1 radian = $\frac{180^0}{\pi}$

Yechilishi.

$$arctg\,1,007=arctg(1+0,007)\Rightarrow\begin{cases}x=1;\\ \Delta x=0,007.\end{cases}\qquad y(x)=arctgx\Rightarrow y(1)=arctg1=45^0;$$

$$y'(x)=\frac{1}{1+x^2}\Rightarrow y'(1)=\frac{1}{2}=0,5.$$

Formulaga asosan:

$$arctg\,1,007=45^0+0,5\cdot 0,007=45^0+0,0035\ radian=45^0+0,0035\cdot 1radian=$$

$$=45^0+0,0035\cdot\frac{180^0}{\pi}=45^0+0,2006=45^0+0,2006\cdot 1^0=45^0+0,2006\cdot 60'=$$

$$=45^0+12'=45^0 12'.$$

Misol 644. $y(x)=\sqrt[3]{\frac{8-x}{27+x}}$ funksiyaning $x=0,15$ nuqtadagi qiymatini differensial yordamida hisoblang.

Yechilishi.

$$x=0,15\Rightarrow\begin{cases}x=0;\\ \Delta x=0,15.\end{cases}\quad y(0)=\sqrt[3]{\frac{8-0}{27-0}}=\frac{2}{3};\quad y'(x)=\frac{1}{3\sqrt[3]{(\frac{8-x}{27+x})^2}}*(\frac{8-x}{27+x})'=$$

$$=\frac{1}{3}\sqrt[3]{(\frac{27+x}{8-x})^2}\cdot\frac{(-35)}{(27+x)^2}=-\frac{35}{3(27+x)^2}\sqrt[3]{(\frac{27+x}{8-x})^2};\qquad y'(0)=-\frac{35}{3\cdot 27^2}\sqrt[3]{\frac{3^6}{2^6}}=$$

$$=-\frac{35}{3\cdot 3^6}\cdot\frac{3^2}{2^2}=-\frac{35}{3^5\cdot 2^2}=-\frac{35}{243\cdot 4}=-\frac{35}{972}=-0,036.$$

Formulaga asosan:

$$y(0,15)=\frac{2}{3}+(-0,036)\cdot 0,15=0,6666-0,0054=0,6613.$$

Misol 645. $y(x)=ln(1+e^{10x})+arctge^{5x}$ funksiyaning *x=0,2* nuqtadagi qiymatini differensial yordamida taqribiy hisoblang.

Yechilishi.

$$x=0,2\Rightarrow\begin{cases}x=0;\\ \Delta x=0,2.\end{cases}\quad y(0)=ln(1+e^{10\cdot 0})+arctge^{5\cdot 0}=ln2+arctg1=\frac{lg2}{lge}+\frac{\pi}{4}=$$

$$=\frac{0,301}{0,4342}+\frac{3,14}{4}=0,6932289+0,785=1,4782289.$$

$$y'(x)=\frac{1}{1+e^{10x}}\cdot(1+e^{10x})'+\frac{1}{1+(e^{5x})^2}\cdot(1+e^{5x})=\frac{10\cdot e^{10x}}{1+e^{10x}}+\frac{5e^{5x}}{1+e^{10x}}=\frac{5\cdot e^{5x}(2e^{5x}+1)}{1+e^{10x}};$$

$$y'(0)=7,5.$$

Formulaga asosan: $y(0,2)=1,4782289+7,5\cdot 0,2=2,9782.$

Misol 646. $2^{0,09}$ ni toping.

Yechilishi.

$$x=0,09\Rightarrow\begin{cases}x=0;\\ \Delta x=0,09.\end{cases}\quad y(x)=2^x\Rightarrow y(0)=1;\ y'(x)=2^x ln2\Rightarrow y'(0)=ln2=$$

$$=\frac{lg2}{lge}=\frac{0,301}{0,4342}=0,6932.$$

Formulaga asosan: $2^{0,09}=1+0,6932\cdot 0,69=1,0624.$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi funksiyaning differensialini toping

647. $y=\left(x^2+1\right)^2$ Javobi: $dy=4x\left(x^2+1\right)dx.$

648. $y=sin\left(x^3-2\right).$ Javobi: $dy=3x^2\cdot cos\left(x^3-2\right)dx.$

649. $y=sin\left(sin\,x\right).$ Javobi: $dy=cos\left(sin\,x\right)\cdot cos\,xdx.$

650. $y=sin\left(cos\,x\right)$ Javobi: $dy=-sin\,x\cdot cos\left(cos\,x\right)dx.$

651. $y=cos\left(x^3-5\right).$ Javobi: $dy=-3x^2\,sin\left(x^3-5\right)dx.$

652. $y=cos\left(x^2+3\right).$ Javobi: $dy=-2x\,sin\left(x^2+3\right)dx.$

653. $y = log_2 4x + cos\left(x^2 + 3x\right).$ Javobi: $dy = \frac{1}{x\,ln 2}dx - \left(2x+3\right)sin\left(x^2+3x\right)dx.$

654. $y = ln\left(x^2 - 3\,sin\,x\right).$ Javobi: $dy = \frac{2x - 3cos\,x}{x^2 - 3\,sin\,x}dx.$

655. $y = ln\left(x^2 + 3\,sin\,x\right).$ Javobi: $dy = \frac{2x + 3cos\,x}{x^2 + 3\,sin\,x}dx.$

656. $y = e^{sin 2x}.$ Javobi: $dy = 2cos\,2x \cdot e^{sin 2x}dx.$

Quyidagi misollarni differensial yordamida taqribiy hisoblang:

657. $\sqrt{36,36} = ?$ Javobi: 6,03.

658. $\sqrt{55,55} = ?$ Javobi: 7,453.

659. $\sqrt[3]{520,5} = ?$ Javobi: 8,044.

660. $\sqrt[3]{997} = ?$ Javobi: 9,99.

661. $lg\,42,19 = ?$ Javobi: 1,6252.

662. $lg\,79,71 = ?$ Javobi: 1,9016.

663. $sin\,16^0 = ?$ Javobi: 0,2756.

664. $sin\,42^0 = ?$ Javobi: 0,6691.

665. $cos\,32^0 = ?$ Javobi: 0,848.

666. $cos\,44^0 = ?$ Javobi: 0,7193.

666. $\sqrt{44,44} = ?$ Javobi: 6,666.

667. $\sqrt{70,69} = ?$ Javobi: 8,407.

668. $\sqrt[3]{625} = ?$ Javobi: 8,55.

669. $\sqrt[3]{901,4} = ?$ Javobi: 9,66.

670. $lg\,60,28 = ?$ Javobi: 1,7802.

671. $lg\,99,99 = ?$ Javobi: 1,1.

672. $sin\,28^0 = ?$ Javobi: 0,4695.

673. $sin\,61^0 = ?$ Javobi: 0,8746.

674. $cos\,29^0 = ?$ Javobi: 0,8746.

675. $cos\,59^0 = ?$ Javobi: 0,515.

20-MAVZU. FUNKSIYANI TEKSHIRISH

FUNKSIYA EKSTREMUMINI BIRINCHI TARTIBLI HOSILA YORDAMIDA ANIQLASH

1. Funksiyaning aniqlanish sohasi topiladi.
2. Funksiyadan hosila olinadi.
3. Hosila nolga tenglanib, kritik nuqtalar topiladi.
4. Kritik nuqtalar yordamida funksiyaning aniqlanish sohasi intervallarga ajratiladi.
5. Hosilaning har bir intervaldagi ishorasi aniqlanadi.
6. Hosilaning ishorasi musbat bo’lgan intervalda funksiya o’suvchi, manfiy bo’lgan intervalda esa kamayuvchi bo’ladi.
7. Hosilaning ishorasi kritik nuqtadan o’tishda minusdan plyusga almashsa, funksiya ushbu kritik nuqtada minimumga erishadi.
8. Hosilaning ishorasi kritik nuqtadan o’tishda plyusdan minusga almashsa, funksiya ushbu kritik nuqtada maksimumga erishadi.
9. Funksiya o’zining eng katta va eng kichik qiymatlariga kritik nuqtalarda yoki oraliqning chetki nuqtalarida erishishi mumkin.

Misol 667. $f(x) = \frac{3}{4}\cdot x^4 - x^3 - 9x^2 + 7$ funksiyaning ekstremumini aniqlang.

Yechilishi:

1. $D(f(x)) = (-\infty; +\infty);$

2. $f^I(x) = \frac{3}{4}\cdot 4\cdot x^3 - 3x^2 - 9\cdot 2\cdot x + 0 = 3x^3 - 3x^2 - 18x = 3x(x^2 - x - 6)$

3. $f^I(x) = 0 \Leftrightarrow 3x(x^2 - x - 6) = 0 \Rightarrow \begin{cases} 3 \neq 0; \\ x = 0; \\ x^2 - x - 6 = 0. \end{cases}$

$$x^2 - x - 6 = 0 \Rightarrow x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{P}{2}\right)^2 - q} = -\frac{-1}{2} \pm \sqrt{\left(\frac{-1}{2}\right)^2 - (-6)} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 6} = \frac{1}{2} \pm \sqrt{\frac{25}{4}} =$$

$$= \frac{1}{2} \pm \frac{5}{2} \Rightarrow \begin{cases} x_1 = \frac{1}{2} - \frac{5}{2} = \frac{1-5}{2} = \frac{-4}{2} = -2; \\ x_2 = \frac{1}{2} + \frac{5}{2} = \frac{1+5}{2} = \frac{6}{2} = 3; \end{cases} \Rightarrow \begin{cases} x_1 = -2; \\ x_2 = 3; \end{cases}$$

Demak, kritik nuqtalar quyidagicha bo'ladi:

4. $x = -2, 0, 3 \in (-\infty; +\infty)$;

$D(f) = (-\infty; +\infty) = (-\infty; -2) \cup (-2; 0) \cup (0; 3) \cup (3; +\infty)$.

1. $x = -3 \in (-\infty; -2) \Rightarrow f'(-3) = 3 \cdot (-3) \cdot \left[(-3)^2 - (-3) - 6\right] = -9(9 + 3 - 6) = -54 < 0;$
2. $x = -1 \in (-2; 0) \Rightarrow f'(-1) = 3 \cdot (-1)\left[(-1)^2 - (-1) - 6\right] = -3(1 + 1 - 6) = 12 > 0;$
3. $x = 1 \in (0; 3) \Rightarrow f'(1) = 3 \cdot 1(1^2 - 1 - 6) = -18 < 0;$
4. $x = 4 \in (3; +\infty) \Rightarrow f'(4) = 3 \cdot 4(4^2 - 4 - 6) = 72 > 0;$
5. $x = -2 \Rightarrow f(-2) = \frac{3}{4} \cdot (-2)^4 - (-2)^3 - 9 \cdot (-2)^2 + 7 = \frac{3}{4} \cdot 16 + 8 - 36 + 7 = 9;$

$x = 0 \Rightarrow f(0) = \frac{3}{4} \cdot 0^4 - 0^3 - 9 \cdot 0^2 + 7 = 7;$

$x = 3 \Rightarrow f(3) = \frac{3}{4} \cdot 3^4 - 3^3 - 9 \cdot 3^2 + 7 = \frac{3}{4} \cdot 81 - 27 - 81 + 7 = \frac{243}{4} - 101 = \frac{243 - 404}{4} = \frac{-161}{4} = -40{,}25$

funksiya *x=0* nuqtada o'zining eng katta, *x*=3 nuqtada esa eng kichik qiymatiga erishadi

	$(-\infty;-2)$	-2	$(-2;0)$	0	$(0;3)$	3	$(3;+\infty)$
$f^I(x)$	-	0	+	0	-	0	+
$f(x)$	↘	min	↗	max	↘	min	↗

Demak, berilgan funksiya uchta ekstremumga ega. Ulardan ikkitasi minimum, bittasi maksimum nuqtalar.

FUNKSIYA EKSTREMUMINI IKKINCHI TARTIBLI HOSILA YORDAMIDA ANIQLASH

1. Funksiyaning birinchi tartibli hosilasi nolga tenglanib, *x* kritik nuqtalar topiladi.
2. Funksiyaning ikkinchi tartibli hosilasi topiladi.
3. Ikkinchi tartibli hosilaning x_0 kritik nuqtadagi qiymati aniqlanadi:

1. $f''(x_0) < 0$ bo'lsa, funksiya x_0 nuqtada maksimumga ega bo'ladi.
2. $f''(x_0) > 0$ bo'lsa, funksiya x_0 nuqtada minimumga ega bo'ladi.

Misol 668. $f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 + 7$ funksiyaning ekstrimumini ikkinchi tartibli hosila yordamida aniqlang:

Yechilishi. $f'(x) = 3x^3 - 3x^2 - 18x$ hamda kritik nuqtalarning $x = -2;, 0, 3 \in (-\infty; +\infty)$ ekanligi ma'lum. $f''(x) = 9x^2 - 6x - 18$.

I. $x = -2 \Rightarrow f''(-2) = 9(-2)^2 - 6(-2) - 18 = 36 + 12 - 18 == 30 > 0;$

II. $x = 0 \Rightarrow f''(0) = 9 \cdot 0^2 - 6 \cdot 0 - 18 = -18 < 0;$

III. $x = 3 \Rightarrow f''(3) = 9 \cdot 3^2 - 6 \cdot 3 - 18 = 81 - 18 - 18 = 45 > 0;$

Demak, funksiya *x*=-2, 3 nuqtalarda minimumga, *x*=0 nuqtada maksimumga ega.

Funksiya grafigining qavariq qismini botiq qismidan ajratuvchi nuqtani funksiya grafigining egilish nuqtasi deyiladi.

Egilish nuqtani topish uchun funksiyaning ikkinchi hosilasi nolga tenglanib ikkinchi tur kritik nuqtalar aniqlanadi. Bu kritik nuqtalar berilgan funksiyaga qo'yilib egilish nuqtaning ***y*** ordinatasi topiladi.

Funksiya grafigining qavariqlik oralig'i $f''(x)<0$, botiqlik oralig'i $f''(x)>0$ deb topiladi.

Misol 669. $f(x)=\frac{1}{4}x^4-6x^2+5$ funksiyaning egilish nuqtasini, qavariqlikini va botiqlik oraliqlarini toping.

Yechilishi.

$$f'(x)=x^3-12x; \quad f(x)''=3x^2-12 \Rightarrow f''(x)=0 \Rightarrow 3x^2-12=0 \Rightarrow 3(x^2-4)=0 \Rightarrow$$

$$\begin{cases} 3\neq 0 \\ x^2-4=0 \end{cases} \Rightarrow x^2=4 \Rightarrow x=\pm 2 \Rightarrow \begin{cases} x_1=-2; \\ x_2=2. \end{cases}$$

$$f(-2)=\frac{1}{4}(-2)^2-6(-2)^2+5=-15; \qquad f(2)=\frac{1}{4}\cdot 2^2-6\cdot 2^2+5=-15.$$

Demak, egilish nuqtalar: (-2;-15), (2;-15).

Qavariqlik oralig'i:

$$f''(x)<0 \Rightarrow 3x^2-12<0 \Rightarrow 3(x^2-4)<0 \Rightarrow x^2-4<0 \Rightarrow x^2<4 \Rightarrow |x|<2 \Rightarrow$$

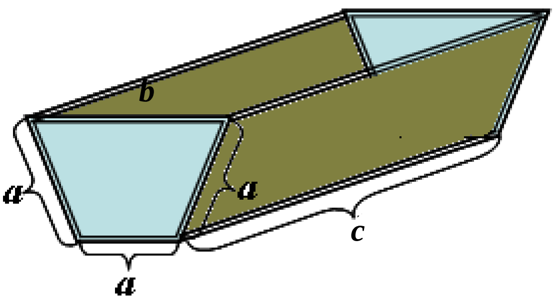
$$\Rightarrow -2<x<2 \Rightarrow (-2;2).$$

Botiqlik oralig'i:

$$f''(x)>0 \Rightarrow 3x^2-12>0 \Rightarrow 3(x^2-4)>0 \Rightarrow x^2-4>0 \Rightarrow x^2>4 \Rightarrow |x|>2 \Rightarrow$$

$$\Rightarrow \begin{cases} x>2 \\ x<-2 \end{cases} \Rightarrow (-\infty;-2)\cup(2;+\infty).$$

Misol 670. Eni *a*, bo'yi *c* *(a<c)* bo'lgan 3 ta taxtani kesmasdan eng katta hajmli oxur yasash talab etilsin. Buning uchun oxur kichik yon yog'i katta asosining o'lchami qanday bo'lishini va yuzini hosila yordamida aniqlang.

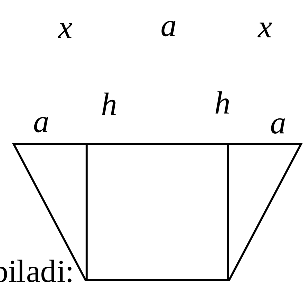


Yechilishi. Kichik yon yoq trapetsiya shaklida bo'lganligi uchun uning yuzi $S=\frac{a+b}{2}\cdot h$ (1) formula bilan topiladi.

Chizmadan

$$b=a+2x, \quad h^2=a^2-x^2 \Rightarrow h=\sqrt{a^2-x^2}.$$

Bularni formulaga qo'yilsa trapetsiyaning yuzi *x* ning funksiyasiga aylanadi:



$$S(x)=\frac{a+(a+2x)}{2}\cdot\sqrt{a^2-x^2}=(a+x)\cdot\sqrt{a^2-x^2} \Rightarrow$$

$$\Rightarrow S(x)=(a+x)\cdot\sqrt{a^2-x^2}$$

Bu funksiyadan hosila olinib va nolga tenglanib kritik nuqtalar topiladi:

1) $S'(x)=(a+x)'\cdot\sqrt{a^2-x^2}+(a+x)\cdot\left(\sqrt{a^2-x^2}\right)'=\sqrt{a^2-x^2}+\frac{1}{2}\cdot\frac{a+x}{\sqrt{a^2-x^2}}\cdot\left(a^2-x^2\right)'=$

$$=\sqrt{a^2-x^2}-\frac{(a+x)\cdot x}{\sqrt{a^2-x^2}}=\frac{a^2-x^2-ax-x^2}{\sqrt{a^2-x^2}}=\frac{a^2-ax-2x^2}{\sqrt{a^2-x^2}}.$$

2) $S'(x)=0\Rightarrow\frac{a^2-ax-2x^2}{\sqrt{a^2-x^2}}=0\;\Rightarrow\;a^2-ax-2x^2=0\;\Rightarrow\;\begin{cases}x_1=-a,\\ x_2=\frac{a}{2}.\end{cases}$

Masofa manfiy bo'lmaganligi sababli $x_1=-a$ chet ildiz, $x_2=\frac{a}{2}$ ildiz bo'ladi.

Ikkinchi tartibli hosila yordamida Ikkinchi tartibli hosila yordamida $x=\frac{a}{2}$ ning maksimum yoki minimum nuqta ekanligini aniqlash qulfy:

3) $S''(x)=\left(\frac{a^2-ax-2x^2}{\sqrt{a^2-x^2}}\right)'=\frac{\left(a^2-ax-2x^2\right)'\cdot\sqrt{a^2-x^2}-\left(\sqrt{a^2-x^2}\right)'}{\left(\sqrt{a^2-x^2}\right)^2}=$

$$=\frac{-(a+4x)\cdot\sqrt{a^2-x^2}+\frac{\left(a^2-ax-x^2\right)\cdot x}{\sqrt{a^2-x^2}}}{a^2-x^2}=\frac{-(a+4x)\cdot\left(a^2-x^2\right)+a^2x-ax^2-x^3}{\left(a^2-x^2\right)^{\frac{3}{2}}}=$$

$$=\frac{-a^3-4a^2x+ax^2+4x^3+a^2x-ax^2-x^3}{\sqrt{\left(a^2-x^2\right)^3}}=\frac{3x^3-3a^2x-a^3}{\sqrt{\left(a^2-x^2\right)^3}}.$$

Ikkinchi tartibli hosilaning $x=\frac{a}{2}$ kritik nuqtadagi qiymati aniqlanadi:

4) $S''\left(\frac{a}{2}\right)=\frac{3\cdot\frac{a^3}{8}-3a^2\cdot\frac{a}{2}-a^3}{\sqrt{\left(a^2-\frac{a^2}{4}\right)^3}}=\frac{\frac{3a^3-12a^3-8a^3}{8}}{\sqrt{\frac{27a^6}{64}}}=\frac{-17a^3}{3\sqrt{3}a^3}=-\frac{17}{3\sqrt{3}}<0.$

Demak, $x=\frac{a}{2}$ maksimum nuqta ekan.

U holda eng katta yuz

5) $S\left(\frac{a}{2}\right)=\left(a+\frac{a}{2}\right)\cdot\sqrt{a^2-\frac{a^2}{4}}=\frac{3a}{2}\cdot\sqrt{\frac{3a^2}{4}}=\frac{3\sqrt{3}}{4}a^2\;(\boldsymbol{kv.birlik})$ bo'ladi.

Xulosa: kichik asosi va yon tomonlari a ga teng bo'lgan teng yonli trapetsiyaning eng katta yuzga ega bo'lishi uchun, uning katta asosi $b=a+2x=a+2\cdot\frac{a}{2}=2a$ bo'lishi kerak.

Misol 671. Sigirlardan bir sutka davomida sog'ib olinadigan sut miqdori y bilan, sigirning yoshi x orasidagi bog'lanish quyidagi ishlab chiqarish funksiyasi bilan berilgan:

$y = -0{,}49x^2 + 6{,}86x - 9{,}53, \quad x > 2$. Sigirlarning yoshi qanday b'lganda sutkalik sut miqdori eng ko'p bo'ladi?

Yechilishi. Bu masalani yechish uchun funksiya ekstremumini topish qoidasini qo'llaymiz:

1) $y' = -0{,}98x + 6{,}86;$

2) $y' = 0 \Rightarrow -0{,}98x + 6{,}86 = 0 \quad \Rightarrow \quad x = 7.$

2) ***x=7*** kritik nuqta atrofida hosila ishorasini tekshiramiz:

$y'(6) = -0{,}98 \cdot 6 + 6{,}86 = 0{,}98 > 0, \; y'(8) = -0{,}98 \cdot 8 + 6{,}86 = -0{,}98 < 0.$

Demak, ***x=7*** nuqtada funksiya maksimumga ega bo'ladi.

$y(7) = -0{,}49 \cdot 7^2 + 6{,}86 \cdot 7 - 9{,}53 = 14{,}48$.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi funksiyalarning ekstremumini birinchi va ikkinchi tartibli hosilalar yordamida toping:

672. $f(x) = 2x^2 + x - 6;$ *Javobi:* $x = -\frac{1}{4}$ ***da minimumga ega.***

673. $f(x) = (1 - x^2)^3;$ *Javobi:* $x = 1$ ***da minimumga ega.***

674. $y = \frac{x}{2} + \frac{2}{x}, x \neq 0;$ *Javobi:* $x = -2$ ***da maksimum,*** $x = 2$ ***da minimumga ega.***

Quyidagi funksiyalarning o'sish oralig'ini toping:

675. $f(x) = -x^2 + 2x - 1;$ *Javobi:* $(-\infty; 1].$

676. $f(x) = x^2 + 2x + 4;$ *Javobi:* $[-1; +\infty).$

677. $f(x) = x^2 - 2x + 3;$ *Javobi:* $[1; +\infty).$

678. $f(x) = -\frac{1}{3}x^3 - x^2 + 3x - 5;$ *Javobi:* $[-3; 1].$

679. $f(x) = x^2 + 1;$ *Javobi:* $[0; +\infty).$

Quyidagi funksiyalarning kamayish oralig'ini toping:

680. $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + 12x + 1;$ *Javobi:* $[-4; 3].$

681. $f(x) = x^2 - 2;$ *Javobi:* $(-\infty; 0].$

682. $f(x) = 2x^3 + 3x^2 - 12x + 7;$ *Javobi:* $[-2; 1].$

Quyidagi funksiyalarning ko'rsatilgan oraliqdagi eng katta, eng kichik qiymatlarini toping:

683. $y = 1 + \cos x, \; \left[\frac{\pi}{3}; \frac{\pi}{2}\right];$ *Javobi:* 1 ***va*** $\frac{3}{2}$.

684. $y = x^2 - 3x + 1.25, \; [-1; 1];$ *Javobi:* $-0{,}75$ ***va*** $5{,}25.$

685. $y = 2\sin x - 1, \; \left[0; \frac{\pi}{6}\right];$ *Javobi:* -1 ***va*** $0.$

686. $f(x) = 3x^2 - 6x - 4, \; [0; 3];$ *Javobi:* -7 ***va*** $5.$

687. $f(x) = 2 - 2\sin x, \; \left[0; \frac{\pi}{6}\right];$ *Javobi:* 1 ***va*** $2.$

688. $f(x) = \frac{2}{3}x^3 + 8x$ funksiyaning maksimumini toping. *Javobi:* ***mavjud emas.***

689. $f(x) = 3x - x^2$ funksiyaning maksimumini toping. *Javobi: 3.*

690. $y=-4x^3+12x$ funksiyaning minimumini toping. *Javobi:* ±1.

691. $g(x)=12x-x^3$ funksiyaning minimumini toping. *Javobi:* ±2.

692. $y=2\sin x+\cos x$ funksiyaning eng katta qiymatini toping. *Javobi:* $\sqrt{5}$.

693. $y=\dfrac{x^2-5}{x^2+5}$ funksiyaning eng kichik qiymatini toping. *Javobi:* -1.

Quyidagi funksiyalarning egilish nuqtalarini, qavariqlik va botiqlik oraliqlarini toping:

694. $y=xe^x$ *Javobi*: $(-2;-2e^{-2})$; $(-\infty;-2)$ da qavariq; $(-2;+\infty)$ da botiq.

695. $y=x^5+5x-6$. *Javobi*: $(0;6)$; $(-\infty;0)$ da qavariq; $(0;+\infty)$ da botiq.

696. $y=(x-4)^5+4x+4$. *Javobi*: $(4;20)$; $(-\infty;4)$ da botiq; $(4;+\infty)$ da qavariq.

697. $y=e^{-\frac{x^2}{2}}$. *Javobi:* $\left(-1;e^{-\frac{1}{2}}\right)$ *va* $\left(1;e^{-\frac{1}{2}}\right)$; $(-\infty;-1)$ *va* $(1;+\infty)$ da botiq; $(-1;1)$ da qavariq.

21-MAVZU. ANIQMAS INTEGRAL

Ta'rif. Agar ***F(x)*** funksiyaning hosilasi ***f(x)*** funksiyaga teng, ya'ni $F'(x)=f(x)$ bo'lsa, u holda ***F(x)*** funksiyaga ***f(x)*** funksiyaning boshlang'ich funksiyasi deyiladi.

Ta'rif. ***f(x)*** funksiyaning barcha boshlang'ich funksiyalaridan iborat {***F(x)+c***} to'plamni ***f(x)*** funksiyaning ANIQMAS INTEGRALI deyiladi va

$$\int f(x)dx=F(x)+c$$

ko'rinishda yoziladi. ***c*** **– o'zgarmas son**

INTEGRALLASH QOIDALARI

$$I.\ \int f(x)dx=F(x)+c$$

$$II.\int kf(x)dx=k\int f(x)dx+c.\quad k-const.,\quad k\neq 0$$

$$III.\ \int[f(x)\pm g(x)]dx=\int f(x)dx\pm\int g(x)dx+c$$

$$IV.\ \int dF(x)=F(x)+c.,\quad d[\int f(x)dx]=f(x)dx.$$

ASOSIY INTEGRALLAR JADVALI

1. $\int dx=x+c;$
2. $\int x^m dx=\dfrac{x^{m+1}}{m+1}+c\ (m\neq-1);$
3. $\int a^x dx=\dfrac{a^x}{\ln a}+c\ \ (a>0;a\neq1);$
4. $\int e^x dx=e^x+c;$
5. $\int\dfrac{dx}{x}=\ln|x|+c;$
6. $\int\sin x dx=-\cos x+c;$
7. $\int\cos dx=\sin x+c;$
8. $\int\dfrac{dx}{\sin^2 x}=-ctgx+c;$
9. $\int\dfrac{dx}{\cos^2 x}=tgx+c;$
10. $\int tgxdx=-\ln|\cos x|+c.$
11. $\int ctgxdx=\ln|\sin x|+c.$
12. $\int\dfrac{dx}{\sqrt{1-x^2}}=\arcsin x+c.$
13. $\int\dfrac{dx}{1+x^2}=arctgx+c;$
14. $\int f(ax+b)dx=\dfrac{1}{a}F(ax+b)+c.$

Misol 698. $\int(x+1)^2dx$ ni integrallang.

Yechilishi.

$$\int (x+1)^2 dx = \int (x^2+2x+1)dx = \int x^2dx + 2\int xdx + \int dx = \frac{x^{2+1}}{2+1} + 2\cdot\frac{x^{1+1}}{1+1} + x + c =$$

$$= \frac{x^3}{3} + x^2 + x + c$$

Misol 699. $\int (\cos 3x + 2e^x + \frac{3}{\sin^2 x})dx$ ni integrallang.

Yechilishi.

$$\int (\cos 3x + 2e^x + \frac{3}{\sin^2 x})dx = \int \cos 3x \cdot dx + 2\int e^x dx + 3\int \frac{dx}{\sin^2 x} = \frac{1}{3}\sin 3x + 2e^x - 3ctgx + c.$$

Misol 700. $\int (x^2+5)^3 dx$ ni integrallang.

Yechilishi.

$$\int (x^2+5)^3 dx = \int [(x^2)^3 + 3(x^2)^2 * 5 + 3x^2 * 5^2 + 5^3]dx = \int (x^6 + 15x^4 + 75x^2 + 125)dx =$$

$$= \int x^6 dx + \int 15x^4 dx + \int 75x^2 dx + \int 125 dx = \int x^6 dx + 15\int x^4 dx + 75\int x^2 dx + 125\int dx =$$

$$= \frac{x^{6+1}}{6+1} + 15\frac{x^{4+1}}{4+1} + 75\frac{x^{2+1}}{2+1} + 125x + c = \frac{x^7}{7} + 3x^5 + 25x^3 + 125x + c.$$

Misol 701. $\int \cos 4x dx$ ni integrallang.

Yechilishi. $\int \cos 4x dx = \frac{1}{4}\sin 4x + c.$

ANIQMAS INTEGRALDA O'ZGARUVCHINI ALMASHTIRISH

$$\int f(x)dx = \int f(\varphi(t))\,\varphi'(t)\cdot dt = F(\varphi(t)) + c$$

Misol 702. $\int (2+3x)^5 dx$ ni integrallang.

Yechilishi.

$$\int (2+3x)^5 dx = \left| \begin{array}{l} 2+3x = t \Rightarrow 3x = t-2 \Rightarrow \\ \Rightarrow x = \frac{t-2}{3} \Rightarrow x = \frac{1}{3}\cdot t - \frac{2}{3} \Rightarrow \\ \Rightarrow dx = \frac{1}{3}dt \end{array} \right| = \int t^5 \cdot \frac{1}{3}\cdot dt = \frac{1}{3}\int t^5 \quad dt = \frac{1}{3}\cdot\frac{t^{5+1}}{5+1} + c = \frac{t^6}{18} + c = \frac{(2+3x)^6}{18} + c.$$

Misol 703. $\int (x-3)^5 dx$ ni integrallang.

Yechilishi.

$$\int (x-3)^5 dx = \left| \begin{array}{l} t = x-3; \\ x = t+3; \\ dx = dt \end{array} \right| = \int t^5 dt = \frac{t^{5+1}}{5+1} + c = \frac{(x-3)^6}{6} + c.$$

Misol 704. $\int \cos(5x-3)dx$ ni integrallang.

Yechilishi.

$$\int \cos(5x-3)dx = \left| \begin{array}{l} t = 5x-3 \\ 5x = t+3; \\ x = \frac{1}{5}t + \frac{3}{5}; \\ dx = \frac{1}{5}dt. \end{array} \right| = \int \cos t * \frac{1}{5}dt = \frac{1}{5}\int \cos tdt = \frac{1}{5}\sin t + c = \frac{1}{5}\sin(x-3) + c.$$

ANIQMAS INTEGRALLARNI BO'LAKLAB INTEGRALLASH

u=u(x) va ***v=v(x)*** funksiyalarning uzluksiz ***u′(x)*** va ***v′(x)*** hosilalari mavjud bo'lsin. U holda ko'paytmadan hosila olish qoidasiga ko'ra,

$$d(u \cdot v) = udv + vdu;$$

$$\int u \cdot dv = \int d(u \cdot v) - \int v \cdot du;$$

$$\int u \cdot dv = u \cdot v - \int vdu;$$

Misol 705. $\int xe^x dx$ ni integrallang.

Yechilishi.

$$\int x\ e^x dx = \left| \begin{array}{l} u = x \Rightarrow dx = du; \\ dv = e^x dx; \int dv = \int e^x dx; \\ v = e^x. \end{array} \right| = x \cdot e^x - \int e^x dx = x \cdot e^x - e^x + c.$$

Misol 706. $\int \ln xdx$ ni integrallang.

Yechilishi.

$$\int ln\, xdx = \left| \begin{array}{l} u = ln\, x;\ du = \frac{1}{x}dx; \\ dv = dx; \quad \int dv = \int dx; \\ v = x. \end{array} \right| = x \cdot ln\, x - \int x \cdot \frac{1}{x} \cdot dx = x\, ln\, x - x + c.$$

Misol 707. $\int x\cos xdx$ ni integrallang.

Yechilishi.

$$\int x\cos xdx = \left| \begin{array}{l} u = x \Leftrightarrow du = dx; \\ dv = \cos xdx; \\ \int dv = \int \cos xdx; \\ v = \sin x \end{array} \right| = x * \sin x - \int \sin xdx = x\sin x - (-\cos x) + c =$$

$$= x\sin x + \cos x + c.$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi integrallarni toping:

708. $\int\left(x^3+2x+\frac{4}{x}\right)dx$; Javobi: $\frac{x^4}{4}+x^2+4\ln x+c$.

709. $\int(1-\frac{1}{\cos^2 3x})dx$; Javobi: $x-\frac{1}{3}tg3x+c$.

710. $\int\frac{x^2+2x+2}{\sqrt{x^3}}dx$; Javobi: $\frac{2}{3}\sqrt{x^3}+4\sqrt{x}-\frac{4}{\sqrt{x}}+c$.

711. $\int 2\sin 3x dx$; Javobi: $-\frac{2}{3}\cos 3x+c$.

712. $\int(\frac{1}{3}x^2-6x+\frac{3}{4})dx$; Javobi: $\frac{x^3}{9}-3x^2+\frac{3}{4}x+c$

713. $\int\sqrt[3]{x^2}dx$; Javobi: $\frac{3}{5}\sqrt[3]{x^5}+c.$

714. $\int 3\cos 3x dx$. Javobi: $\sin 3x+c.$

Aniqmas integralda o'zgaruvchini almashtirishga doir misollar:

715. $\int\sqrt{2x}dx;$ Javobi: $\frac{1}{3}\sqrt{(2x)^3}+c.$

716. $\int\sqrt{3x+5}dx;$ Javobi: $\frac{2}{9}\sqrt{(3x+5)^3}+c.$

717. $\int\sin(5x-3)dx;$ Javobi: $-\frac{1}{5}\cos(5x-3)+c.$

718. $\int\cos(16x+5)dx;$ Javobi: $\frac{1}{16}\sin(16x+5)+c.$

719. $\int\frac{xdx}{x^2+1};$ Javobi: $\frac{1}{2}\ln(x^2+1)+c.$

720. $\int\ln(4x-6)dx;$ Javobi: $\frac{4}{4x-6}+c.$

Aniqmas integralni bo'laklab integrallashga doir misollar:

721. $\int x\sin x dx$; Javobi: $-x\cos x-\sin x+c.$

722. $\int x\cos 2x dx;$ Javobi: $\frac{1}{2}x\sin 2x+\cos 2x+c.$

723. $\int(2x+1)\sin 3x dx;$ Javobi: $(2x+1)(-\frac{1}{3}\cos 3x)+\frac{2}{9}\sin 3x+c.$

724. $\int x\,arctgx dx;$ Javobi: $\frac{x^2}{2}arc\,tgx-\frac{x}{2}+\frac{1}{2}arctgx+c.$

22-MAVZU. RATSIONAL KASRLARNI VA TRIGONOMETRIK FUNKSIYALARNI INTEGRALLASH

RATSIONAL KASRLARNI INTEGRALLASH

$$R(x)=\frac{Q_m(x)}{P_n(x)}$$

ko'rinishdagi funksiyaga ***kasr ratsional funksiya*** yoki ***ratsional kasr*** deyiladi. Bunda ***m*** va ***n*** $Q_m(x)$ va $P_n(x)$ ko'phadlarning daraja ko'rsatkichlari bo'lib, ular natural sonlardan iborat. ***m<n*** da ***R(x)*** kasr ratsional funksiya to'g'ri kasr, $m\geq n$ da esa noto'g'ri kasr bo'ladi.

Quyidagi to'g'ri kasrlar eng sodda kasrlar deyiladi.

I. $\frac{A}{x-\alpha}$

II. $\frac{A}{(x-\alpha)^k}$, bunda $k\geq 2$ - butun son.

III. $\frac{Ax+B}{x^2+px+q}$, bunda $D=p^2-4q<0$.

IV. $\frac{Ax+B}{(x^2+px+q)^S}$, bunda $S\geq 2$ - butun son, $D=p^2-4q<0$.

A,B,p,q, α - haqiqiy sonlar.

To'g'ri ratsional kasrlarni integrallash eng sodda integrallashga keltiriladi.

V. $\int\frac{Adx}{x-\alpha}=A\ln|x-\alpha|+c$

VI. $\int\frac{Adx}{(x-\alpha)^k}=\frac{A}{1-k}\cdot(x-\alpha)^{1-k}+C=\frac{A}{(1-k)(x-\alpha)^{k-1}}+C$

VII. $\int\frac{Ax+B}{x^2+px+q}dx=\frac{A}{2}\ln\left|x^2+px+q\right|+(B-\frac{Ap}{2})\cdot\frac{1}{\sqrt{q-\frac{p^2}{4}}}arctg\frac{x+\frac{p}{2}}{\sqrt{q-\frac{p^4}{4}}}+C$ yoziladi.

VIII. $\int\frac{Ax+B}{(x^2+px+q)^S}dx=\int\frac{\frac{A}{2}(2x+p)+B-\frac{Ap}{2}}{\left(x^2+px+q\right)^S}dx=$

$$=\frac{A}{2}\cdot\frac{1}{(1-S)\left(x^2+px+q\right)^{S-1}}+\left(B-\frac{Ap}{2}\right)\int\frac{d\left(x+\frac{p}{2}\right)}{\left(\left(x+\frac{p}{2}\right)^2+q-\frac{p^2}{4}\right)}.$$

Bunda $a^2=q-\frac{p^2}{4}$, $t=x+\frac{p}{2}$ belgilashlar kiritib, ikkinchi integral $I_S=\int\frac{dt}{(t^2+a^2)^S}$ ko'rinishga keltiriladi va quyidagi rekkurrent formula yordamida topiladi:

$$I_S=\frac{t}{2(S-1)a^2(t^2+a^2)^{S-1}}+\frac{2S-3}{2(S-1)a^2}I_{S-1}.$$

Misol 725. $R(x)=\dfrac{15x^2-4x-81}{(x-3)(x+4)(x-1)}$ ratsional kasrni integrallang.

Yechilishi. Berilgan R(x) ratsional kasr to'g'ri kasr. Maxraj ildizlarining ***3,-4,1*** ekanligini e'tiborga olgan holda ratsional kasrni quyidagicha yozish mumkin:

$$R(x)=\frac{15x^2-4x-81}{(x-3)(x+4)(x-1)}=\frac{A}{x-3}+\frac{B}{x+4}+\frac{C}{x-1}$$

bunda A,B,C –noma'lum koeffitsientlar.

Kasr qatnashgan tenglikda umumiy maxraj berilib, maxraj tashlab yuborilsa,

$15x^2-4x-81=A(x+4)(x-1)+B(x-3)(x-1)+C(x-3)(x+4)$ bo'ladi.

$$15x^2-4x-81=A\left(x^2+3x-4\right)+B\left(x^2-4x+3\right)+C\left(x^2+x-12\right);$$

$$15x^2-4x-81=\left(A+B+C\right)x^2+\left(3A-4B+C\right)x-4A+3B-12C;$$

$$\begin{cases} A+B+C=15; \\ 3A-4B+C=-4; \\ -4A+3B-12C=-81. \end{cases}$$

$$\Delta=\begin{vmatrix}1 & 1 & 1\\ 3 & -4 & 1\\ -4 & 3 & -12\end{vmatrix}=48-4+9-16+36-3=70;$$

$$\Delta_A=\begin{vmatrix}15 & 1 & 1\\ -4 & -4 & 1\\ -81 & 3 & -12\end{vmatrix}=720-81-12-324-48-45=210;$$

$$\Delta_B=\begin{vmatrix}1 & 15 & 1\\ 3 & -4 & 1\\ -4 & -81 & -12\end{vmatrix}=48-60-243-16+540+81=350;$$

$$\Delta_C=\begin{vmatrix}1 & 1 & 15\\ 3 & -4 & -4\\ -4 & 3 & -81\end{vmatrix}=324+16+135-240+243+12=490.$$

$$A=\frac{\Delta_A}{\Delta}=\frac{210}{70}=3;\quad B=\frac{\Delta_B}{\Delta}=\frac{350}{70}=5;\quad C=\frac{\Delta_C}{\Delta}=\frac{490}{70}=7.$$

Hosil qilingan tenglamalar sistemasidan: ***A=3, B=5, C=7.***

U holda, $R(x)=\dfrac{15x^2-4x-81}{(x-3)(x+4)(x-1)}=\dfrac{3}{x-3}+\dfrac{5}{x+4}+\dfrac{7}{x-1}$.

Bundan

$$\int R(x)dx=\int\frac{15x^2-4x-81}{(x-3)(x+4)(x-1)}dx=\int\frac{3}{x-3}dx+\int\frac{5}{x+4}dx+\int\frac{7}{x-1}dx=$$

$$=3\int\frac{d(x-3)}{x-3}+5\int\frac{d(x+4)}{x+4}+7\int\frac{d(x-1)}{x-1}=3\ln|x-3|+5\ln|x+4|+7\ln|x-1|+\ln C=$$

$$=\ln(x-3)^3+\ln(x+4)^5+\ln(x-1)^7+\ln C=\ln C\cdot(x-3)^3(x+4)^5(x-1)^7.$$

Misol 726. $R(x)=\dfrac{x}{(x-1)(x+1)^2}$ ratsional kasrni integrallang.

Yechilishi: Bu ratsional kasrni quyidagicha yoyib yozish mumkin:

$$R(x)=\frac{x}{(x-1)(x+1)^2}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}.$$

Bundan $x=A(x+1)^2+B(x-1)(x+1)+C(x-1)$;

$$x=(A+B)x^2+(2A+C)x+A-B-C \Rightarrow \begin{cases} A+B=0 \\ 2A+C=1 \\ A-B-C=0 \end{cases} \Rightarrow \begin{cases} A=-B \\ 2A+C=1 \\ C=A-B \end{cases} \Rightarrow \begin{cases} B=-A \\ 4A=1 \\ C=2A \end{cases} \Rightarrow$$

$$\Rightarrow A=\frac{1}{4}, \quad B=-\frac{1}{4}, \quad C=\frac{1}{2}.$$

Demak, $R(x)=\dfrac{x}{(x-1)(x+1)^2}=\dfrac{1}{4(x-1)}-\dfrac{1}{4(x+1)}+\dfrac{1}{2(x+1)^2}$.

$$\int R(x)dx=\int\frac{xdx}{(x-1)(x+1)^2}=\frac{1}{4}\int\frac{dx}{x-1}-\frac{1}{4}\int\frac{dx}{x+1}+\frac{1}{2}\int\frac{dx}{(x+1)^2}=$$

$$=\frac{1}{4}ln|x-1|-\frac{1}{4}ln|x+1|-\frac{1}{2(x+1)}+C=\frac{1}{4}ln\left|\frac{x-1}{x+1}\right|-\frac{1}{2(x+1)}+C.$$

Misol 727. $R(x)=\dfrac{x^4+4x^3+11x^2+12x+8}{(x+1)(x^2+2x+3)^2}$ ratsional kasrni integrallang.

Yechilishi. $R(x)=\dfrac{x^4+4x^3+11x^2+12x+8}{(x+1)(x^2+2x+3)^2}=\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+2x+3}+\dfrac{Dx+E}{(x^2+2x+3)^2}$.

Umumiy maxraj berilgandan so'ng:

$$x^4+4x^3+11x^2+12x+8=A(x^2+2x+3)^2+(Bx+C)(x+1)(x^2+2x+3)+(Dx+E)(x+1)=$$

$$=Ax^4+4Ax^3+10Ax^2+12Ax+9A+Bx^4+3Bx^3+5Bx^2+3Bx+Cx^3+3Cx^2+5Cx+3C+$$

$$+Dx^2+Dx+Ex+E$$

Bundan

$$x^4+4x^3+11x^2+12x+8=(A+B)x^4+(4A+3B+C)x^3+(10A+5B+3C+D)x^2+$$

$$+(12A+3B+5C+D+E)x+9A+3C+E \Rightarrow$$

$$\Rightarrow\begin{cases} A+B=1; \\ 4A+3B+C=4; \\ 10A+5B+3C+D=11; \\ 12A+3B+5C+D+E=12; \\ 9A+3C+E=8. \end{cases} \Rightarrow A=1,\ B=0,\ C=0,\ D=1,\ E=-1.$$

Demak,

$$R(x)=\frac{x^4+4x^3+11x^2+12x+8}{(x+1)(x^2+2x+3)^2}=\frac{1}{x+1}+\frac{x-1}{(x^2+2x+3)^2}.$$

U holda

$$\int R(x)=\int\frac{dx}{x+1}+\int\frac{x-1}{(x^2+2x+3)^2}dx=\int\frac{d(x+1)}{x+1}+\frac{1}{2}\int\frac{d(x^2+2x+3)-2}{(x^2+2x+3)^2}=$$

$$=ln|x+1|+\frac{1}{2}\int\frac{d(x^2+2x+3)}{(x^2+2x+3)^2}-\int\frac{dx}{(x^2+2x+3)^2}=ln|x+1|+\frac{1}{2}\int(x^2+2x+3)^{-2}d(x^2+2x+3)-$$

$$-\int\frac{dx}{\left[(x+1)^2+2\right]^2}=\ln|x+1|-\frac{1}{x^2+2x+3}-\int\frac{dx}{\left[(x+1)^2+2\right]^2}.$$

Oxirgi integralda $t=x+1,\ a^2=2$ almashtirish olinib $I_2=\int\frac{dt}{\left(t^2+a^2\right)^2}$ ko'rinishga keltiriladi va quyidagi rekkurent formula yordamida yechiladi:

$$I_s=\frac{t}{2(s-1)\cdot a^2\cdot\left(t^2+a^2\right)^{s-1}}+\frac{2S-3}{2(S-1)\cdot a^2}\cdot I_{S-1}$$

$$I_2=\frac{x+1}{2\cdot(2-1)\cdot 2\left[(x+1)^2+2\right]^{2-1}}+\frac{2\cdot 2-3}{2\cdot(2-1)\cdot 2}\cdot I_{2-1}=\frac{x+1}{4\left[(x+1)^2+2\right]}+\frac{1}{4}\cdot I_1.$$

Demak,

$$\int R(x)dx=\ln|x+1|-\frac{1}{x^2+2x+3}+\frac{x+1}{4\left(x^2+2x+3\right)}+\frac{1}{4}\cdot I_1.$$

Misol 728. $\int\frac{x^4+3x^2-5}{x^3+2x^2+5x}dx$ ratsional kasrni integrallang.

Yechilishi. Surat maxrajga bo'linib, to'g'ri ratsianal kasrga keltiriladi:

$$\frac{x^4+3x^2-5}{x^3+2x^2+5x}=x-2+\frac{2x^2+10x-5}{x^3+2x^2+5x}.$$

Oxirgi kasr sodda kasrlar ko'rinishida yoziladi:

$$\frac{2x^2+10x-5}{x(x^2+2x+5)}=\frac{A}{x}+\frac{Mx+N}{x^2+2x+5}.$$

Bu tenglikning o'ng qismini umumiy maxrajga keltirilib, kasrlarning suratlari tenglanadi:

$$2x^2+10x-5=Ax^2+2Ax+5A+Mx^2+Nx;\quad 2x^2+10x-5=$$

$$=(A+M)x^2+(2A+N)x+5A;\Rightarrow\begin{cases}A+M=2\\2A+N=10\\5A=-5\end{cases}\Rightarrow\begin{cases}A=-1;\\M=3;\\N=12.\end{cases}$$

U holda

$$\int\frac{2x^2+10x-5}{x^3+2x^2+5x}dx=\int(x-2-\frac{1}{x}+\frac{3x+12}{x^2+2x+5})dx=\int(x-2)d(x-2)-\int\frac{dx}{x}+\frac{3}{2}\int\frac{2x+2+6}{x^2+2x+5}dx=\frac{(x-2)^2}{2}-$$

$$-\ln|x|+\frac{3}{2}\int\frac{d(x^2+2x+5)}{x^2+2x+5}+\frac{3}{2}*6*\int\frac{dx}{(x+1)^2+1}=\frac{(x-2)^2}{2}-\ln|x|+\frac{3}{2}\ln|x^2+2x+5|+\frac{9}{2}arctg\frac{x+1}{2}+c.$$

Misol 729. $\int\frac{3x-1}{x^2+4x+8}dx$ integralni toping.

Yechilishi:

$$\int\frac{3x-1}{x^2 4x+8}dx=\int\frac{\frac{3}{2}(2x-4)+6-1}{x^2-4x+8}dx=\frac{3}{2}\int\frac{2x-4}{x^2-4x+8}dx+5\int\frac{dx}{x^2-4x+8}=\frac{3}{2}\ln\left|x^2-4x+8\right|+$$

$$+5\int\frac{dx}{(x-2)^2+2^2}=\frac{3}{2}\ln\left|x^2-4x+8\right|+\frac{5}{2}arctg\frac{x-2}{2}+C$$

Misol 730. $\int\frac{x^5+1}{x^4-8x^2+16}dx$ integralni toping.

Yechilishi: Berilgan ratsional kasr noto'g'ri kasr bo'lganligi uchun uning butun qismi ajratiladi.

Demak,

$$\frac{x^5+1}{x^4-8x^2+16}=x+\frac{8x^3-16x+1}{x^4-8x^2+16}=x+\frac{8x^3-16x+1}{(x-2)^2(x+2)^2}=x+\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{x+2}+\frac{D}{(x+2)^2}$$

to'g'ri kasr eng sodda kasrlar yig'indisiga yoyiladi.

$$8x^3-16x+1=A(x+2)^2(x-2)+B(x+2)^2+C(x-2)^2(x+2)+D(x-2)^2.$$

Noma'lum koeffitsientlarni topish uchun tenglamalar sistemasi tuziladi:

$$\begin{array}{r|l} x=2 & 33=16B \\ x=-2 & -31=16D \\ x^3 da & 8=A+C \\ x^2 da & 0=2A+B-2C+D \end{array}$$

Bu sistemani yechib koeffitsientlar topiladi. $A=\frac{127}{32}, B=\frac{33}{16}, C=\frac{129}{32}, D=-\frac{31}{16}$.

Demak,

$$\int\frac{(x^5+1)dx}{x^4-8x^2 16}=\int\left(x+\frac{\frac{127}{32}}{x-2}+\frac{\frac{33}{16}}{(x-2)^2}+\frac{\frac{129}{32}}{x+2}-\frac{\frac{31}{16}}{(x+2)^2}\right)dx=\frac{x^2}{2}+\frac{127}{32}\ln|x-2|-\frac{33}{16(x-2)}+$$

$$+\frac{129}{32}\ln|x+2|+\frac{31}{16(x+2)}+C$$

TRIGONOMETRIK FUNKSIYALARNI INTEGRALLASH

$\int sin^n x\,cos^m x dx$ ko'rinishdagi integrallar quyidagicha topiladi:

a) Agar ***n>0*** toq bo'lsa, $cos\, x=t,\quad sin\, xdx=-dt$ o'rniga qo'yish integralni ratsionallashtiradi.

Misol 731. $\int\sin^3 x\cdot\cos^2 xdx$ integralni toping.

Yechilishi: $\sin^3 x$ da bitta $\sin x$ ko'paytuvchini ajratamiz va uni differentsial ostiga kiritamiz:

$$\int sin^3 x\cdot cos^2 xdx=\int sin^2 x\cdot cos^2 x\,sin\,xdx=-\int sin^2 x\,cos^2 xd(cos\,x)=-\int(1-cos^2 x)cos^2 xd(cos\,x)=$$

$$=-\int(cos^2 x-cos^4 x)d(cos\,x)=C-\frac{1}{3}cos^3 x+\frac{1}{5}cos^5 x.$$

b) Agar ***m>0*** toq bo'lsa, u holda $\sin x=t,\ \cos xdx=dt$ o'rniga qo'yish integralni ratsionallashtiradi.

Misol 732. $\int\frac{\cos^3 xdx}{\sqrt[3]{\sin^4 x}}$ integralni toping.

Yechilishi:

$$\int\frac{cos^3 xdx}{sin^{\frac{4}{3}} x}=\int\frac{cos^2 x\,cos\,xdx}{sin^{\frac{3}{4}} x}=\int\frac{(1-sin^2 x)d(sin\,x)}{sin^{\frac{4}{3}} x}=\int(sin^{-\frac{4}{3}} x-sin^{\frac{2}{3}} x)d(sin\,x)=$$

$$=-3sin^{-\frac{1}{3}} x-\frac{3}{5}sin^{\frac{5}{3}} x+C=C-\frac{3}{\sqrt[3]{sin\,x}}-\frac{3}{5}\sqrt[3]{sin^5 x}.$$

Misol 733. $\int\sin^4 xdx$ integralni toping.

Yechilishi:

$$\int sin^4 xdx = \int (sin^2 x)^2 dx = \int\left(\frac{1-cos\,2x}{2}\right)^2 dx = \frac{1}{4}(1-2cos\,2x+cos^2 2x)dx =$$

$$=\frac{1}{4}(x - sin\,2x + \int cos^2 2xdx) = \frac{1}{4}(x - sin\,2x + \frac{1}{2}\int(1+cos\,4x)dx) =$$

$$=\frac{1}{4}(x - sin\,2x + \frac{1}{2}x + \frac{1}{8}sin\,4x) + C = \frac{1}{4}\left(\frac{3x}{2} - sin\,2x + \frac{1}{8}sin\,4x\right) + C.$$

Misol 734. $\int cos\,x * cos\,2x * cos\,4xdx$ integralni toping.

Yechilishi. Ko'paytmadan yig'indiga o'tish formulasi ikki marta qo'llaniladi:

$$\int cos\,x * cos\,2x * cos\,4xdx = \frac{1}{2}\int [cos\,3x + cos(-x)]\,cos\,4xdx = \frac{1}{2}\int cos\,3x\,cos\,4xdx + \frac{1}{2}\int cos\,x\,cos\,4xdx =$$

$$=\frac{1}{4}\int [cos\,7x + cos(-x)]dx + \frac{1}{4}\int [cos\,5x + cos(-3x)]dx = \frac{1}{4}(\frac{1}{7}sin\,7x + \frac{1}{5}sin\,5x + \frac{1}{3}sin\,3x + sin\,x) + c.$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi integrallarni toping:

735. $\int \frac{x^4-3x^2-3x-2}{x^3-x^2-2x}dx.$ *Javobi:* $\frac{(x+1)^2}{2}+\frac{1}{3}\ln\frac{|x|^3}{(x-2)^2(x+1)}+C\cdot$

736. $\int \frac{2x^2-3x+3}{x^3-2x^2+x}dx$ *Javobi:* $C-\frac{2}{x-1}+ln\frac{|x|^2}{|x-1|}.$

737. $\int \frac{x-4}{(x-2)(x-3)}dx$ *Javobi:* $\ln\frac{C(x-)^2}{x-3}.$

738. $\int \frac{(x+1)^3}{x^2-x}dx$ *Javobi:* $\frac{x^2}{2}+4x+\ln\frac{(x-1)^8}{|x|}+C\cdot$

739. $\int \frac{x^3+1}{x(x-1)^3}dx;$ *Javobi :* $C-\frac{x}{(x-1)^2}+ln\frac{(x-1)^2}{|x|}.$

740. $\int cos^3 x * sin^6 xdx$ *Javobi :* $C+\frac{1}{3sin^3 x}-\frac{1}{5sin^5 x}.$

741. $\int sin^3 xdx$ *Javobi :* $C-cos\,x+\frac{cos^3 x}{3}+C.$

742. $\int \frac{sin^2 x}{cos^6 x}$ *Javobi :* $\frac{1}{5}tg^5 x+\frac{1}{3}tg^3 x+C.$

3-mavzu. ANIQ INTEGRAL

INTEGRALLASH FORMULALARI

c - o'zgarmas son

Integrallash qoidalari:

$$I.\int f(x)dx = F(x)+c$$

$$II.\int kf(x)dx = k\int f(x)dx + c.\quad k-const.,\ k\neq 0$$

$$III.\int [f(x)\pm g(x)]dx = \int f(x)dx \pm \int g(x)dx + c$$

$$IV.\int dF(x) = F(x)+c.,\ d[\int f(x)dx] = f(x)dx.$$

INTEGRALLAR JADVALI VA BOSHQA MUHIM FORMULALAR:

1. $\int dx = x + c.$

2. $\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1.$

3. $\int \frac{dx}{x} = \ln|x| + c.$

4. $\int a^x dx = \frac{a^x}{\ln a} + c.$

5. $\int e^x dx = e^x + c.$

6. $\int \sin x dx = -\cos x + c.$

7. $\int \cos x dx = \sin x + c.$

8. $\int \frac{dx}{\cos^2 x} = tgx + c.$

9. $\int \frac{dx}{\sin^2 x} = -ctgx + c.$

10. $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + c.$

11. $\int \frac{dx}{1+x^2} = arctgx + c.$

12. $\int f(ax+b)dx = \frac{1}{a}F(ax+b) + c.$

13. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \ (a < c < b).$

14. $\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a).$

15. $\int_a^b f(x)dx = \int_\alpha^\beta f(\varphi(t))\varphi'(t)dt.$ 16. $\int_a^b udv = uv\Big|_a^b - \int_a^b vdu.$

17. $V = \pi\int_a^b f^2(x)dx.$

NYUTON – LEYBNITS FORMULASI

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a).$$

Misol 743.

$$\int_{-1}^{2}(x-3)^2dx = \int_{-1}^{2}(x^2-6x+9)dx = \int_{-1}^{2}x^2dx - \int_{-1}^{2}6xdx + \int_{-1}^{2}9dx =$$

$$= \int_{-1}^{2}x^2dx - 6\int_{-1}^{2}xdx + 9\int_{-1}^{2}dx = \frac{x^{2+1}}{2+1}\Big|_{-1}^{2} - 6\frac{x^{1+1}}{1+1}\Big|_{-1}^{2} + 9x\Big|_{-1}^{2} =$$

$$= \frac{x^3}{3}\Big|_{-1}^{2} - 6\frac{x^2}{2}\Big|_{-1}^{2} + 9x\Big|_{-1}^{2} = \frac{1}{3}*x^3\Big|_{-1}^{2} - 3x^2\Big|_{-1}^{2} + 9x\Big|_{-1}^{2} =$$

$$= \frac{1}{3}[2^3-(-1)^3] - 3[2^2-(-1)^2] + 9[2-(-1)] = \frac{1}{3}(8+1) - 3(4-1) + 9(2+1) =$$

$$= \frac{1}{3}*9 - 3*3 + 9*3 = 3 - 9 + 27 = 21.$$

Misol 744.

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin 6x dx = \frac{1}{6}(-\cos 6x)\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = -\frac{1}{6}\cos 6x\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = -\frac{1}{6}[\cos 6*\frac{\pi}{2} - \cos 6*\frac{\pi}{3}] =$$

$$= -\frac{1}{6}[\cos 3\pi - \cos 2\pi] = -\frac{1}{6}[-1-1] = \frac{1}{3}.$$

O'ZGARIVCHILARNI ALMASHTIRISH

$$\int_a^b f(x)dx = \int_\alpha^\beta f(\varphi(t))\varphi'(t)dt$$

Misol 745.

$$\int_0^1 \frac{x^2dx}{(x+1)^2} = \left| \begin{array}{l} 1. O'zgaruvchi\ almashtiriladi: \\ x+1=t \Leftrightarrow x=t-1 \Leftrightarrow \\ \Leftrightarrow dx = dt; \\ 2. Chegaralar\ almashtiriladi: \\ x=0 \Rightarrow t=1;\ x=1 \Rightarrow t=2. \end{array} \right| = \int_1^2 \frac{(t-1)^2}{t^2}dt = \int_1^2 \frac{t^2-2t+1}{t^2}dt =$$

$$= \int_1^2 (1 - \frac{2}{t} + \frac{1}{t^2})dt = \int_1^2 dt - 2\int_1^2 \frac{dt}{t} + \int_1^2 t^{-2}dt = t\Big|_1^2 - 2\ln|t|\Big|_1^2 + \frac{t^{-2+1}}{-2+1}\Bigg|_1^2 = t\Big|_1^2 - 2\ln|t|\Big|_1^2 - \frac{1}{t}\Bigg|_1^2 =$$

$$= 2-1-2[\ln|2| - \ln|1|] - [\frac{1}{2} - \frac{1}{1}] = 1 - 2[\ln 2 - 0] - \frac{1}{2} + 1 = 2 - \frac{1}{2} - 2\ln 2 = \frac{3}{2} - 2\ln 2.$$

Misol 746.

$$\int_0^{\frac{\pi}{2}} \cos^2 x \sin x dx = \left| \begin{array}{l} 1.\ t = \cos x; \\ dt = -\sin x dx; \\ \sin x dx = -dt; \\ 2. x=0 \Rightarrow t=1; \\ x = \frac{\pi}{2} \Rightarrow t = 0. \end{array} \right| = \int_1^0 t^2(-dt) = -\int_1^0 t^2 dt = \int_0^1 t^2 dt = \frac{t^3}{3}\Bigg|_0^1 = \frac{1}{3}t^3\Bigg|_0^1 =$$

$$= \frac{1}{3}[1^3 - 0^3] = \frac{1}{3}.$$

BO'LAKLAB INTEGRALLASH

$$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu.$$

Misol 747.

$$\int_0^{\pi} x\sin x dx = \left| \begin{array}{l} u = x \Leftrightarrow du = dx; \\ dv = \sin x dx \Leftrightarrow \\ \Leftrightarrow \int dv = \int \sin x dx \Rightarrow \\ \Rightarrow v = -\cos x. \end{array} \right| = -x*\cos x\Big|_0^{\pi} + \int_0^{\pi} \cos x dx = -x*\cos x\Big|_0^{\pi} + \sin x\Big|_0^{\pi} =$$

$$= -[\pi * \cos\pi - 0*\cos 0] + \sin\pi - \sin 0 = -(\pi*(-1) - 0) + 0 - 0 = \pi.$$

Misol 748.

$$\int_0^{\frac{\pi}{2}} x^2\cos x dx = \left| \begin{array}{l} u = x^2 \Leftrightarrow du = 2xdx; \\ dv = \cos x dx \Leftrightarrow \\ \int dv = \int \cos x dx \Rightarrow \\ \Rightarrow v = \sin x. \end{array} \right| = x^2 \sin x\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \cdot 2xdx = x^2\sin x\Big|_0^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}} x\sin x dx =$$

$$= \left| \begin{array}{l} u = x \Leftrightarrow du = dx; \\ dv = \sin x dx \Leftrightarrow \\ \Leftrightarrow \int dv = \sin x dx \Rightarrow \\ \Rightarrow v = -\cos x. \end{array} \right| = x^2\sin x\Big|_0^{\frac{\pi}{2}} - 2[-x\cos x\Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x dx] = x^2\sin x\Big|_0^{\frac{\pi}{2}} + 2x\cos x\Big|_0^{\frac{\pi}{2}} -$$

$$-2\sin x\Big|_0^{\frac{\pi}{2}} = (\frac{\pi}{2})^2 \cdot \sin\frac{\pi}{2} - 0^2 \cdot \sin 0 + 2[\frac{\pi}{2}\cos\frac{\pi}{2} - 0\cdot\cos 0] - 2[\sin\frac{\pi}{2} - \sin 0] = \frac{\pi^2}{4} - 2.$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi aniq integrallarni hisoblang:

749. $\int_0^3 x^2 dx$; **Javobi: 9.**

750. $\int_1^2 \frac{1}{x^3} dx$ **Javobi:** $\frac{3}{8}$**.**

751. $\int_{-2}^3 2xdx$ **Javobi:5.**

752. $\int_4^9 \frac{1}{\sqrt{x}} dx$ **Javobi: 2.**

753. $\int_{-2}^{-1} (5-4x)dx$; **Javobi: 11.**

754. $\int_{-1}^{1} (x^2+1)dx$; **Javobi:** $\frac{2}{3}$**.**

755. $\int_0^{\frac{\pi}{2}} \sin(2x+\frac{\pi}{3})dx$; **Javobi: 0,5.**

756. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx$; **Javobi: 2.**

Quyidagi aniq integrallarni o'zgarivchini almashtirish orqali hisoblang:

757. $\int_0^1 \frac{x^3 dx}{(x-1)^2}$, $(x-1=t)$; **Javobi: -1,5.**

760. $\int_0^5 x\sqrt{x+4}dx$, $(t=\sqrt{x+4})$; **Javobi:** $33\frac{11}{15}$**.**

758. $\int_4^9 \frac{dx}{\sqrt{x}-1}$, $(\sqrt{x}=t)$; **Javobi:** $2(1+\ln 2)$.

761. $\int_0^1 x\sqrt{1+x^2}dx$, $(t=1+x^2)$; **Javobi:** $\frac{2\sqrt{2}-1}{3}$.

759. $\int_3^8 \frac{x}{\sqrt{1+x}}dx$, $(\sqrt{1+x}=t)$; **Javobi:** $\frac{32}{3}$.

762. $\int_0^{\frac{\pi}{4}} \frac{dx}{2\cos x+3}$, $(t=tg\frac{x}{2})$; **Javobi:** $\frac{2}{\sqrt{5}}arctg\frac{1}{\sqrt{5}}$.

Quyidagi aniq integrallarni bo'laklab integrallash orqali hisoblang:

763. $\int_0^e xe^x dx$, $(u=x)$; **Javobi:** $e^e(e-1)+1$.

766. $\int_0^1 arctgx dx$, $(u=arctgx)$; **Javobi:** $\frac{\pi}{4}-\ln\sqrt{2}$.

764. $\int_0^{\pi} x^2 \ln x dx$, $(u=\ln x)$; **Javobi:** 2.

767. $\int_0^{\pi} x\cos x dx$, $(u=x)$; **Javobi:** 2.

765. $\int_0^{\frac{\pi}{2}} x\cos x dx$, $(u=x)$; **Javobi:** -2.

768. $\int_1^8 x\sin x dx$, $(u=x)$; **Javobi:** π.

24-MAVZU. ANIQ INTEGRALNING TATBIQI

Aniq integral yordamida yuzni xisoblash

1. $f(x)\geq 0, x=a, x=b, \ (0x)$ *bo'lsa*, $S=\int_a^b f(x)dx.$

2. $f(x)\leq 0, x=a, x=b, \ (0x)$ *bo'lsa*, $S=\left|\int_a^b f(x)dx\right|=-\int_a^b f(x)dx.$

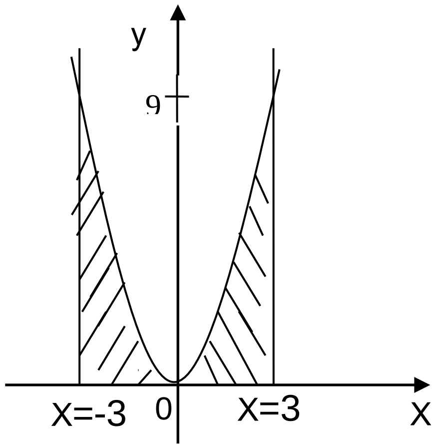
3. $f(x)\geq g(x), x=a, x=b,$ *bo'lsa*, $S=\int_a^b [f(x)-g(x)]dx.$

Misol 769. $y=x^2$ parabola, $x=-3$ va $x=3$ to'g'ri chiziqlar, **(0*x*)** abssissa o'qi bilan chegaralangan yuzni hisoblang.

Yechilishi.

x							
-3	-2	-1	0	1	2	3	
$u=x^2$	9	4	1	0	1	4	9

1- usul.



$$S=\int_a^b f(x)dx=\int_{-3}^{3}x^2dx=\frac{1}{3}x^3\Big|_{-3}^{3}=\frac{1}{3}[3^3-(-3)^3]=$$

$$=\frac{1}{3}(27+27)=\frac{54}{3}=18\ \ kv.\ birlik.$$

2- usul.

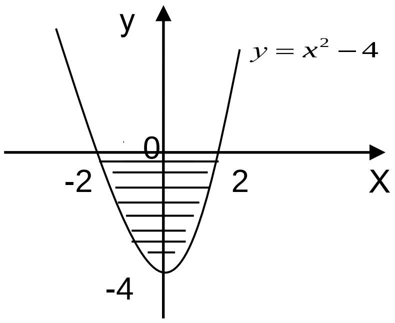
$$S=2\int_0^3 x^2dx=2*\frac{1}{3}x^3\Big|_0^3=\frac{2}{3}[3^3-0^3]=$$

$$=\frac{54}{3}=18\ kv.\ birlik.$$

Misol 770. $u=x^2-4$ parabola va $(0x)$ o'qi bilan chegaralangan yuzni xisoblang.

Yechilishi.

x	-3	-2	-1	0	1	2	3
$u=x^2-4$	5	0	-3	-4	3	0	5



Intervalning a va b chegaralarini topish uchun $y=x^2-4$ funksiyani, $(0x)$ o'qining tenglamasi $y=0$ bilan birgalikda sistema qilib yechish kerak.

$$\begin{cases} y=x^2-4 \\ y=0 \end{cases} \Rightarrow x^2-4=0 \Rightarrow x^2=4 \Rightarrow \sqrt{x^2}=\sqrt{4} \Rightarrow |x|=2 \Rightarrow$$

$$\Rightarrow \pm x=2 \Rightarrow x=\pm 2 \Rightarrow \begin{cases} x_1=-2 \\ x_2=2 \end{cases} \Rightarrow \begin{cases} a=-2; \\ b=2. \end{cases}$$

1-usul

$$S=\left|\int_a^b f(x)dx\right|=\left|\int_{-2}^{2}(x^2-4)dx\right|=\left|\int_{-2}^{2}x^2dx-4\int_{-2}^{2}dx\right|=\left|\frac{1}{3}x^3\Big|_{-2}^{2}-4x\Big|_{-2}^{2}\right|=$$

$$=\left|\frac{1}{3}[2^3-(-2)^3]-4[2-(-2)]\right|=$$

$$=\left|\frac{16}{3}-16\right|=\left|\frac{16-3*16}{3}\right|=\left|-\frac{32}{3}\right|=\frac{32}{3}=10\frac{2}{3}\ kv.\ birlik.$$

2-usul.

$$S=-2\int_0^2(x^2-4)dx=-2\int_0^2 x^2dx+8\int_0^2 dx=-2*\frac{x^3}{3}\Bigg|_0^2+8x\Big|_0^2=-\frac{2}{3}[2^3+0^3]+8[2-0]=$$

$$=-\frac{16}{3}+16=\frac{-16+48}{3}=\frac{32}{3}=10\frac{2}{3}\ kv.birlik$$

Misol 771. $y=x^2-2x$ egri chiziq, $x=-1$, $x=1$ to'g'ri chiziqlar va $(0x)$ o'qi bilan chegaralangan shakilning yuzini hisoblang.

Yechilishi.

$$S=\int_{-1}^{0}(x^2-2x)dx-\int_0^1(x^2-2x)dx=\left(\frac{x^3}{3}-x^2\right)\Bigg|_{-1}^{0}-\left(\frac{x^3}{3}-x^2\right)\Bigg|_0^1=(\frac{1}{3}+1)-(\frac{1}{3}-1)=\frac{1}{3}+1-\frac{1}{3}+1=2.$$

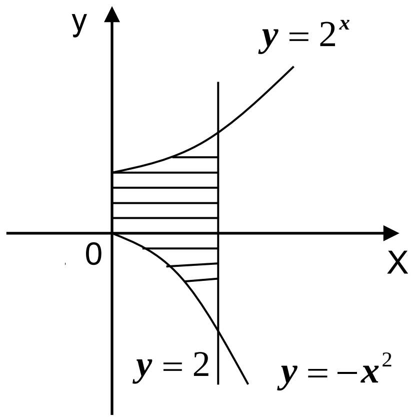
Misol 772. $f(x) = 2^x$ *va* $g(x) = -x^2$ funksiyalar, **(0*x*)** o'qi va ***x=2*** to'g'ri chiziq bilan chegaralangan yuzni hisoblang.

x	0	1	2	*x*	0	1	2
$f(x)=2^x$	1	2	4	$g(x)=-x^2$	0	-1	-4

$$S = \int_a^b [f(x) - g(x)]dx = \int_0^2 [2^x - (-x^2)]dx = \int_0^2 2^x dx + \int_0^2 x^2 dx =$$

$$\frac{2^x}{\ln 2}\Bigg|_0^2 + \frac{1}{3}x^3\Bigg|_0^2 = \frac{2^2}{\ln 2} - \frac{2^0}{\ln 2} + \frac{8}{3} = \frac{4-1}{\ln 2} + \frac{8}{3} = \frac{3}{\ln 2} + \frac{8}{3} = \frac{9+8\ln 2}{3\ln 2} =$$

$$= \frac{9+\ln 2^8}{\ln 2^3} = \frac{9+\ln 256}{\ln 8} \; kv.\, birlik.$$



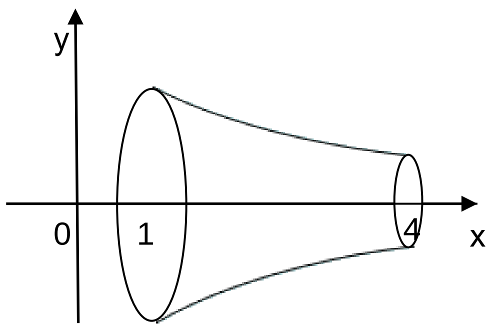
Aniq integral yordamida hajmni hisoblash

$$V = \pi \int_a^b f^2(x)dx$$

Misol 773. ***xy=4, x=1, x=4*** chiziqlar bilan chegaralangan figuraning **(*0x*)** o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini hisoblang.

Yechilishi.

x	1	2	4
$y = \frac{4}{x}$	4	2	1



$$V = \pi \int_1^4 \left(\frac{4}{x}\right)^2 dx = \pi \int_1^4 \frac{16}{x^2} dx = 16\pi \int_1^4 x^{-2} dx = 16\pi \frac{x^{-2+1}}{-2+1}\Bigg|_1^4 = -16\pi\, x^{-1}\Big|_1^4 = -16\pi * \frac{1}{x}\Bigg|_1^4 =$$

$$= -16\pi\left[\frac{1}{4} - 1\right] = -16\pi * \frac{1-4}{4} = 12\pi \; kub\, birlik.$$

Misol 774. $y^2 + x - 4 = 0$ **va *x*=0** chiziqlar bilan chegaralangan figuraning (*0x*) o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini toping.

Yechilishi.

$$\begin{cases} x = 4 - y^2 \\ x = 0 \end{cases} \Rightarrow 4 - y^2 = 0 \Rightarrow \begin{cases} y_1 = -2 \\ y_2 = 2 \end{cases} \Rightarrow \begin{cases} a = -2; \\ b = 2. \end{cases}$$

$$V=\pi\int_{-2}^{2}(4-y^2)^2dy=2\pi\int_{0}^{2}(16-8y^2+y^4)dy=2\pi(16y-\frac{8}{3}y^3+\frac{1}{5}y^5)\Big|_0^2=$$

$$=2\pi(16*2-\frac{8}{3}2^3+\frac{1}{5}2^5)=2\pi(32-\frac{64}{3}+\frac{32}{5})=2\pi\frac{480-320+96}{15}=$$

$$=2\pi\frac{265}{15}=\frac{512\pi}{15}=34\frac{2}{15}\pi \ \textit{kub birlik}.$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Qo'yidagi chiziqlar bilan chegaralangan figuraning yuzini hisoblang:

775. $y=x^2,\ y=0,\ x=0,\ x=2;$ Javobi: $2\frac{2}{3}$. kv. birlik.

776. $y=x^2,\ y=0,\ x=-2;$ Javobi: $2\frac{2}{3}$. kv. birlik.

777. $y=x^3,\ y=0,\ x=2;$ Javobi: 4. kv. birlik.

778. $y=\sqrt{x},\ y=0,\ x=4;$ Javobi: $5\frac{1}{3}$. kv. birlik.

779. $y=\frac{1}{\sqrt{x}},\ y=0,\ x=1, x=4;$ Javobi: 2. kv. birlik.

780. $y=\frac{3}{\sqrt{x}},\ y=0,\ x=1;\ x=4;$ Javobi: 6. kv. birlik.

781. $y=x^2,\ y=2x.$ Javobi: $1\frac{1}{3}$. kv. birlik.

Quyidagi chiziqlar bilan chegaralangan figuraning ko'rsatilgan kordinata o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini hisoblang.

782. $y=x-1,\ x=-3, x=0,\ y=0;\ (0x).$ **Javobi:** 21π ***kub birlik.***

783. $y=x+2,\ x=-3, x=0,\ y=0;\ (0x).$ **Javobi:** 3π ***kub birlik.***

784. $y=x-1,\ x=-1, x=2,\ y=0;\ (0x).$ **Javobi:** 3π ***kub birlik.***

785. $y=x^3,\ \ x=4,\ (0x).$ **Javobi:** 64π ***kub birlik.***

786. $y=2x-x^2,\ \ y=0,\ (0x).$ **Javobi:** $18{,}9\pi$ ***kub birlik.***

787. $y=|x+1|,\ x=-3,\ x=0,\ y=0;\ (0x).$ **Javobi:** $1\frac{1}{3}\pi$ ***kub birlik.***

788. $x\cdot y=9,\ \ y=3,\ \ y=10,\ \ (0y).$ **Javobi:** 3π ***kub birlik.***

789. $y=|x+2|,\ x=-3,\ x=0,\ y=0;\ (0x).$ **Javobi:** 3π ***kub birlik.***

790. $y=|x-1|,\ x=-1,\ x=2,\ y=0;\ (0x).$ **Javobi:** 3π ***kub birlik.***

25-MAVZU. KO'P ARGUMENTLI FUNKSIYALAR

IKKI ARGUMENTLI FUNKSIYANING XUSUSIY HOSILALARI

Agar $z=f(x;\,y)$ ikki argumentli funksiyaning x argumentiga Δx orttirma berilib, y argumenti o'zgarishsiz qoldirilsa, u holda z funksiya $\Delta_x z$ orttirma oladi. Bu orttirmani z funksiyaning x o'zgaruvchi bo'yicha xususiy orttirmasi deyiladi va

$\Delta_x z = f(x+\Delta x; y) - f(x; y)$ ko'rinishda yoziladi. Xuddi shuningdek y o'zgaruvchi bo'yicha xususiy orttirma $\Delta_y z = f(x; y+\Delta y) - f(x; y)$ ko'rinishda bo'ladi.

Agar $\lim\limits_{\Delta x \to 0} \frac{\Delta_x z}{\Delta x}$ chekli limit mavjud bo'lsa, uni z funksiyaning x o'zgaruvchi bo'yicha xususiy hosilasi deyiladi va $\frac{\partial z}{\partial x}$ yoki $f_x'(x; y)$ ko'rinishda yoziladi.

$$\lim_{\Delta x \to 0} \frac{\Delta_x z}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x; y) - f(x; y)}{\Delta x} = \frac{\partial z}{\partial x} = f_x'(x; y).$$

Xuddi shuningdek

$$\lim_{\Delta y \to 0} \frac{\Delta_y z}{\Delta y} = \lim_{\Delta y \to 0} \frac{f(x; y+\Delta y) - f(x; y)}{\Delta y} = \frac{\partial z}{\partial y} = f_y'(x; y).$$

z=f(x; y) funksiyaning to'la orttirmasi $\Delta z = f(x+\Delta x; y+\Delta y) - f(x; y)$ formula yordamida topiladi. Boshqacha aytganda to'la orttirma, funksiyaning xususiy orttirmalari yig'indisiga teng.

z=f(x; y) funksiyaning ***(x₀; y₀)*** nuqtadagi limiti: $\lim\limits_{\substack{x \to x_0 \\ y \to y_0}} f(x; y) = A.$

z=f(x; y) funksiyaning ***(x₀; y₀)*** nuqtadagi uzluksizligi: $\lim\limits_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \Delta z = 0.$

z=f(x; y) funksiyaning to'la differensiali xususiy hosilalarning erkli o'zgaruvchilar differensiallari ko'paytmalarining yig'indisiga teng:

$$dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial y} \cdot dy.$$

Shuningdek

$$d^2 z = \frac{\partial^2 z}{\partial x^2} dx^2 + 2\frac{\partial^2 z}{\partial x \partial y} dxdy + \frac{\partial^2 z}{\partial y^2} dy^2.$$

Misol 791. $z = x^2 + y^2$ funksiyaning xususiy orttirmalarini, to'la orttirmasini, uzluksizligini, xususiy hosilalarini, to'la differensialini, *(2; 3)* nuqtadagi limitini aniqlang.

Yechilishi. **Xususiy orttirmalar:**

$$\Delta_x z = (x+\Delta x)^2 + y^2 - (x^2 + y^2) = x^2 + 2x \cdot \Delta x + (\Delta x)^2 + y^2 - x^2 - y^2 = 2x \cdot \Delta x + (\Delta x)^2.$$

$$\Delta_y z = x^2 + (y+\Delta y)^2 - (x^2 + y^2) = x^2 + y^2 + 2y \cdot \Delta y + (\Delta y)^2 - x^2 - y^2 = 2y \cdot \Delta y + (\Delta y)^2.$$

To'la orttirma:

$$\Delta z = (x+\Delta x)^2 + (y+\Delta y)^2 - (x^2 + y^2) = x^2 + 2x \cdot \Delta x + (\Delta x)^2 + y^2 + 2y \cdot \Delta y + (\Delta y)^2 - x^2 - y^2 =$$

$$= 2x \cdot \Delta x + (\Delta x)^2 + 2y \cdot \Delta y + (\Delta y)^2.$$

Uzluksizlik:

$$\lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \Delta z = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \left[2x \cdot \Delta x + (\Delta x)^2 + 2y \cdot \Delta y + (\Delta y)^2\right] = 2x \cdot 0 + 0^2 + 2y \cdot 0 + 0^2 = 0.$$

Limit:

$$\lim_{\substack{x \to 2 \\ y \to 3}} z = \lim_{\substack{x \to 2 \\ y \to 3}} (x^2 + y^2) = 2^2 + 3^2 = 13.$$

Xususiy hosilalar:

$$\frac{\partial z}{\partial x} = 2x, \qquad \frac{\partial z}{\partial y} = 2y.$$

To'la differensial:

$$dz = 2xdx + 2ydy.$$

Misol 792. $z = arcsin\frac{x}{y}$ funksiyaning xususiy hosilalarini va to'la differensialini toping.

Yechilishi. y o'zgarmas deb, x bo'yicha xususiy hosila topiladi:

$$\frac{\partial z}{\partial x} = \frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}}\cdot\left(\frac{x}{y}\right)'_x = \frac{1}{\sqrt{1-\frac{x^2}{y^2}}}\cdot\frac{1}{y} = \frac{1}{\sqrt{\frac{y^2-x^2}{y^2}}}\cdot\frac{1}{y} = \frac{1}{\sqrt{y^2-x^2}}.$$

x o'zgarmas deb, y bo'yicha xususiy hosila topiladi:

$$\frac{\partial z}{\partial y} = \frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}}\cdot\left(\frac{x}{y}\right)'_y = \frac{1}{\sqrt{1-\frac{x^2}{y^2}}}\cdot\left(-\frac{x}{y^2}\right) = \frac{1}{\sqrt{\frac{y^2-x^2}{y^2}}}\cdot\left(-\frac{x}{y^2}\right) = -\frac{x}{y\sqrt{y^2-x^2}}.$$

Bulardan foydalanib to'la differensial yoziladi:

$$dz = \frac{1}{\sqrt{y^2-x^2}}dx - -\frac{x}{y\sqrt{y^2-x^2}}dy.$$

Misol 793. $u = \sqrt{1-x^2-y^2-z^2}$ funksiyaning xususiy hosilalarini toping.

Yechilishi. y va z larni o'zgarmas deb, x bo'yicha xususiy hosila topiladi:

$$\frac{\partial u}{\partial x} = \frac{1}{2\sqrt{1-x^2-y^2-z^2}}\cdot\left(\sqrt{1-x^2-y^2-z^2}\right)'_x = \frac{1}{2\sqrt{1-x^2-y^2-z^2}}\cdot(-2x) = -\frac{x}{\sqrt{1-x^2-y^2-z^2}}.$$

x va z larni o'zgarmas deb, z bo'yicha xususiy hosila topiladi:

$$\frac{\partial u}{\partial y} = \frac{1}{2\sqrt{1-x^2-y^2-z^2}}\cdot\left(\sqrt{1-x^2-y^2-z^2}\right)'_y = \frac{1}{2\sqrt{1-x^2-y^2-z^2}}\cdot(-2y) = -\frac{y}{\sqrt{1-x^2-y^2-z^2}}.$$

x va y larni o'zgarmas deb, z bo'yicha xususiy hosila topiladi:

$$\frac{\partial u}{\partial z} = \frac{1}{2\sqrt{1-x^2-y^2-z^2}}\cdot\left(\sqrt{1-x^2-y^2-z^2}\right)'_z = \frac{1}{2\sqrt{1-x^2-y^2-z^2}}\cdot(-2z) = -\frac{z}{\sqrt{1-x^2-y^2-z^2}}.$$

Misol 794. Ushbu $z = 5-x-y$ funksiyaning $(x;y)\to(1;2)$ dagi limitini hisoblang.

Yechilishi. $\lim\limits_{(x;y)\to(1;2)}(5-x-y) = 5-1-2 = 2.$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi funksiyalarning xususiy orttirmalarini, to'la orttirmasini, uzluksizligini, *(1; 1)* nuqtadagi limitini aniqlang:

795. $z = (x+y)^2.$
796. $z = x^4 - 2x^3y^2 + 3y^2.$
797. $z = x^3 + y^3.$
798. $z = x^3 - y^3.$
799. $z = x^2 + xy + y^3.$
800. $z = \frac{x}{y}.$
801. $z = (x-y)^2.$
802. $z = x^2 + xy^2 + y^3.$
803. $z = x^2 - y^2.$
804. $z = x^2 - xy + y^3.$
805. $z = \frac{y}{x}.$

Quyidagi funksiyalarning xususiy hosilalarini va to'la differensialini toping:

806. $z=\frac{xy}{x^2+y^2}$. *Javobi :* $dz=\frac{y\left(y^2-x^2\right)}{\left(x^2+y^2\right)^2}dx+\frac{x\left(x^2-y^2\right)}{\left(x^2+y^2\right)^2}dy.$

807. $z=arcsin\frac{y}{x}$. *Javobi :* $dz=-\frac{x^2y}{\sqrt{x^2-y^2}}dx+\frac{1}{\sqrt{x^2-y^2}}dy$.

808. $z=ln\left(4-x^2+y^2\right)$. *Javobi :* $dz=-\frac{2x}{4-x^2+y^2}dy+\frac{2y}{4-x^2+y^2}dy.$

809. $z=\sqrt{y^2-2x+4}$. *Javobi :* $dz=-\frac{1}{\sqrt{y^2-2x+4}}dx+\frac{y}{\sqrt{y^2-2x+4}}dy.$

810. $z=\sqrt{4-x^2+y^2}$ *Javobi :* $-\frac{x}{\sqrt{4-x^2+y^2}}dx+\frac{y}{\sqrt{4-x^2+y^2}}dy.$

811. $z=ln\left(x+\sqrt{x^2+y^2}\right)$. *Javobi :* $dz=dx+\frac{y}{\left(x+\sqrt{x^2+y^2}\right)\cdot\sqrt{x^2+y^2}}dy.$

812. $z=\sqrt{x^2-y^2-9}$. *Javobi :* $dz=\frac{x}{\sqrt{x^2-y^2-9}}dx-\frac{y}{\sqrt{x^2-y^2-9}}dy.$

813. $z=\left(x^3+y^3-xy^2\right)^2$. *Javobi :* $dz=3\left(x^3+y^3-xy^2\right)^2\cdot\left(\left(3x^2-y^2\right)dx+\left(3y^2-2xy\right)dy\right)$

Quyidagi funksiyalar xususiy hosilalarining *M₀(3; 4)* nuqtadagi qiymatini hisoblang:

814. $z=x+y+\sqrt{x^2+y^2}$. *Javobi :* $\left.\frac{\partial z}{\partial x}\right|_{(3;4)}=1,6,\quad \left.\frac{\partial z}{\partial y}\right|_{(3;4)}=1,8.$

815. $z=ln\sqrt{x^2+y^2}$. *Javobi :* $\left.\frac{\partial z}{\partial x}\right|_{(3;4)}=0,12,\quad \left.\frac{\partial z}{\partial y}\right|_{(3;4)}=0,16.$

Quyidagi limitlarni hisoblang:

816. $\lim\limits_{(x;y)\to(0;0)}\frac{2-\sqrt{xy+4}}{xy}$. *Javobi :* $-\frac{1}{4}$. **817.** $\lim\limits_{(x;y)\to(0;0)}\frac{sin\left(x\cdot y\right)}{xy}$. *Javobi :* 1.

818. $\lim\limits_{(x;y)\to(0;0)}\frac{x}{x+y}$. *Javobi : mavjud emas.* **819.** $\lim\limits_{(x;y)\to(0;0)}\frac{tg\left(xy\right)}{y}$. *Javobi :* 3.

26-MAVZU. KO'P O'ZGARUVCHILI FUNKSIYANING EKSTREMUMI

$z=f(x,y)$ funksiya ekstremumi mavjudligining zaruriy sharti: $M_0(x_0,y_0)$ ekstremum nuqta bo'lsa, $f_x^1(x_0,y_0)=f_y^1(x_0,y_0)=0$ bo'ladi.

Ekstremum mavjudligining yetarli sharti: Ikkinchi tartibli xususiy hosilalarning $M_0(x_0,y_0)$ ekstremum nuqtadagi qiymatlari

$A=f_{xx}''(x_0,y_0),\quad B=f_{xy}''(x_0,y_0),\quad C=f_{yy}^{11}(x_0,y_0)$ bo'lsin. Bulardan $\Delta=AC-B^2$ diskriminant tuziladi:

1) $\Delta>0, A<0, C<0$ da z funksiya M_0 nuqtada maksimumga erishadi;
2) $\Delta>0, A>0, C>0$ da z funksiya M_0 nuqtada minimumga erishadi;
3) $\Delta<0$ bo'lsa, M_0 nuqtada ekstremum mavjud emas;

4) $\Delta = 0$ bo'lsa, ekstremum mavjud bo'lishi ham, bo'lmasligi ham mumkin.

Misol 820. $z = x^2 - xy + y^2 + 9x - 6y + 20$ funksiyaning ekstremum qiymatini toping.

Yechilishi. z funksiyaning xususiy hosilalari topiladi;

$$\frac{\partial z}{\partial x} = 2x - y + 9; \qquad \frac{\partial z}{\partial y} = 2y - x - 6.$$

Xususiy hosilalar nolga tenglanib kritik nuqtalar topiladi:

$$\begin{cases} 2x - y + 9 = 0 \\ 2y - x - 6 = 0 \end{cases} \Rightarrow \begin{cases} y = 2x + 9 \\ 2(2x + 9) - x - 6 = 0 \end{cases} \Rightarrow \begin{cases} y = 2x + 9 \\ x = -4 \end{cases} \Rightarrow \begin{cases} x = -4 \\ y = 1 \end{cases} \Rightarrow M_0(-4;1).$$

Ikkinchi tartibli xususiy hosilalar topiladi:

$$\frac{\partial^2 z}{\partial x^2} = 2; \qquad \frac{\partial^2 z}{\partial x \partial y} = -1; \qquad \frac{\partial^2 z}{\partial y^2} = 2.$$

Demak, A=2, B= -1; C=2.

U holda $\Delta = AC - B^2 = 2 \cdot 2 - (-1)^2 = 3 > 0.$

Bulardan $\Delta = 3 > 0,$ $A = 2 > 0,$ $C = 2 > 0.$

Demak, z funksiya $M_0(-4;1)$ kritik nuqtada minimumga ega:

$z_{\min} = (-4)^2 - (-4) \cdot 1 + 1^2 + 9 \cdot (-4) - 6 \cdot 1 + 20 = -1.$

Misol 821. $z = y \cdot \sqrt{x} - y^2 - x + 6y$ funksiyaning ekstremum qiymatini toping.

Yechilishi: 1. $\frac{\partial z}{\partial x} = \frac{y}{2\sqrt{x}} - 1;$ $\qquad \frac{\partial z}{\partial y} = \sqrt{x} - 2y + 6.$

2. $\begin{cases} \frac{y}{2\sqrt{x}} - 1 = 0 \\ \sqrt{x} - 2y + 6 = 0 \end{cases} \Rightarrow \begin{cases} \frac{y - 2\sqrt{x}}{2\sqrt{x}} = 0 \\ \sqrt{x} - 2y + 6 = 0 \end{cases} \Rightarrow \begin{cases} 2\sqrt{x} \neq 0 \\ y - 2\sqrt{x} = 0 \\ \sqrt{x} - 2y + 6 = 0 \end{cases} \Rightarrow \begin{cases} y = 2\sqrt{x} \\ \sqrt{x} = 2 \end{cases} \Rightarrow \begin{cases} x = 4 \\ y = 4 \end{cases} \Rightarrow M_0(4;4).$

3. $\frac{\partial^2 z}{\partial x^2} = -\frac{y}{4\sqrt{x^3}};$ $\qquad \frac{\partial^2 z}{\partial x \partial y} = \frac{1}{2\sqrt{x}};$ $\qquad \frac{\partial^2 z}{\partial y^2} = -2.$

4. $A = -\frac{4}{4\sqrt{4^3}} = -\frac{1}{8};$ $\qquad B = \frac{1}{2\sqrt{4}} = \frac{1}{4};$ $\qquad C = -2.$

5. $\Delta = A \cdot C - B^2 = -\frac{1}{8} \cdot (-2) - (\frac{1}{4})^2 = \frac{1}{4} - \frac{1}{16} = \frac{3}{16}.$

6. $\Delta = \frac{3}{16} > 0,$ $\qquad A = -\frac{1}{8} < 0,$ $\qquad C = -2 < 0.$

Demak, z funksiya $M_0(4;4)$ kritik nuqtada maksimumga ega.

$$z_{max} = 4 \cdot \sqrt{4} - 4^2 - 4 + 6 \cdot 4 = 12.$$

Misol 822. $z = 2xy - 4x - 2y$ funksiyaning ekstremumini aniqlang.

Yechilishi.

1. $\frac{\partial z}{\partial x} = 2y - 4;$ $\qquad \frac{\partial z}{\partial y} = 2x - 2.$

2. $\begin{cases} 2y - 4 = 0 \\ 2x - 2 = 0 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 2 \end{cases} \Rightarrow M_0(1;2).$

3. $\frac{\partial^2 z}{\partial x^2}=0;\qquad \frac{\partial^2 z}{\partial x\partial y}=2;\qquad \frac{\partial^2 z}{\partial y^2}=0.$

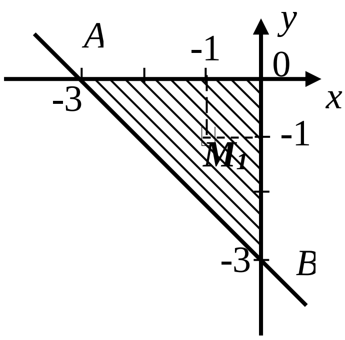
4. $A=0, B=2, C=0.$

5. $\Delta = AC-B^2=-4<0.$

Demak, ekstremum mavjud emas.

Misol 823. $z=x^2+y^2-xy+x+y$ funksiyaning x=0, y=0, x+y=-3 chiziqlar bilan chegaralangan sohadagi eng katta va eng kichik qiymatlarini toping.

Yechilishi: D soha ΔAOB dan iborat. Bu sohaga tegishli kritik nuqta topilib, z funksiyaning undagi qiymati aniqlanadi:



$$\frac{\partial z}{\partial x}=2x-y+1;\qquad \frac{\partial z}{\partial y}=2y-x+1.$$

$$\begin{cases}2x-y+1=0\\ 2y-x+1=0\end{cases}\Rightarrow\begin{cases}x=-1\\ y=-1\end{cases}\Rightarrow M_1(-1;-1)\Rightarrow$$

$$\Rightarrow z_1=z(-1;-1)=(-1)^2+(-1)^2-(-1)(-1)-1=-1.$$

Berilgan z funksiya D sohaning chegaralaridagi kritik nuqtalarda va uchlarida tekshiriladi:

$$(0x): y=0\Rightarrow z=x^2+x\Rightarrow z_x^1=2x+1\Rightarrow 2x+1=0\Rightarrow x-\frac{1}{2}\Rightarrow M_2(-\frac{1}{2};0)\Rightarrow$$

$$\Rightarrow z_2=z(-\frac{1}{2};0)=-\frac{1}{4}.$$

$$(0y): x=0\Rightarrow z=y^2+y\Rightarrow z_y^1=2y+1\Rightarrow 2y+1=0\Rightarrow y-\frac{1}{2}\Rightarrow M_3(0;-\frac{1}{2})\Rightarrow$$

$$\Rightarrow z_3=z(0;-\frac{1}{2})=-\frac{1}{4}.$$

$$(AB): x+y=-3\Rightarrow y=-x-3\Rightarrow z=z(-x-3)=3x^2+9x+6\Rightarrow z_x^1=6x+9\Rightarrow$$

$$\Rightarrow 6x+9=0\Rightarrow x=-\frac{3}{2}\Rightarrow y=\frac{3}{2}-3=-\frac{3}{2}\Rightarrow M_4(-\frac{3}{2};-\frac{3}{2})\Rightarrow z_4=z(-\frac{3}{2};-\frac{3}{2})=-\frac{3}{4}.$$

$A(-3;0)\Rightarrow z_5=z(-3;0)=6.\qquad O(0;0)\Rightarrow z_6=z(0;0)=0.\qquad B(0;-3)\Rightarrow z_7=z(0;-3)=6.$

Demak, berilgan z funksiya $A(-3;0)$ va $B(0;-3)$ nuqtalarda eng katta, $M_1(-1;-1)$ nuqtada eng kichik qiymatga erishadi.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi ikki argumentli funksiyalarning ekstremum qiymatlarini aniqlang:

824. $z=xy(x+y-2).$ J.: $z_{\min}=-\frac{8}{27}.$

825. $z=e^{\frac{x}{2}}(x+y^2).$ J.: $z_{\min}=-\frac{2}{e}.$

826. $z=x^2+y^2+xy-6x-9y.$ J.: $z_{\min}=-21.$

827. $z=x^3+8y^3-6xy+1.$ J.: $z_{min}=0.$

828. $z=x^2+y^2-2x-4\sqrt{xy}-2y+8.$ J.: $z_{min}=0.$

829. $z=x^3+y^3-3xy.$ J.: $z_{\min}=-1.$

830. $z=xy(x+y-2).$ J.: $z_{\min}=-\frac{8}{28}.$

831. $z=y\sqrt{x}-y^2-x+6y.$ J.: $z_{max}=12.$

832. $z=3x+6y-x^2-xy-y^2.$ J.: $z_{min}=9.$

833. $z=2x^3-xy^2+5x^2+y^2.$ J.: $z_{min}=0.$

27-MAVZU. IKKI KARRALI INTEGRALLAR

$a \le x \le b$ va $y_1(x) \le y \le y_2(x)$ chiziqlar bilan chegaralangan D sohaning yuzi quyidagi ikki karrali integral yordamida hisoblanadi:

$$S=\iint\limits_D f(x,y)dxdy=\int_a^b dx\int_{y_1(x)}^{y_2(x)} f(x,y)dy.$$

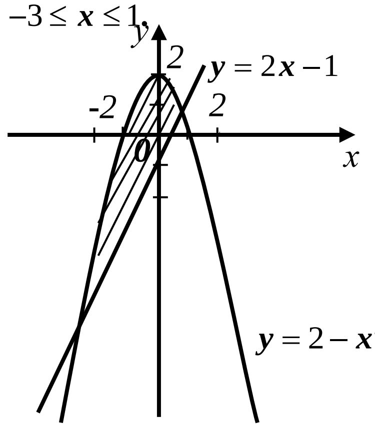
$c \le y \le d$ va $x_1(y) \le x \le x_2(y)$ chiziqlar bilan chegaralangan D sohaning yuzi quyidagi ikki karrali integral yordamida hisoblanadi:

$$S=\iint\limits_D f(x,y)dx\,dy=\int_c^d dy\int_{x_1(y)}^{x_2(y)} f(x,y)dx.$$

Misol 834. $\iint\limits_D (x-y)dxdy$ ikki karrali integralni hisoblang. Bunda D soha $y=2-x^2$ *va* $y=2x-1$ chiziqlar bilan chegaralangan .

Yechilishi. D soha chiziladi. Berilgan funksiyalar sistema qilib yechilib, integral chegaralari aniqlanadi:

$$\begin{cases} y=2-x^2 \\ y=2x-1 \end{cases} \Rightarrow \begin{cases} y=y \\ 2-x^2=2x-1 \Rightarrow x^2+2x-3=0 \Rightarrow \begin{cases} x_1=-3 \\ x_2=1 \end{cases} \Rightarrow -3 \le x \le 1, \end{cases}$$



D sohadagi chiziqlarning ($0x$) o'qiga nisbatan joylashishi e'tiborga olingan holda , ikkinchi integralning chegaralari aniqlanadi:

$$2x-1 \le y \le 2-x^2$$

Demak, integral chegaralari quyidagicha bo'ladi:

$$\begin{cases} -3 \le x \le 1; \\ 2x-1 \le y \le 2-x^2. \end{cases}$$

Endi ikki karrali integral hisoblanadi:

$$\iint\limits_D (x-y)dxdy=\int_{-3}^{1}dx\int_{2x-1}^{2-x^2}(x-y)dy=\int_{-3}^{1}dx\left[\int_{2x-1}^{2-x^2}xdy-\int_{2x-1}^{2-x^2}ydy\right]=\int_{-3}^{1}dx\left[x*y\Big|_{2x-1}^{2-x^2}-\frac{1}{2}*y^2\Big|_{2x-1}^{2-x^2}\right]=$$

$$=\int_{-3}^{1}dx\left\{x\left[2-x^2-(2x-1)\right]-\frac{1}{2}\left[(2-x^2)^2-(2x-1)^2\right]\right\}=\int_{-3}^{1}dx\left[x(2-x^2-2x+1)\right]-$$

$$-\frac{1}{2}\left[4-4x^2+x^4-(4x^2-4x+1)\right]=\int_{-3}^{1}dx\left[3x-x^3-2x^2-2+2x^2-\frac{1}{2}x^4+2x^2-2x+\frac{1}{2}\right]=$$

$$=\int_{-3}^{1}\left(-\frac{1}{2}x^4-x^3+2x^2+x-\frac{3}{2}\right)dx=\left[-\frac{1}{2}*\frac{1}{5}x^5-\frac{1}{4}x^4+2*\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{3}{2}x\right]\Bigg|_{-3}^{1}=-\frac{1}{10}\left[1^5-(-3)^5\right]-$$

$$-\frac{1}{4}\left[1^4-(-3)^4\right]+\frac{2}{3}\left[1^3-(-3)^3\right]+\frac{1}{2}\left[1^2-(-3)^2\right]-\frac{3}{2}[1-(-3)]=-\frac{1}{10}*244+\frac{1}{4}*80+\frac{2}{3}*28-\frac{1}{2}*8-\frac{3}{2}*4=$$

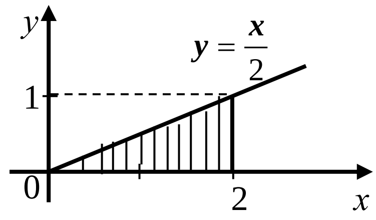
$$=-\frac{122}{5}+20+\frac{56}{3}-4-6=10-\frac{122}{5}+\frac{56}{3}=\frac{150-366+280}{15}=\frac{64}{15}\ kv.\ birlik.$$

Misol 835. $\iint\limits_D (x^2+y^2)dxdy$ ikki karrali integralni hisoblang.

Bunda D soha $y = 0,\ x = 2$ va $y = \dfrac{x}{2}$ to'g'ri chiziqlar bilan chegaralangan uchburchakdan iborat.

Yechilishi. 1- usul. Masala sharti va rasmdan:

$0 \le x \le 2;\quad 0 \le y \le \dfrac{x}{2}.$



$$S = \iint_D (x^2 + y^2)dxdy = \int_0^2 dx \int_0^{\frac{x}{2}} (x^2 + y^2)dy =$$

$$\int_0^2 dx \left[\int_0^{\frac{x}{2}} x^2 dy + \int_0^{\frac{x}{2}} y^2 dy \right] = \int_0^2 dx \left[x^2 \cdot y \Big|_0^{\frac{x}{2}} + \frac{1}{3} \cdot y^3 \Big|_0^{\frac{x}{2}} \right] =$$

$$\int_0^2 dx [x^2 (\frac{x}{2} - 0) + \frac{1}{3}(\frac{x^3}{8} - 0^3)] =$$

$$= \int_0^2 (\frac{1}{2}x^3 + \frac{1}{24}x^3)dx = \frac{13}{24}\int_0^2 x^3 dx = \frac{13}{24} * \frac{1}{4}x^4\Big|_0^2 = \frac{13}{96} * 2^4 = \frac{13}{6}\ kv.\ birlik.$$

2- usul. Masala sharti va rasmdan:

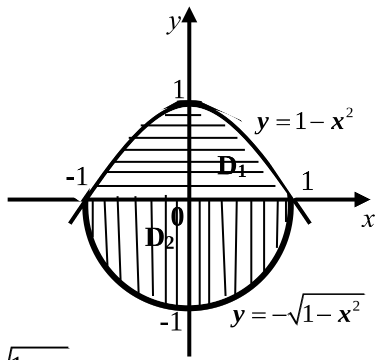
$$\begin{cases} y = 0 \\ x = 2 \\ y = \dfrac{x}{2} \end{cases} \Rightarrow \begin{cases} y = 0 \\ y = 1 \end{cases} \Rightarrow 0 \le y \le 1; \qquad \begin{cases} x = 2 \\ y = \dfrac{x}{2} \end{cases} \Rightarrow \begin{cases} x = 2 \\ x = 2y \end{cases} \Rightarrow 2y \le x \le 2.$$

$$S = \iint_D (x^2 + y^2)dxdy = \int_0^1 dy \int_{2y}^2 (x^2 + y^2)dx = \int_0^1 dy \left[\int_{2y}^2 x^2 dx + \int_{2y}^2 y^2 dx \right] = \int_0^1 dy \left[\frac{1}{3}x^3 \Big|_{2y}^2 + y^2 * x\Big|_{2y}^2 \right] =$$

$$= \int_0^1 dy \left\{ \frac{1}{3}\left[2^3 - (2y)^3\right] + y^2\left[2 - 2y\right] \right\} = \int_0^1 dy \left[\frac{1}{3}(8 - 8y^3) + 2y^2 - 2y^3 \right] = \int_0^1 (\frac{8}{3} - \frac{8}{3}y^3 + 2y^2 - 2y^3)dy =$$

$$= \int_0^1 (-\frac{14}{3}y^3 + 2y^2 + \frac{8}{3})dy = -\frac{14}{3} * \frac{1}{4} * y^4\Big|_0^1 + \frac{2}{3} * y^3\Big|_0^1 + \frac{8}{3} * y\Big|_0^1 = -\frac{7}{6} + \frac{2}{3} + \frac{8}{3} = \frac{-7 + 4 + 16}{6} = \frac{13}{16}\ kv.\ birlik.$$

Misol 836 $\displaystyle\int_{-1}^{1} dx \int_{-\sqrt{1-x^2}}^{1-x^2} f(x,y)dy$ integralda integrallash tartibini o'zgartiring.



Yechilishi. $D: \begin{cases} -1 \le x \le 1; \\ -\sqrt{1 - x^2} \le y \le 1 - x^2. \end{cases}$

Bu soha chiziladi:

$$D = D_1 \cup D_2. \quad D_1: \begin{cases} 0 \le y \le 1 \\ y = 1 - x^2 \Rightarrow x^2 = 1 - y \Rightarrow |x| = \sqrt{1 - y} \Rightarrow x = \pm\sqrt{1 - y}. \end{cases}$$

$$D_1: \begin{cases} 0 \le y \le 1; \\ -\sqrt{1 - y} \le x \le \sqrt{1 - y}. \end{cases} \quad D_2: \begin{cases} -1 \le y \le 0; \\ y = -\sqrt{1 - x^2} \Rightarrow y^2 = 1 - x^2 \Rightarrow x^2 = 1 - y^2 \Rightarrow x = \pm\sqrt{1 - y^2}. \end{cases}$$

$$D_2 = \begin{cases} -1 \le y \le 0; \\ -\sqrt{1 - y^2} \le x \le \sqrt{1 - y^2}. \end{cases}$$

U holda $\int\limits_{-1}^{1} dx \int\limits_{-\sqrt{1-x^2}}^{1-x^2} f(x,y)dy = \int\limits_{0}^{1} dy \int\limits_{-\sqrt{1-y}}^{\sqrt{1+y}} f(x,y)dx + \int\limits_{-1}^{0} dy \int\limits_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)dx.$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Integrallarni hisoblang:

837. $\int\limits_{1}^{2} dx \int\limits_{x^3}^{x} (x-y)dy.$ Javobi: $112\frac{8}{105}$ ***kv.birlik*** . **838.** $\int\limits_{0}^{4} dx \int\limits_{1}^{e} x \ln y dy.$ Javobi: 8 ***kv.birlik*** .

839. $\int\limits_{-3}^{8} dy \int\limits_{y^2-4}^{5} (x+2y)dx.$ Javobi: $50\frac{2}{5}$ ***kv.birlik*** .

Quyidagi integrallarni hisoblang:

840. $\iint\limits_{D} (x^2+y)dxdy.$ Bunda D soha $y=x^2$ va $y^2=x$ chiziqlar bilan chegaralangan.

Javobi: $\frac{33}{140}$ ***kv.birlik*** .

841. $\iint\limits_{D} \frac{x^2}{y^2} dxdy.$ Bunda D soha $x=2, \quad y=x, \quad x \cdot y = 1$ chiziqlar bilan chegaralangan.

Javobi: $\frac{9}{4}$ ***kv.birlik*** .

842. $y=x^2-2x, \quad y=x$ chiziqlar bilan chegaralangan yuzini hisoblang.

Javobi: $\frac{9}{2}$ ***kv.birlik*** .

843. $y=2-x, \quad y^2=4x+4$ chiziqlar bilan chegaralangan yuzini hisoblang.

Javobi: $\frac{64}{3}$ ***kv.birlik***.

844. $\iint\limits_{D} (\cos 2x + \sin y)dxdy.$ Bunda D soha $x=0, \quad y=0, \quad 4x+4y-\pi=0$ chiziqlar bilan chegaralangan. Javobi: $\frac{1}{4}\left(\pi+1-2\sqrt{2}\right)$ ***kv.birlik*** .

845. $\iint\limits_{D} y \ln x dxdy.$ Bunda D soha $x \cdot y = 1, \quad y=\sqrt{x}, \quad x=2$ chiziqlar bilan chegaralangan. Javobi: $\frac{5}{8}$***(***2 ***ln*** $2-1$***)*** ***kv.birlik*** .

846. $\iint\limits_{D} \sin(x+y)dxdy.$ Bunda D soha $x=0, \quad y=\frac{\pi}{2}, \quad y=x$ chiziqlar bilan chegaralangan. Javobi: 1 ***kv.birlik*** .

847. $\iint\limits_{D} xdxdy.$ Bunda D soha uchlari $A(2;3), \quad B(2;7), \quad C(4;5)$ nuqtalarda bo'lgan uchburchak. Javobi: 26 ***kv.birlik*** .

Integrallash tartibini o'zgartiring:

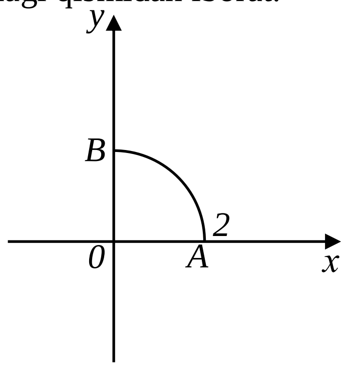
848. $\int\limits_{-6}^{2} dx \int\limits_{\frac{x^2}{4}-1}^{2-x} f(x,y)dy.$ **849.** $\int\limits_{0}^{1} dx \int\limits_{\frac{1-x^2}{2}}^{\sqrt{1-x^2}} f(x,y)dy.$ **850.** $\int\limits_{0}^{1} dy \int\limits_{2-y}^{1+\sqrt{1-y^2}} f(x,y)dx.$

28-MAVZU. EGRI CHIZIQLI INTEGRALLAR

BIRINCHI TUR EGRI CHIZIQLI INTEGRAL

$$\int\limits_{\overset{\smile}{AB}} f(x,y)dt=\int\limits_a^b f(x,y(x))\cdot\sqrt{1+y'^2}\,dx.$$

Misol 851. $\int\limits_{\overset{\smile}{AB}}\sqrt{x^2+y^2}\,dt$ integralni hisoblang. Bunda $\overset{\smile}{AB}$ markazi $O(0{,}0)$ nuqtada va radiusi $r=2$ bo'lgan aylananing birinchi kvadrantdagi qismidan iborat.



Yechilishi. $x^2+y^2=2^2\Rightarrow y=\sqrt{4-x^2};\ \ 0\le x\le 2.$

U holda

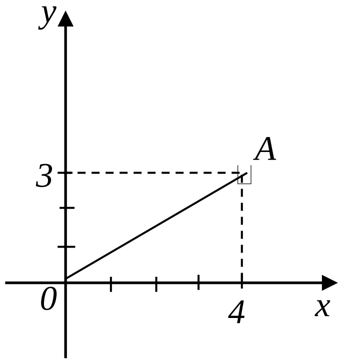
$$\int\limits_{\overset{\smile}{AB}}\sqrt{x^2+y^2}\cdot dt=\int\limits_0^2\sqrt{x^2+4-x^2}\cdot\sqrt{1+(\sqrt{4-x^2})'^2}\,dx=$$

$$=2\int\limits_0^2\sqrt{1+\left(\frac{-2x}{2\sqrt{4-x^2}}\right)^2}\,dx=2\int\limits_0^2\sqrt{1+\frac{x^2}{4-x^2}}\,dx=$$

$$=2\int\limits_0^2\sqrt{\frac{4-x^2+x^2}{4-x^2}}\,dx=2\int\limits_0^2\frac{2dx}{\sqrt{4-x^2}}=2\int\limits_0^2\frac{\frac{2}{2}\cdot dx}{\frac{\sqrt{4-x^2}}{2}}=$$

$$=2\int\limits_0^2\frac{dx}{\sqrt{1-(\frac{x}{2})^2}}=2\int\limits_0^2\frac{2\cdot d(\frac{x}{2})}{\sqrt{1-(\frac{x}{2})^2}}=4\arcsin\frac{x}{2}\Big|_0^2=4[\arcsin 1-\arcsin 0]=4\left[\frac{\pi}{2}-0\right]=2\pi.$$

Misol 852. $\int\limits_k (x-y)dt$ integralni hisoblang. Bunda k uchlari $O(0;0)$ va $A(4;3)$ nuqtalarda bo'lgan to'g'ri chiziq kesmasi.



Yechilishi. $\frac{x-0}{4-0}=\frac{y-0}{3-0}\Rightarrow y=\frac{3}{4}\cdot x.\qquad 0\le x\le 4.$

U holda

$$\int\limits_k (x-y)dt=\int\limits_0^4(x-\frac{3}{4}x)\sqrt{1+(\frac{3}{4}x)'^2}\,dx=\int\limits_0^4\frac{1}{4}\cdot x\sqrt{1+\frac{9}{16}}\,dx=$$

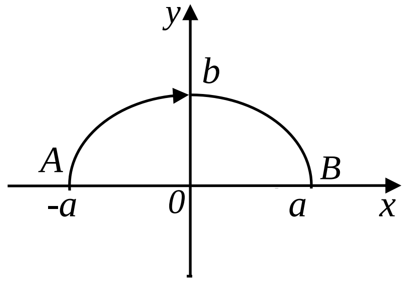
$$=\frac{1}{4}\int\limits_0^4 x\cdot\frac{5}{4}dx=\frac{5}{16}\cdot\frac{1}{2}x^2\Big|_0^4=\frac{5}{32}\cdot 4^2=\frac{5}{2}.$$

IKKINCHI TUR EGRI CHIZIQLI INTEGRAL

$$\int\limits_{\overset{\smile}{AB}} P(x,y)dx+Q(x,y)dy$$

Misol 853. $\int\limits_L y^2dx+x^2dy$ egri chiziqli integralni hisoblang. Bunda L kontur $x=a\cdot\cos t,\ y=b\cdot\sin t$ ellipsning soat mili harakati bo'yicha aylanib o'tiladigan yuqori yarmi.

Yechilishi.



$$\int\limits_L y^2dx+x^2dy=\int\limits_{\overset{\smile}{AB}} y^2dx+x^2dy=$$

$$=\int\limits_{\pi}^{0}(b\cdot\sin t)^2 d(a\cos t)+(a\cos t)^2 d(b\sin t)=$$

$$=\int_{\pi}^{0} b^2 \sin^2 t(-a\sin t)dt + a^2\cos^2 t\cdot b\cos dt = -ab^2\int_{\pi}^{0}\sin^3 tdt + a^2b\int_{\pi}^{0}\cos^3 tdt = ab^2\int_{0}^{\pi}\sin^3 tdt - a^2b\int_{0}^{\pi}\cos^3 tdt =$$

$$= -ab^2\int_{0}^{\pi}(1-\cos^2 t)d(\cos t) - a^2b\int_{0}^{\pi}(1-\sin^2 t)d(\sin t) =$$

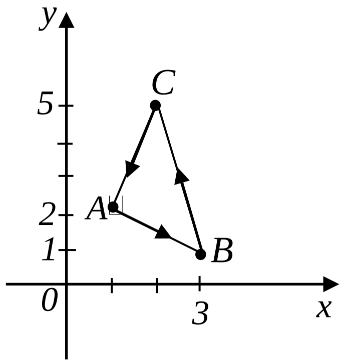
$$= -ab^2(\cos t - \frac{1}{3}\cos^3 t)\Big|_0^{\pi} - a^2b(\sin t - \frac{1}{3}\sin^3 t)\Big|_0^{\pi} = -ab^2\left\{\cos\pi - \cos 0 - \frac{1}{3}\left[(\cos\pi)^3 - (\cos 0)^3\right]\right\} - 0 =$$

$$= -ab^2\left[-1-1-\frac{1}{3}(-1-1)\right] = -ab^2(-2+\frac{2}{3}) = \frac{4}{3}ab^2.$$

YOPIQ KONTUR BO'YICHA EGRI CHIZIQLI INTEGRAL

$$\oint P(x,y)dx + Q(x,y)dy.$$

Misol 854. $\oint_L 2xdy - 3ydx$ egri chiziqli integralni hisoblang. Bunda L uchlari $A(1;2)$, $B(3;1)$ va $C(2;5)$ nuqtalarda bo'lgan uchburchak konturi.



Yechilishi. To'g'ri chiziqlar tenglamalari tuziladi:

$$(AB):\ y = -\frac{1}{2}\cdot x + \frac{5}{2};\qquad (BC):\ y = -4x + 13;$$

$$(AC):\ y = 3x - 1.$$

U holda berilgan integral quyidagicha yoziladi:

$$\oint_L 2xdy - 3ydx = \int_{AB} 2xdy - 3ydx + \int_{BC} 2xdy - 3ydx + \int_{CA} 2xdy - 3ydx.$$

Har bir integral alohida-alohida hisoblanadi:

$$\int_{AB} 2xdy - 3ydx = \int_1^3 2\cdot xd(-\frac{1}{2}x + \frac{5}{2}) - 3(-\frac{1}{2}x + \frac{5}{2})dx = \int_1^3 2x\cdot(-\frac{1}{2})dx + (\frac{3}{2}x - \frac{15}{2})dx =$$

$$= \int_1^3 (-x + \frac{3}{2}x - \frac{15}{2})dx = \int_1^3 (\frac{1}{2}x - \frac{15}{2})dx = \frac{1}{2}\int_1^3 (x-15)dx = \frac{1}{2}\left[\frac{1}{2}x^2 - 15x\right]\Big|_1^3 =$$

$$= \frac{1}{2}\left[\frac{1}{2}(3^2 - 1^2) - 15(3-1)\right] = \frac{1}{2}(4-30) = -13;$$

$$\int_{BC} 2xdy - 3ydx = \int_3^2 2xd(-4x+13) - 3(-4x+13)dx = \int_3^2 2x(-4)dx - 3(-4x+13) =$$

$$= \int_3^2 (-8x + 12x - 39)dx = \int_3^2 (4x - 39)dx = (2x^2 - 39x)\Big|_3^2 = 2\left[2^2 - 3^2\right] - 39\left[2-3\right] = -10 + 39 = 29;$$

$$\int_{CA} 2xdy - 3ydx = \int_2^1 2xd(3x-1) - 3(3x-1)dx = \int_2^1 2x\cdot 3\cdot dx - 3(3x-1)dx = \int_2^1 (6x - 9x + 3)dx =$$

$$\int_2^1 (-3x+3)dx = 3\cdot\int_2^1 (1-x)dx = 3\cdot(x - \frac{1}{2}x^2)\Big|_2^1 = 3\left[1 - 2 - \frac{1}{2}(1^2 - 2^2)\right] = 3(-1 + \frac{3}{2}) = \frac{3}{2}.$$

Demak, $\oint 2xdy - 3ydx = -13 + 29 + \frac{3}{2} = 16 + \frac{3}{2} = \frac{35}{2}.$

GRIN FORMULASI

$$\oint P(x;y)dx + Q(x;y)dy = \iint_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})dxdy.$$

Misol 855. Grin formulasidan foydalanib $\oint\limits_L y(1-x^2)dx+(1+y^2)xdy$ integralni hisoblang. Bunda L kontur $x^2+y^2=2^2$ aylanadan iborat bo'lib, u musbat yo'nalishda aylanib o'tiladi.

Yechilishi. Grin formulasi bo'yicha ikki karrali integralga o'tiladi:

$$\oint\limits_L y(1-x^2)dx+(1+y^2)xdy=\iint\limits_D(1+y^2-1+x^2)dxdy=\iint\limits_D(x^2+y^2)dxdy.$$

Bunda D soha $x^2+y^2\le 4$ tengsizlik bilan aniqlanadigan doira. Integralni hisoblash uchun qutb koordinatalarga o'tiladi: $x=r\cos\varphi,\quad y=r\sin\varphi$.

Bu yerda

$\iint\limits_D f(x,y)dxdy=\iint\limits_{D^1} f(r\cos\varphi,r\sin\varphi)rdrd\varphi$ ni e'tiborga olinsa

$$\iint\limits_D(x^2+y^2)dxdy=\iint\limits_{D^1}(r^2\cos^2\varphi+r^2\sin^2\varphi)rdrd\varphi=\iint\limits_{D^1}r^3drd\varphi=\int\limits_0^{2\pi}d\varphi\int\limits_0^2 r^3dr=\int\limits_0^{2\pi}\frac{1}{4}\cdot r^4\Big|_0^2 d\varphi=$$

$$=\frac{1}{4}\int\limits_0^{2\pi}2^4d\varphi=4\cdot\varphi\Big|_0^{2\pi}=8\pi.$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

856. $\int\limits_L(x+y)dt$ egri chiziqli integralni hisoblang. Bunda L uchlari $A(2;2)$ va $B(2;0)$ nuqtalarda bo'lgan to'g'ri chiziq kesmasi. Javobi: $4\sqrt{2}.$

857. $\int\limits_C(x+y)ds$ egri chiziqli integralni hisoblang. Bunda C - uchlari $O(0;0),\quad A(1;0),\quad B(0;1)$ nuqtalarda bo'lgan uchburchakning konturi. Javobi: $1+\sqrt{2}.$

858. $\int\limits_L\frac{dl}{x-y}$ egri chiziqli integralni hisoblang. Bunda L kontur $y=\frac{1}{2}x-2$ to'g'ri chiziqning $A(0;-2)$ va $B(4;0)$ nuqtalari orasidagi kesmasi. Javobi: $\sqrt{5}\ln 2.$

859. $\int\limits_L xydl$ egri chiziqli integralni hisoblang. Bunda L - uchlari A(0;0), B(4;0), C(4;2), D(0;2) nuqtalarda bo'lgan to'g'ri to'rtburchak konturi. Javobi: 24

860. Agar $x=\sqrt{\cos t},\, y=\sqrt{\sin t},\quad 0\le t\le\frac{\pi}{2}$ bo'lsa, $\int\limits_k x^2ydy-y^2xdx$ egri chiziqli integralni hisoblang. Javobi: $\frac{\pi}{4}.$

861. Agar G: $y=x^2$ parabola, $-1\le x\le 1$ bo'lsa, $\int\limits_G(x^2-2xy)dx+(y^2-2xy)dy$ egri chiziqli integralni hisoblang. Javobi: $-\frac{14}{15}.$

862. $\int\limits_L(x^2-2xy)dx+(2xy+y^2)dy$ egri chiziqli integralni hisoblang. Bunda L kontur $y=x^2$ parabolaning A(1;1) nuqtasidan B(2;4) nuqtasigacha bo'lgan yoyi. Javobi: $40\frac{19}{30}.$

863. Agar L - A(0;0) va B(1;1) nuqtalarni tutashtiruvchi ciziq: $a)\, y=x;\quad b)\, y=x^2;\quad m)\, y^2=x;\quad n)\, y=x^3$ tenglamalar bilan berilgan bo'lsa, $\int\limits_L xydx+(y-x)dy$ egri chiziqli integralni hisoblang.

Javoblari: $a)\frac{1}{3};b)\frac{1}{12};m)\frac{17}{30};n)-\frac{1}{20}.$

864. $\oint\limits_L \boldsymbol{ydx-xdy}$ egri chiziqli integralni hisoblang. Bunda L kontur musbat yo'nalishda aylanib o'tiladigan $x=a\cos t,\quad y=b\sin t$ ellips. Javobi: $-2\pi ab$

865. $\oint\limits_L 2(x^2+y^2)dx+(x+y)^2dy$ egri chiziqli integralni hisoblang. Bunda L - uchlari A(1;1), B(2;2), C(1;3) nuqtalarda bo'lgan uchburchak konturi. Javobi: $-\frac{4}{3}$.

29-MAVZU. UCH KARRALI INTEGRALLAR

Agar Ω soha, ushbu

$$\begin{cases} a\le x\le b, \\ y_1(x)\le y\le y_2(x), \\ z_1(x,y)\le z\le z_2(x,y) \end{cases}$$

tengsizliklar sistemasi bilan aniqlangan bo'lsa, u holda uch karrali integral quyidagi formula bo'yicha hisoblanadi:

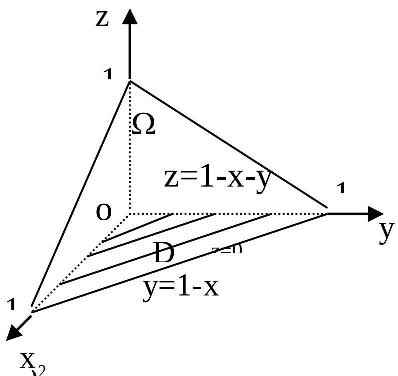
$$\iiint\limits_{\Omega} f(x,y,z)dxdydz=\int\limits_a^b dx\int\limits_{y_1(x)}^{y_2(x)}dy\int\limits_{z_1(x,y)}^{z_2(x,y)}f(x,y,z)dz$$

yoki

$$\iiint\limits_{\Omega} f(x,y,z)dxdydz=\iint\limits_D dxdy\int\limits_{z_1(x,y)}^{z_2(x,y)}f(x,y,z)dx.$$

Agar ***f(x; y; z)=1*** bo'lsa, Ω sohaning hajmi hisoblanadi:

Misol 866. Ushbu $\iiint\limits_{\Omega} zdxdydz$ integralni hisoblang, bu yerda Ω soha ***x+y+z=1, z=0, y=0, x=0*** tekisliklar bilan chegaralangan.



Yechilishi: Integrallash sohasi Ω ni chiziladi. Bu soha ushbu tengsizliklar sistemasi orqali aniqlanadi:

$$\begin{cases} 0\le \boldsymbol{x}\le 1, \\ 0\le \boldsymbol{y}\le 1-\boldsymbol{x}, \\ 0\le \boldsymbol{z}\le 1-\boldsymbol{x}-\boldsymbol{y}. \end{cases}$$

2. Berilgan uch karrali integral quyidagicha hisoblanadi:

$$\iiint\limits_{\Omega} zdxdydz=\int\limits_0^1 dx\int\limits_0^{1-x}dy\int\limits_0^{1-x-y}zdz=\int\limits_0^1 dx\int\limits_0^{1-x}dy\cdot\frac{1}{2}\cdot z^2\Big|_0^{1-x-y}=\frac{1}{2}\int\limits_0^1 dx\int\limits_0^{1-x}dy(1-x-y)^2=$$

$$=\frac{1}{2}\int\limits_0^1 dx\int\limits_0^{1-x}\left[-(1-x-y)^2d(1-x-y)\right]=-\frac{1}{2}\int\limits_0^1 dx\frac{1}{3}\cdot(1-x-y)^3\Big|_0^{1-x}=-\frac{1}{6}\int\limits_0^1 dx\left[(1-x-1+x)^3-(1-x-0)^3\right]=$$

$$=\frac{1}{6}\int\limits_0^1(1-x)^3dx=\frac{1}{6}\int\limits_0^1\left[-(1-x)^3d(1-x)\right]=-\frac{1}{6}\cdot\frac{1}{4}\cdot(1-x)^4\Big|_0^1=-\frac{1}{24}\left[(1-1)^4-(1-0)^4\right]=\frac{1}{24}.$$

Misol 867. $z=x^2+y^2,\ z=2x^2+2y^2,\ \ y=x,\ \ y=x^2$ sirtlar bilan chegaralangan jismning hajmini hisoblang.

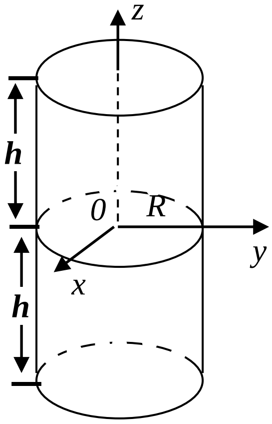
Yechilishi. $V=\iiint\limits_{V'}dxdydz.$

$$\begin{cases} y = x \\ y = x^2 \end{cases} \Rightarrow x^2 = x \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow \begin{cases} x = 0 \\ x = 1 \end{cases} \Rightarrow 0 \le x \le 1.$$

Demak, $V' = \{0 \le x \le 1;\ x^2 \le y \le x;\ x^2 + y^2 \le z \le 2(x^2 + y^2)\}.$

$$\boldsymbol{V} = \int_0^1 \boldsymbol{dx} \int_{x^2}^{x} \boldsymbol{dy} \int_{x^2+y^2}^{2(x^2+y^2)} \boldsymbol{dz} = \int_0^1 \boldsymbol{dx} \int_{x^2}^{x} \boldsymbol{dy} \cdot \boldsymbol{z} \Big|_{x^2+y^2}^{2(x^2+y^2)} = \int_0^1 \boldsymbol{dx} \int_{x^2}^{x} \boldsymbol{dy} [2\boldsymbol{x}^2 + 2\boldsymbol{y}^2 - \boldsymbol{x}^2 - \boldsymbol{y}^2] =$$

$$= \int_0^1 \boldsymbol{dx} \int_{x^2}^{x} (\boldsymbol{x}^2 + \boldsymbol{y}^2)\boldsymbol{dy} =$$



$$= \int_0^1 \boldsymbol{dx} \left\| \left[\boldsymbol{x}^2 \boldsymbol{y} + \frac{\boldsymbol{y}^2}{3} \right]_{x^2}^{x} = \int_0^1 \boldsymbol{dx} [(\boldsymbol{x}^2 \cdot \boldsymbol{x} - \boldsymbol{x}^2 \cdot \boldsymbol{x}^2) + \frac{1}{3}(\boldsymbol{x}^3 - (\boldsymbol{x}^2)^3] = \right.$$

$$= \int_0^1 \boldsymbol{dx} [\boldsymbol{x}^3 - \boldsymbol{x}^4 + \frac{1}{3}\boldsymbol{x}^3 - \frac{1}{3}\boldsymbol{x}^6) = \int_0^1 [\frac{4}{3}\boldsymbol{x}^3 - \boldsymbol{x}^4 - \frac{1}{3}\boldsymbol{x}^6]\boldsymbol{dx} = (\frac{4}{3} \cdot \frac{\boldsymbol{x}^4}{4} -$$

$$- \frac{1}{5}\boldsymbol{x}^5 - \frac{1}{3} \cdot \frac{1}{7}\boldsymbol{x}^7] \Big|_0^1 = \frac{1}{3} - \frac{1}{5} - \frac{1}{21} = \frac{35 - 21 - 5}{105} = \frac{9}{105} = \frac{3}{35} (\textbf{\textit{kub birlik}}).$$

JISMNING INERSIYA MOMENTI

Massasi ***m*** bo'lgan ***M(x,y,z)*** nuqtaning ***Ox, Oy, Oz*** koordinata o'qlariga nisbatan inersiya momenti mos ravishda quyidagi formulalar bilan ifodalanadi:

$$I_{xx} = (y^2 + z^2)m; \quad I_{yy} = (x^2 + z^2)m; \quad I_{zz} = (y^2 + x^2)m.$$

Jismning inersiya momenti mos integrallar bilan ifodalanadi. Masalan, ***Oz*** o'qqa nisbatan jismning inersiya momenti:

$$I_{ZZ} = \iiint_V (x^2 + y^2)\gamma(x, y, z)dxdydz,$$

bunda $\boldsymbol{\gamma}$ (***x,y,z***) moddaning zichligi

Misol 868. To'g'ri doiraviy silindrning balandligi 2h va radiusi R, zichligi o'zgarmas γ_0 ga teng. O'rta kesimining diametriga nisbatan silindrning inersiya momentini toping.

Yechilishi. Silindir o'qi ***(Oz)*** bilan, simmetriya markazi koordinatalar boshi bilan ustma-ust tushiriladi. Natijada silindrning inersiya momentini ***(Ox)*** o'qqa nisbatan hisoblashga keltiriladi: $J_{xx} = \iiint (y^2 + z^2)\gamma_0 dxdydz.$ Silindrik koordinatalarga o'tib, quyidagi olinadi:

$$J_{xx} = \gamma_0 \int_0^{2\pi} \left\{ \int_0^R \left[\int_{-h}^{h} (z^2 + \rho^2 \sin^2\theta)dz \right] \rho d\rho \right\} d\theta = \gamma_0 \int_0^{2\pi} \left\{ \int_0^R \left[\frac{2h^3}{3} + 2h\rho^2 \sin^2\theta \right] \rho d\rho \right\} d\theta =$$

$$= \gamma_0 \int_0^{2\pi} \left\{ \frac{2h^3}{3} \cdot \frac{R^2}{2} + \frac{2h \cdot R^4}{4} \sin^2\theta \right\} d\theta = \gamma_0 \left[\frac{2h^3 R^2}{6} \cdot 2\pi + \frac{2hR^4}{4} \cdot \pi \right] = \gamma_0 \pi h R^2 \left[\frac{2}{3}h^2 + \frac{R^2}{2} \right].$$

JISM OG'IRLIK MARKAZINING KOORDINATALARI

Yassi shakldagi jism og'irlik markazining koordinatalari ushbu formulalar bilan ifodalanadi:

$$x_c = \frac{\iiint\limits_V x\gamma(x,y,z)dxdydz}{\iiint\limits_V \gamma(x,y,z)dxdydz}; \quad y_c = \frac{\iiint\limits_V y\gamma(x,y,z)dxdydz}{\iiint\limits_V \gamma(x,y,z)dxdydz};$$

$$z_c = \frac{\iiint\limits_V z\gamma(x,y,z)dxdydz}{\iiint\limits_V \gamma(x,y,z)dxdydz};$$

bunda γ (***x,y,z***) – zichlik.

Misol 869. Markazi koordinatalar boshida bo'lgan R radiusli sharning γ_0 zichligi o'zgarmas bo'lganda, yuqori yarim shar og'irlik markazining koordinatalarini toping.

Yechilishi. Yarim shar $z = \sqrt{R^2 - x^2 - y^2}$, $z = 0$ sirtlar bilan chegaralangan.

Uning og'irlik markazining aplikatasi $z_c = \frac{\iiint\limits_v z\gamma_0 dxdydz}{\iiint\limits_v \gamma_0 dxdydz}$ formula yordamida aniqlanadi.

Sferik koordinatalarga o'tiladi:

$$z_c = \frac{\gamma_0 \int\limits_0^{2\pi} \left\{ \int\limits_0^{\frac{\pi}{2}} \left[\int\limits_0^R r\cos\varphi \cdot r^2 \sin\varphi dr \right] d\varphi \right\} d\theta}{\gamma_0 \int\limits_0^{2\pi} \left\{ \int\limits_0^{\frac{\pi}{2}} \left[\int\limits_0^R r^2 \sin\varphi dr \right] d\varphi \right\} d\theta} = \frac{2\pi \frac{R^2}{4} \cdot \frac{1}{2}}{\frac{2}{3}\pi R^3} = \frac{3}{8} \cdot R.$$

Yarim sharlar simmetrik bo'lganligidan Demak, $\left(0;0;\frac{3}{8}R\right)$.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR
Quyidagi uch karrali integrallarni hisoblang:

870. $\int\limits_0^1 dx \int\limits_0^x dy \int\limits_0^{xy} x^3 y^3 z dz.$ Javobi: $\frac{1}{110}$. **871.** $\int\limits_0^3 dx \int\limits_0^{2x} dy \int\limits_0^{\sqrt{xy}} z dz.$ Javobi: $\frac{81}{4}$.

872. $\int\limits_0^a dx \int\limits_0^x dy \int\limits_0^y xyz dz.$ Javobi: $\frac{a^6}{48}$.

873. $\iiint\limits_\Omega xydxdydz$ uch o'lchovli integralni hisoblang. Bunda Ω soha $z = xy$ giperbolik paraboloid hamda $x + y = 1$ va $z = 0$ $(z \geq 0)$ tekisliklar bilan chegaralangan. *Javobi:* $\frac{1}{180}$.

874. $\iiint\limits_\Omega \boldsymbol{xyzdxdydz}$ uch o'lchovli integralni hisoblang. Bunda Ω soha $y = x^2$, $x = y^2$, $z = xy$, $z = 0$ sirtlar bilan chegaralangan. *Javobi:* $\frac{1}{96}$.

875. $\iiint\limits_\Omega (2x + y)dxdydz$ uch o'lchovli integralni hisoblang. Bunda Ω soha $y = x$, $x = 1$, $z = 1$ va $z = 1 + x^2 + y^2$ sirtlar bilan chegaralangan. *Javobi:* $\frac{41}{60}$.

876. $z = x + y,\quad z = x\cdot y,\quad x + y = 1,\ x = 0,\ y = 0$ sirtlar bilan chegaralangan jismning hajmini hisoblang. *Javobi:* $\frac{7}{24}$.

877. $(x-a)^2 + (y-b)^2 = 2a^2$ doira yuzining (Oy) o'qqa nisbatan inersiya momentini hisoblang. *Javobi:* $3\pi\cdot a^4$.

878. $y^2 = ax$ parabola va $x = a$ to'g'ri chiziq bilan chegaralangan shakl yuzining $y = -a$ to'g'ri chiziqqa nisbatan inersiya momentini hisoblang. *Javobi:* $\frac{8}{5}\cdot a^4$.

879. $x^2 + y^2 = a^2$ doira ustki yarmi og'irlik markazining koordinatalarini toping. *Javobi:* $x_c = 0,\quad y_c = \frac{4a}{3\pi}$.

880. $x - a(t-\sin t),\qquad y = a(1-\cos t)$ sikloida bitta arki yuzi og'irlik markazi koordinatalarini toping. *Javobi:* $x_c = a\pi,\qquad y_c = \frac{5a}{6}$.

30-MAVZU. QATORLAR

SONLI QATORLAR

$a_1, a_2, a_3, ...a_n, ...$ (1) sonli ketma-ketlik.

$$\boldsymbol{a}_1 + \boldsymbol{a}_2 + \boldsymbol{a}_3 + ... + \boldsymbol{a}_n + ... = \sum_{n=1}^{\infty} \boldsymbol{a}_n \qquad (2)$$

sonli qator. a_1 - qatorning birinchi hadi, a_n esa umumiy hadi deyiladi.

$$S_n = a_1 + a_2 + a_3 + ... + a_n \qquad (3)$$

(2) qatorning dastlabki n ta hadi yig'indisi. Buni (2) ning n-qismiy yig'indisi deyiladi.

Agar $\lim\limits_{n\to\infty} S_n = S$ chekli limit mavjud bo'lsa, (2) qator yaqinlashuvchi, bunda S ni uning yig'indisi deyiladi.

Agar $\lim\limits_{n\to\infty} S_n = \infty$ bo'lsa yoki mavjud bo'lmasa (2) qator uzoqlashuvchi deyiladi.

Misol 881. $\sum\limits_{n=1}^{\infty} \frac{n}{(n+1)^2}$ qatorni yoyib yozing.

Yechilishi. n ning o'rniga natural sonlar qo'yilib qo'shiladi:

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)^2} = \frac{1}{2^2} + \frac{2}{3^2} + \frac{3}{4^2} + ...$$

Misol 882. $\frac{1}{2\cdot 7} + \frac{1}{7\cdot 12} + \frac{1}{12\cdot 17} + \frac{1}{17\cdot 22} + ...$ qatorning umumiy hadini toping.

Yechilishi. $a_n = a_1 + d(n-1)$ formuladan foydalanib quyidagi ketma-ketliklarning umumiy hadlari topiladi:

$$2, 7, 12, 17, ... \Rightarrow 2 + 5(\boldsymbol{n}-1) = 5\boldsymbol{n} - 3; \qquad 7, 12, 17, 22, ... \Rightarrow 7 + 5(\boldsymbol{n}-1) = 5\boldsymbol{n} + 2$$

U holda $a_n = \frac{1}{(5n-3)\cdot(5n+2)}$. Shuningdek

$$a_{n+1} = \frac{1}{[5(n+1)-3]\cdot[5(n+1)+2]} = \frac{1}{(5n+2)(5n+7)}.$$

Misol 883. $\boldsymbol{ln}2 + \boldsymbol{ln}\frac{3}{2} + \boldsymbol{ln}\frac{4}{3} + ... = \sum\limits_{n=1}^{\infty} \boldsymbol{ln}\frac{\boldsymbol{n}+1}{\boldsymbol{n}}$ qatorning yaqinlashishini tekshiring.

Yechilishi.

$$S_n = \ln 2 + \ln\frac{3}{2} + \ln\frac{4}{3} + ... + \ln\frac{n}{n-1} + \ln\frac{n+1}{n} = \ln 2\cdot\frac{3}{2}\cdot\frac{4}{3}...\frac{n}{n-1}\cdot\frac{n+1}{n} = \ln(n+1) \Rightarrow$$

$$\Rightarrow \lim_{n\to\infty} S_n = \lim_{n\to\infty}\ln(n+1) = \ln\infty = \infty.$$ Demak, qator uzoqlashuvchi.

Misol 884. $\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}$ qatorning yaqinlashishini tekshiring.

Yechilishi. $a_n = \frac{1}{(2n-1)(2n+1)} = \frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right).$

$$S_n = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + ... + \frac{1}{(2n-1)(2n+1)} = \frac{1}{2}\left(1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + ... + \frac{1}{2n-1} - \frac{1}{2n+1}\right) =$$

$$= \frac{1}{2}\left(1 - \frac{1}{2n+1}\right) \Rightarrow \lim_{n\to\infty} Sn = \lim_{n\to\infty}\frac{1}{2}\left(1 - \frac{1}{2n+1}\right) = \frac{1}{2}.$$ Demak, qator yaqinlashuvchi.

QATOR YAQINLASHISHINING ZARURIY SHARTI

$$\lim_{n\to\infty} a_n = 0.$$

Misol 885. $1 + \frac{6}{4} + \frac{9}{5} + \frac{12}{6} + ...$ qator yaqinlashishining zaruriy shartini tekshiring.

Yechilishi.

$$1 + \frac{6}{4} + \frac{9}{5} + \frac{12}{6} + ... = \frac{3}{3} + \frac{6}{4} + \frac{9}{5} + \frac{12}{6} + ... \Rightarrow a_n = \frac{3n}{n+2} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{3n}{n+2} = \lim_{n\to\infty}\frac{3}{1+\frac{2}{n}} = 3 \neq 0.$$

MUSBAT HADLI QATORLARNING YAQINLASHISH VA UZOQLASHISH ALOMATLARI

TAQQOSLASH ALOMATI

$\sum_{n=1}^{\infty} a_n$ va $\sum_{n=1}^{\infty} b_n$ qatorlar uchun $a_n \le b_n$ bo'lsin. U holda $\sum_{n=1}^{\infty} b_n$ qator yaqinlashishidan $\sum_{n=1}^{\infty} a_n$ qatorning yaqinlashishi, $\sum_{n=1}^{\infty} a_n$ qatorning uzoqlashishidan $\sum_{n=1}^{\infty} b_n$ qatorning uzoqlashishi kelib chiqadi.

Misol 886. $\sum_{n=1}^{\infty}\frac{1}{n\cdot 2^n} = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 2^2} + \frac{1}{3\cdot 2^3} + ...$ qator yaqinlashishini taqqoslash alomatidan foydalanib tekshiring.

Yechilishi. $a_n = \frac{1}{n\cdot 2^n} \le \frac{1}{2^n} = b_n.$

$\sum_{n=1}^{\infty}\frac{1}{2^n} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ...$. Bu maxraji $q = \frac{1}{2} < 1$ bo'lgan geometrik progressiya hadlari yig'indisidan iborat va u yaqinlashuvchi. Demak, taqqoslash alomatiga ko'ra berilgan qator ham yaqinlashuvchi.

DALAMBER ALOMATI

$\sum_{n=1}^{\infty} a_n$ qator uchun $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = d$ limit mavjud bo'lib, d<1 bo'lsa qator yaqinlashuvchi, d>1 bo'lsa uzoqlashuvchi bo'ladi. d=1 da ma'lum emas.

Misol 887. $\frac{1}{5} + \frac{2}{11} + \frac{3}{29} + ... + \frac{n}{3^n + 2} + ...$ qatorning yaqinlashishini tekshiring.

Yechilishi. $a_n = \frac{n}{3^n + 2}$, $a_{n+1} = \frac{n+1}{3^{n+1} + 2}$.

$$\frac{a_{n+1}}{a_n}=\frac{n+1}{3^{n+1}+2}\cdot\frac{3^u+2}{n}=\frac{n+1}{n}\cdot\frac{3^n+2}{3^{n+1}+2}=\frac{1+\frac{1}{n}}{1}\cdot\frac{1+\frac{2}{3^n}}{\frac{3^n\cdot 3}{3^n}+\frac{2}{3^n}}=\left(1+\frac{1}{n}\right)\cdot\frac{1+\frac{2}{3^n}}{3+\frac{2}{3^n}}.$$

$\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)\cdot\frac{1+\frac{2}{3^n}}{3+\frac{2}{3^n}}=1\cdot\frac{1}{3}=\frac{1}{3}<1.$ Demak qator yaqinlashuvchi.

Misol 888. $1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}+...$ garmonik qatorning uzoqlashuvchiligini tekshiring.

Yechilishi. $a_n=\frac{1}{n},\ a_{n+1}=\frac{1}{n+1}\cdot$

$$\lim_{n\to\infty}\frac{a_n+1}{a_n}=\lim_{n\to\infty}\frac{1}{n+1}\cdot\frac{n}{1}=\lim_{n\to\infty}\frac{n}{n+1}=\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}=1.$$

Demak, Dalamber alomati masalani hal etmaydi.

KOSHI ALOMATI

$\sum\limits_{n=1}^{\infty}a_n$ qator uchun $\lim\limits_{n\to\infty}\sqrt[n]{a_n}=k$ limit mavjud bo'lib , k<1 bo'lsa qator yaqinlashuvchi, k>1 bo'lsa uzoqlashuvchi. $\boldsymbol{k}$=1 da ma'lum emas.

Misol 889. $\sum\limits_{n=1}^{\infty}\frac{1}{2^n}(1+\frac{1}{n})^{n^2}$ qator yaqinlashishini tekshiring.

Yechilishi. $\lim\limits_{n\to\infty}\sqrt[n]{\frac{1}{2^n}\left(1+\frac{1}{n}\right)^{n^2}}=\lim\limits_{n\to\infty}\frac{1}{2}\left(1+\frac{1}{n}\right)^n=\frac{1}{2}\cdot e>1.$ Demak, qator uzoqlashuvchi.

KOSHINING INTEGRAL ALOMATI

$\sum\limits_{n=1}^{\infty}a_n$ qator hadlari, quyidagi natural argumentli $f(x)=f(n)$ funksiyadan iborat bo'lsin. U holda $\int\limits_1^{\infty}f(x)dx$ xosmas integral yaqinlashsa, berilgan qator ham yaqinlashadi va aksincha.

Misol 890. $\sum\limits_{n=1}^{\infty}\frac{2n}{(n^2+1)^2}$ qatorning yaqinlashishini tekshiring.

Yechilishi. $f(x)=\frac{2x}{(x^2+1)^2}$ deb olinadi. U holda

$$\int_1^{\infty}\frac{2xdx}{(x^2+1)^2}=\lim_{b\to\infty}\int_1^b\frac{d(x^2+1)}{(x^2+1)^2}=\lim_{b\to\infty}\int_1^b(x^2+1)^{-2}d(x^2+1)=\lim_{b\to\infty}\frac{(x^2+1)^{-1}}{-1}\Big|_1^b=\lim_{b\to\infty}\left(-\frac{1}{x^2+1}\right)\Big|_1^b=$$

$$=-\lim_{b\to\infty}\left(\frac{1}{b^2+1}-\frac{1}{1+1}\right)=-\left[\frac{1}{\infty+1}-\frac{1}{1+1}\right]=\frac{1}{2}.$$

Demak, xosmas integral yaqinlashadi, shuning uchun berilgan qator ham yaqinlashadi.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi qatorlarni yoyib yozing:

891. $\sum\limits_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}.$ **892.** $\sum\limits_{n=1}^{\infty}\frac{1}{n(n+3)}.$ **893.** $\sum\limits_{n=1}^{\infty}\frac{3^n+2^n}{6^n}.$ **894.** $\sum\limits_{n=1}^{\infty}\frac{2n+1}{n^2(n+1)^2}.$

Quyidagi qatorlarning a_n va a_{n+1} hadlarini toping:

895. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\ldots$; **896.** $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}+\frac{7}{8}+\ldots$; **897.** $2+\frac{5}{2!}+\frac{8}{3!}+\frac{11}{4!}+\ldots$;

898. $\frac{2}{15}+\frac{6}{75}+\frac{12}{375}+\frac{20}{1875}+\ldots$ *Ko'rsatma*: $2,\ 2\cdot 3,\ 3\cdot 4,\ 4\cdot 5,\ldots$ - surat;

$15\cdot 5^0$, $15\cdot 5^1$, $15\cdot 5^2$, $15\cdot 5^3$,... - maxraj.

Quyidagi qatorlarning yaqinlashishini tekshiring:

899. $1+\frac{1}{3^2}+\frac{1}{3^4}+\ldots+\frac{1}{3^{2n}}+\ldots$ Javobi: $\frac{9}{8}$, yaqinlashuvchi.

900. $\ln 2+\ln\frac{3}{1}+\ln\frac{4}{2}+\ldots+\ln\frac{n+1}{n-1}+\ldots$ Javobi: uzoqlashuvchi.

901. $\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\ldots+\frac{1}{n(n+2)}+\ldots$ Javobi: $\frac{3}{4}$, yaqinlashuvchi.

Quyidagi qatorlarning yaqinlashuvchi ekanligini isbotlang va ularning yig'indisini toping:

902. $\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}$. Javobi: $S=\frac{1}{2}$. 903. $\sum_{n=1}^{\infty}\frac{1}{(3n-2)(3n+1)}$. Javobi: $S=\frac{1}{3}$.

904. $\sum_{n=1}^{\infty}\frac{3^n+2^n}{6^n}$. Javobi: $S=\frac{3}{2}$. 905. $\sum_{n=1}^{\infty}\frac{2n+1}{n^2(n+1)^2}$. Javobi: $S=1$.

Quyidagi qatorlarning yaqinlashuvchi yoki uzoqlashuvchi ekanligini tekshiring:

906. $\sum_{n=1}^{\infty}\frac{1}{\ln(n+1)}$. Javobi: uzoqlashuvchi. 907. $\sum_{n=1}^{\infty}\frac{n^n}{n!}$. Javobi: uzoqlashuvchi.

908. $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$. Javobi: yaqinlashuvchi. 909. $\sum_{n=1}^{\infty}\frac{n}{2n-1}$. Javobi: uzoqlashuvchi.

910. $\sum_{n=1}^{\infty}\frac{n^2}{n!}$. Javob: yaqinlashuvchi. **911.** $\sum_{n=1}^{\infty} arctg^n\frac{1}{n}$. Javob: yaqinlashuvchi.

31-MAVZU. ISHORA ALMASHINUVCHI QATORLAR

LEYBNITS ALOMATI

$\sum_{n=1}^{\infty}(-1)^{n+1}a_n=a_1-a_2+a_3-a_4+\cdots$ qator uchun $|a_1|>|a_2|>|a_3|>\cdots$ va $\lim_{n\to\infty}a_n=0$ bo'lsa, qator yaqinlashuvchi bo'ladi va uning S yig'indisi $0<S<a_1$ shartni qanoatlantiradi.

Misol 912. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}=1-\frac{1}{2}+\frac{1}{3}-\cdots$ qatorning yaqinlashishini tekshiring.

Yechilishi. $1>\frac{1}{2}>\frac{1}{3}>\cdots$ va $\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{1}{n}=0$. Demak Leybnits alomati o'rinli bo'lganligi uchun qator yaqinlashuvchi.

ABSOLYUT VA SHARTLI YAQINLASHUVCHI QATORLAR

$\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ qatordan tuzilgan $\sum_{n=1}^{\infty}|a_n|$ qator yaqinlashuvchi bo'lsa, berilgan qator absolyut yaqinlashuvchi qator deyiladi.

Agar $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ qator yaqinlashuvchi bo'lib, undan tuzilgan $\sum_{n=1}^{\infty}|a_n|$ uzoqlashuvchi bo'lsa, berilgan o'zgaruvchi ishorali qator shartli yaqinlashuvchi qator deyiladi.

Misol 913. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}$ qator yaqinlashishini tekshiring.

Yechilishi. $$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}=\frac{1}{2}-\frac{2}{5}+\frac{3}{10}-\frac{4}{17}+\cdots \qquad (*)$$

Leybnits alomatiga asosan $\frac{1}{2}>\frac{2}{5}>\frac{3}{10}>\frac{4}{17}>\cdots$ va $\lim_{n\to\infty}\frac{n}{n^2+1}=\lim_{n\to\infty}\frac{\frac{1}{n}}{1+\frac{1}{n^2}}=0$ bo'lganligi uchun qator yaqinlashuvchi.

Endi (*) qator hadlarining absolyut qiymatlaridan to'zilgan
$$\sum_{n=1}^{\infty}\frac{n}{n^2+1}=\frac{1}{2}+\frac{2}{5}+\frac{3}{10}+\frac{4}{17}+\cdots$$
qator tekshiriladi.

Taqqoslash alomatiga ko'ra garmonik qator uzoqlashuvchi bo'lgani uchun bu qator ham uzoqlashadi. Demak, (*) qator shartli yaqinlashuvchi.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi o'zgaruvchi ishorali qatorlarning absolyut yoki shartli yaqinlashishini tekshiring:

914. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{2n+1}$. $J: uzoq-chi$.

915. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n+1}$. $J: shartli\ yaq-chi$.

916. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{2n-1}$. $J: uzoq-chi$.

917. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n!}$. $J: absolyut\ yaq-chi$.

918. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3n-2}{3n-1}$. $J: uzoq-chi$.

919. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\ln(n+1)}$. $J: shartli\ yaq-chi$.

920. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n\cdot 2^n}$. $J: absolyut\ yaq-chi$.

921. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3^n}{n^2}$. $J: uzoq-chi$.

922. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^3}{2^n}$. $J: absolyut\ yaq-chi$.

923. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{6n+5}$. $J: uzoq-chi$.

924. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+1}{n^2+n+1}$. $J: absolyut\ yaq-chi$.

925. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2+4}$. $J: absolyut\ yaq-chi$.

32-MAVZU. FUNKSIONAL QATORLAR

Hadlari funksiyalardan iborat bo'lgan
$$\sum_{n=1}^{\infty}f_n(x)=f_1(x)+f_2(x)+\cdots+f_n(x)+\cdots \qquad (1)$$
qatorga funksional qator deyiladi.

Agar
$$\sum_{n=1}^{\infty}f_n(x_0)=f_1(x_0)+f_2(x_0)+\cdots+f_n(x_0)+\cdots$$

sonli qator yaqinlashuvchi bo'lsa, (1) funksional qator $x=x_0$ nuqtada yaqinlashuvchi deyiladi. Funksional qator yaqinlashuvchi bo'ladigan x ning qiymatlar sohasini, funksional qatorning yaqinlashish sohasi deyiladi.

Funksional qatordan ajratib olingan

$$S_n(x)=f_1(x)+f_2(x)+\cdots+f_n(x)$$

yig'indini, funksional qatorning n-qismiy yig'indisi deyiladi. $\lim\limits_{n\to\infty} S_n(x)=S(x)$ funksiyani funksional qatorning yig'indisi deyiladi.

Misol. 926. $\sum\limits_{n=1}^{\infty} x^{n-1}=1+x+x^2+\cdots$ qatorning yaqinlashish sohasi va yig'indisini toping.

Yechilishi. Qismiy yig'indi ajratiladi va geometrik progressiya n ta hadi yig'indisi formulasi $S_n=\frac{b_1-b_nq}{1-q}$ dan foydalanib, uning yig'indisi topiladi:

$$S_n(x)=1+x+x^2+\cdots+x^{n-1}=\frac{1-x^{n-1}\cdot x}{1-x}=\frac{1-x^n}{1-x}.$$

Agar $x=1$ bo'lsa, $S_n(x)=1+1+1+\cdots+1=n$ bo'ladi.

Bulardan

$$S_n(x)=\begin{cases}\frac{1-x^n}{1-x}, & agar\ x\neq 1\ bo'lsa;\\ n, & agar\ x=1\ bo'lsa.\end{cases}$$

$|x|<1$, ya'ni (-1; 1) da berilgan qator yaqinlashuvchi.

$$S(x)=\lim_{n\to\infty}S_n(x)=\lim_{n\to\infty}\frac{1-x^n}{1-x}=\lim_{n\to\infty}\left[\frac{1}{1-x}-\frac{x^n}{1-x}\right]=\frac{1}{1-x}-\lim_{n\to\infty}\frac{1}{1-x}\cdot x^n=\frac{1}{1-x}.$$

Demak, berilgan qatorning yaqinlashish sohasi (-1;1), yig'indisi $S(x)=\frac{1}{1-x}$ dan iborat.

FUNKSIONAL QATORNING TEKIS YAQINLASHUVCHI BO'LISHINING VEYERSHTRASS ALOMATI

$\sum\limits_{n=1}^{\infty} f_n(x)$ funksional qator uchun hadlari musbat sonlardan iborat shunday yaqinlashuvchi $\sum\limits_{n=1}^{\infty} c_n$ qator mavjud bo'lib, $x\in[a;b]$ *da* $|f_n(x)|\le c_n$ bo'lsa, u holda funksional qator bu $[a;b]$ kesmada tekis yaqinlashadi.

Misol 927. $\sum\limits_{n=1}^{\infty}\frac{\cos nx}{n^2}=\frac{\cos x}{1^2}+\frac{\cos 2x}{2^2}+\frac{\cos 3x}{3^2}+\cdots$ qator $(-\infty;+\infty)$ sohada tekis yaqinlashuvchi ekanligini isbotlang.

Yechilishi. $|f_n(x)|=\left|\frac{\cos nx}{n^2}\right|\le\frac{1}{n^2}$ bo'lib, $\sum\limits_{n=1}^{\infty}\frac{1}{n^2}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$ sonli qator yaqinlashuvchi. Demak, berilgan qator $(-\infty;+\infty)$ sohada tekis yaqinlashuvchi.

DARAJALI QATORLAR

$$\sum_{n=0}^{\infty}a_n(x-x_0)^n=a_0+a_1(x-x_0)+a_2(x-x_0)^2+\cdots$$

ko'rinishdagi funksional qatorga darajali qator deyiladi.

Xususiy holda, $x_0=0$ bo’lganda darajali qator

$$\sum_{n=1}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots$$

ko’rinishni oladi.

ABEL TEOREMASI

1) agar $\sum_{n=1}^{\infty} a_n x^n$ darajali qator birorta $x = x_1 \neq 0$ nuqtada yaqinlashsa, u holda u x ning $|x| < |x_1|$ tengsizlikni qanoatlantiruvchi har qanday qiymatida yaqinlashadi;

2) agar $\sum_{n=1}^{\infty} a_n x^n$ qator birorta $x = x_1$ qiymatda uzoqlashsa, u holda u x ning $|x| > |x_1|$ tengsizlikni qanoatlantiruvchi har qanday qiymatida uzoqlashadi.

$\sum_{n=1}^{\infty} a_n x^n$ darajali qator uchun shunday (-R;R) oraliq mavjud-ki, qator shu oraliq ichida absolyut yaqinlashib, undan tashqarida uzoqlashadi. Bu oraliqni darajali qatorning yaqinlashish intervali deyiladi. Yaqinlashish intervalining chetki nuqtalari $x = \pm R$ da darajali qatorning yaqinlashish yoki uzoqlashishi alohida o’rganiladi.

$\sum_{n=1}^{\infty} a_n x^n$ darajali qatorning yaqinlashish radiusi quyidagi formulalar yordamida aniqlanadi:

$$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| \quad yoki \quad R = \lim_{n\to\infty}\frac{1}{\sqrt[n]{|a_n|}}.$$

$\sum_{n=1}^{\infty} a_n x^{n\cdot p}$ qatorning yaqinlashish radiusi quyidagi formulalar yordamida aniqlanadi:

$$R = \sqrt[p]{\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|} \quad yoki \quad R = \sqrt[p]{\lim_{n\to\infty}\frac{1}{\sqrt[n]{|a_n|}}}.$$

Misol 928. $\sum_{n=0}^{\infty} n!\, x^n = 1 + 1!\, x + 2!\, x^2 + 3!\, x^3 + ... + n!\, x^n + ...$ darajali qatorning yaqinlashish radiusini toping.

Yechilishi:

$a_n = n!; \qquad a_{n+1} = (n+1)!.$

$$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\left|\frac{n!}{(n+1)!}\right| = \lim_{n\to\infty}\left|\frac{n!}{n!(n+1)}\right| = \lim_{n\to\infty}\left|\frac{1}{(n+1)}\right| = 0 \Rightarrow R = 0$$

Demak, berilgan qator faqat $x = 0$ nuqtada yaqinlashadi.

Misol 929. $\sum_{n=1}^{\infty}\frac{1}{n\cdot 2^n} x^n = \frac{1}{1\cdot 2} x + \frac{1}{2\cdot 2^2} x^2 + \frac{1}{3\cdot 2^3} x^3 + ...$ qatorning yaqinlashish intervalini toping va intervalning chetki nuqtalarida qatorning yaqinlashishini tekshiring.

Yechilishi: $a_n = \frac{1}{n\cdot 2^n}; \qquad a_{n+1} = \frac{1}{(n+1)\cdot 2^{n+1}}.$

$$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\left|\frac{1}{n\cdot 2^n}:\frac{1}{(n+1)2^n\cdot 2}\right|=\lim_{n\to\infty}\left|\frac{(n+1)\cdot 2^n\cdot 2}{n\cdot 2^n}\right|=$$

$$=\lim_{n\to\infty}\left|\frac{2n+2}{n}\right|=\lim_{n\to\infty}\frac{2+\frac{2}{n}}{\frac{n}{n}}=2\ \Rightarrow\ R=2\ \Rightarrow(-2;2).$$

Endi topilgan intervalning chegaralarida berilgan qatorning yaqinlashishi yoki uzoqlashishi tekshiriladi:

1) $x=-2\Rightarrow\sum_{n=1}^{\infty}\frac{1}{n\cdot 2^n}\cdot(-2)^n=\sum_{n=1}^{\infty}(-1)^n\cdot\frac{1}{n}=-1+\frac{1}{2}-\frac{1}{3}+...+(-1)^n\cdot\frac{1}{n}+...;$

Leybnits alomatiga asosan bu qator yaqinlashuvchi;

2) $x=2\Rightarrow\sum_{n=1}^{\infty}\frac{1}{n\cdot 2^n}\cdot 2^n=\sum_{n=1}^{\infty}\frac{1}{n}=1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}+...$

bu garmonik qator bo'lib, uzoqlashuvchi.

Demak, berilgan qatorning yaqinlashish oralig'i $[-2;2)$.

DARAJALI QATORNING XOSSALARI

a) yaqinlashish intervali ichida yotuvchi $[a;b]$ kesmada darajali qator tekis yaqinlashadi. Uning yig'indisi yaqinlashish intervalida uzluksiz bo'ladi.

b) darajali qatorni uning yaqinlashish intervalida hadma-had integrallash va differensiallash mumkin.

Misol 930. $\sum_{n=1}^{\infty}\frac{x^{2n-1}}{2^{n-1}}=x+\frac{x^3}{3}+\frac{x^5}{5}+\cdots$

Yechilishi. $a_n=\frac{1}{2n-1};\qquad a_{n+1}=\frac{1}{2(n+1)-1}=\frac{1}{2n+1}.$

x ning darajasida *n* dan tashqari 2 bo'lganligi uchun kvadrat ildiz olinadi:

$$R=\sqrt{\lim_{n\to\infty}\frac{1}{2n-1}:\frac{1}{2n+1}}=\sqrt{\lim_{n\to\infty}\frac{2n+1}{2n-1}}=1.$$

Demak, (-1;1) oraliqda qator yaqinlashadi, shuning uchun uni shu intervalda hadlab differensiallash mumkin. Berilgan qator yig'indisi *S(x)* bilan belgilansa:

$$S'(x)=1+x^2+x^4+\cdots+x^{2n-2}+\cdots$$

Bu qator (-1;1) da yaqinlashadi. Qator geometrik progressiya tashkil etganligi uchun uning yig'indisi $S'(x)=\frac{1}{1-x^2}$ bo'ladi.

U holda berilgan qator yig'indisi

$$S(x)=\int S'(x)dx=\int\frac{dx}{1-x^2}=\frac{1}{2}ln\left|\frac{1+x}{1-x}\right|,\qquad (|x|<1).$$

FUNKSIYANI DARAJALI QATORGA YOYISH

$$f(x)=f(x_0)+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+\cdots+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+R_n(x)$$

- Teylor qatori.

$$f(x)=f(0)+\frac{f^{I}(0)}{1!}x+\frac{f^{II}(0)}{2!}x^2+...+\frac{f^{(n)}(0)}{n!}x^n+...$$ - Makloren qatori.

Misol 931. $f(x)=e^x$ funktsiyani darajali qatorga yoyinig.

Yechilishi

$$f(x)=e^x; \qquad f(0)=1;$$
$$f'(x)=e^x; \qquad f'(0)=1;$$
$$f''(x)=e^x; \qquad f''(0)=1;$$
$$\ldots\ldots \qquad \ldots\ldots$$
$$f^{(n)}(x)=e^x; \qquad f^{(n)}(0)=1.$$

U holda

$$e^x=1+\frac{1}{1!}x+\frac{1}{2!}x^2+...+\frac{1}{n!}x^n+....$$

Misol 932. $f(x)=\sin x$ funktsiyani darajali qatorga yoying.

Yechilishi:

$$f(x)=\sin x; \qquad f(0)=\sin 0=0;$$
$$f'(x)=\cos x=\sin\left(x+\frac{\pi}{2}\right); \qquad f'(0)=\sin\left(0+\frac{\pi}{2}\right)=1;$$
$$f''(x)=\sin\left(x+2\cdot\frac{\pi}{2}\right); \qquad f''(0)=\sin\pi=0;$$
$$f'''(x)=\sin\left(x+3\cdot\frac{\pi}{2}\right); \qquad f'''(0)=\sin\frac{3\pi}{2}=-1;$$

$$f^{(IV)}(x)=\sin\left(x+4\cdot\frac{\pi}{2}\right); \qquad f^{(IV)}(0)=\sin 2\pi=0;$$
$$f^{(V)}(x)=\sin\left(x+5\cdot\frac{\pi}{2}\right); \qquad f^{(V)}(0)=\sin\frac{5\pi}{2}=1;$$
$$f^{(VI)}(x)=\sin\left(x+6\cdot\frac{\pi}{2}\right); \qquad f^{(VI)}(0)=\sin 3\pi=0;$$
$$f^{(VII)}(x)=\sin\left(x+7\cdot\frac{\pi}{2}\right); \qquad f^{(VII)}(0)=\sin\frac{7\pi}{2}=-1.$$
$$\ldots\ldots \qquad \ldots\ldots$$

$$\sin x=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^{(IV)}}{4!}x^4+\cdots=\frac{x}{1!}-\frac{x^3}{3!}+\cdots+(-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!}+\cdots$$

Misol 933. $2^{0,09}$ ni Makloren qatoriga yoyish orqali hisoblang.

Yechilishi.

$$\begin{aligned} f(x)&=2^x & & f(0)=1\\ f'(x)&=2^x\ln 2, & & f'(0)=\ln 2,\\ f''(x)&=2^x\ln^2 2, & \Rightarrow\quad & f''(0)=\ln^2 2,\\ f'''(x)&=2^x\ln^3 2. & & f'''(0)=\ln^3 2.\end{aligned}$$

Bularni hamda $x=0,09$ va $\ln 2=0,6932$ larni Makloren formulasiga qo'ysak

$$2^{0,09}=1+\frac{\ln 2}{1!}\cdot 0,09+\frac{(\ln 2)^2}{2!}\cdot 0,09^2+\frac{(\ln 2)^3}{3!}\cdot 0,09^3=1+0,0624+$$
$$+0,2402\cdot 0,00812+0,555\cdot 0,0007=1,0624+0,0019+0,00003=1,0643.$$

$$(1+x)^m=1+\frac{m}{1!}\cdot x+\frac{m(m-1)}{2!}x^2+\frac{m(m-1)(m-2)}{3!}x^3+\cdots$$ - Binomial qator.

Misol 934. Binomial qatordan foydalanib $\sqrt{90}$ ni hisoblang.

Yechilishi. $\sqrt{90}=\sqrt{81+9}=\sqrt{81\cdot\left(1+\frac{1}{9}\right)}=9\cdot\left(1+\frac{1}{9}\right)^{\frac{1}{2}}$.

Demak, $x=\frac{1}{9},\quad m=\frac{1}{2}$ ekan. Bularni Binomial qator formulasining dastlabki uchta hadiga qo'yib hisoblaymiz:

$$9\cdot\left(1+\frac{1}{9}\right)^{\frac{1}{2}}=9\cdot\left[1+\frac{\frac{1}{2}}{1!}\cdot\frac{1}{9}+\frac{\frac{1}{2}\cdot\left(\frac{1}{2}-1\right)}{2!}\cdot\left(\frac{1}{9}\right)^2+\frac{\frac{1}{2}\cdot\left(\frac{1}{2}-1\right)\cdot\left(\frac{1}{2}-2\right)}{3!}\cdot\left(\frac{1}{9}\right)^3\right]=$$

$$=9\cdot\left[1+\frac{1}{2}\cdot\frac{1}{9}-\frac{1}{8}\cdot\frac{1}{81}+\frac{1}{16}\cdot\frac{1}{729}\right]=9\cdot\left[1+\frac{1}{18}-\frac{1}{648}+\frac{1}{11664}\right]\approx 9,48672.$$

Misol 935. $\sqrt{e}$ ni *0,00001* gacha aniqlikda hisoblang.

Yechilishi. Ma'lumki e^x uchun

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+\cdots$$

o'rinli. $\sqrt{e}=e^{\frac{1}{2}}\ \Rightarrow\ x=\frac{1}{2}$ ekanligini e'tiborga olib x ning bu qiymatini yuqoridagi qatorga qo'yamiz:

$$\sqrt{e}=1+\frac{\frac{1}{2}}{1!}+\frac{\left(\frac{1}{2}\right)^2}{2!}+\frac{\left(\frac{1}{2}\right)^3}{3!}+\cdots+\frac{\left(\frac{1}{2}\right)^n}{n!}+\cdots=$$
$$=1+\frac{1}{1!\cdot 2}+\frac{1}{2!\cdot 2^2}+\frac{1}{3!\cdot 2^3}+\cdots+\frac{1}{m!\cdot 2^m}+\cdots.$$

Taqribiy tenglik xatosining qiymati ***0,00001*** dan jshvasligi uchun n ning qiymatini topamiz. Buning uchun

$$R_n<\frac{x^n}{n!}\cdot\frac{x}{n+1-x}=\frac{\left(\frac{1}{2}\right)^n}{n!}\cdot\frac{\frac{1}{2}}{n+1-\frac{1}{2}}=\frac{1}{n!2^n}\cdot\frac{1}{2n+1}<0,00001$$

xatolikni n ni tanlash usuli bilan baholaymiz.

$$n=3\ \Rightarrow R_3<\frac{1}{3!\cdot 2^3}\cdot\frac{1}{2\cdot 3+1}=\frac{1}{336}\approx 0{,}002976.$$
$$n=5\ \Rightarrow R_5<\frac{1}{5!\cdot 2^5}\cdot\frac{1}{2\cdot 5+1}=\frac{1}{42240}\approx 0{,}00002367.$$
$$n=6\ \Rightarrow R_6<\frac{1}{6!\cdot 2^6}\cdot\frac{1}{2\cdot 6+1}=\frac{1}{599040}\approx 0{,}000001669.$$

Demak, **n=6** ni qabul qilamiz.

$$\sqrt{e}\cong 1+\frac{1}{1!\cdot 2}+\frac{1}{2!\cdot 2^2}+\frac{1}{3!\cdot 2^3}+\frac{1}{4!\cdot 2^4}+\frac{1}{5!\cdot 2^5}+\frac{1}{6!\cdot 2^6}=1+0{,}5+0{,}125+$$
$$+0{,}020833+0{,}002604+0{,}000260+0{,}000022=1{,}648719.$$

$\sqrt{e}\cong 1{,}648719$. Bu yerda har bir qo'shiluvchi 0,00001 aniqlikda hisoblandi. Bunday holda yig'indi topilganda yo'l qo'yiladigan xato 0,0001 dan oshmaydi.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi qatorlarning yaqinlashish sohasini toping:

936. $\sum_{n=1}^{\infty}\frac{1}{n^x}$; *Javobi :* $(1;+\infty)$. **937.** $\sum_{n=1}^{\infty}\frac{(x+1)^n}{(2n-1)!}$; *Javobi :* $(-\infty;+\infty)$.

938. $\sum_{n=1}^{\infty}(nx)^n$; *Javobi :* $x=0$. **939.** $\sum_{n=1}^{\infty}\frac{(n+1)^5 x^{2n}}{2n+1}$; *Javobi :* $(-1;1)$.

Quyidagi qatorlarning yig'indisini toping:

940. $\sum_{n=1}^{\infty}(n+1)\cdot x^n$; *J :* $\frac{1}{(x-1)^2}$, $(-1;1)$. **941.** $\sum_{n=1}^{\infty}\frac{x^n}{n}$; *J :* $-ln(1-x)$, $[-1;1)$.

942. $\sum_{n=1}^{\infty}\frac{x^n}{n^2}$ qatorning $x\in[-1;1]$ oraliqda tekis yaqinlashuvchi ekanligini Veyershtrass alomatidan foydalanib isbotlang.

Quyidagi funksiyalarni darajali qatorlarga yoying:

943. $f(x)=cos\,x$. *Javobi :* $cos\,x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots+(-1)^{n-1}\frac{x^{2n-2}}{2n-2}+\cdots$.

944. $f(x)=ln(x+1)$. *Javobi :* $ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots+(-1)^{n-1}\frac{x^n}{n}+\cdots$.

945. $f(x)=(1+x)^m$. *Javobi :* $(1+x)^m=1+\frac{m}{1!}x+\frac{m(m-1)}{2!}x^2+\cdots+\frac{m(m-1)(m-2)\cdots(m-n+1)}{n!}x^n+\cdots$.

946. $f(x)=\frac{1}{1-x}$. *Javobi :* $f(x)=1+x+x^2+\cdots+x^{n-1}+\cdots$.

Makloren qatoriga yoyish yordamida taqribiy qiymatini hisoblang:

947. $\ln 5$ *Javob* : $\ln 5\cong 1{,}609$ **948.** $e^{0.25}$ *Javobi* : $e^{0.25}\approx 1.28385$.

949. $2^{0,5}$ *javob* : $2^{0,5}\approx 1{,}4136$. **950.** $\sqrt{70}$ *Javob* : $\sqrt{70}\approx 8{,}36664$.

0,00001 gacha aniqlikda hisoblang:

951. $\frac{1}{\sqrt[5]{e}}$ *Javob* : $\frac{1}{\sqrt[5]{e}}\approx 0{,}818734$. **952.** $\ln 1{,}04$ *Javob* : $\ln 1{,}04\approx 0{,}039346$.

953. $\ln 2$ *Javob* : $\ln 2\approx 0{,}69314$. **954.** $cos 18^o$ *Javob* : $cos 18^o\approx 0{,}951058$.

33-MAVZU. BIRINCHI TARTIBLI DIFFERENSIAL TENGLAMA

$F(x,y,y^1)=0$ yoki $y^1 = f(x,y)$ ga birinchi tartibli differensial tenglama deyiladi. $M(x)dx+N(y)dy=0$ ga o'zgaruvchilari ajraladigan differensial tenglama deyiladi.

Misol 955. $2y^1 = y$ differensial tenglamani yeching.

Yechilishi. $y^1 = \frac{1}{2}\cdot y.$ y^1 hosila $\frac{dy}{dx}$ bilan almashtiriladi:

$\frac{dy}{dx}=\frac{1}{2}\cdot y.$ Bunda umumiy maxraj berilib, so'ngra tenglikning ikkala tomoni **y** ga bo'linadi: $\frac{dy}{y}=\frac{1}{2}\cdot dx.$ Tenglik integrallanadi: $\int\frac{dy}{y}=\int\frac{1}{2}\cdot dx.$ Integraldan **ln y** chiqsa, ayrim hollarda o'zgarmas son C o'rniga **ln C** qo'shiladi:

$$\ln y=\frac{1}{2}x+\ln C\Rightarrow \ln y-\ln C=\frac{x}{2}\Rightarrow \ln\frac{y}{C}=\frac{x}{2}\Rightarrow\frac{y}{C}=e^{\frac{x}{2}}\Rightarrow y=C\cdot e^{\frac{x}{2}}.$$

Demak, differensial tenglamaning yechimi funksiyadan iborat bo'lar ekan. Differensial tenglama birinchi tartibli bo'lganligi uchun uning yechimida bitta o'zgarmas son C qatnashadi.

$M_1(x)M_2(y)dx+N_1(x)N_2(y)dy=0$ ko'rinishdagi o'zgaruvchilari ajraladigan differensial tenglama. Bu tenglamaning o'zgaruvchilari $M_2(y)N_1(x)\neq 0$ ga bo'lish orqali ajraladi.

Misol 956. $(1+x^2)dy+ydx=0$ differensial tenglamaning $y|_{x=1}=1$ boshlang'ich shartni qanoatlantiruvchi yechimini toping.

Yechilishi: $\frac{dy}{y}+\frac{dx}{1+x^2}=0.\Rightarrow\int\frac{dy}{y}+\int\frac{dx}{1+x^2}=c.\Rightarrow \ln|y|+arctgx=c.$ Bundan boshlang'ich shart e'tiborga olingan holda C ning qiymati topilib o'rniga qo'yiladi:

$$\ln 1+arctg1=c\Rightarrow 0+\frac{\pi}{4}=c\Rightarrow c=\frac{\pi}{4}.$$

Demak, $\boldsymbol{ln|y|+arctgx=\frac{\pi}{4}\Rightarrow ln|y|=\frac{\pi}{4}-arctgx\Rightarrow y=e^{\frac{\pi}{4}-arctgx}}$

BIR JINSLI DIFFERENSIAL TENGLAMA

Tenglikning bir tomoni bir jinsli funksiyadan iborat differensial tenglamaga bir jinsli differensial tenglama deyiladi. Bunday tenglamada $y=u\cdot x$ almashtirish olish orqali o'zgaruvchilar ajratiladi.

Misol 957. $(x^2+2xy)dx+xydy=0$ tenglamani yeching.

Yechilishi. Tenglama $x\cdot y\cdot dx$ ga bo'linib quyidagi ko'rinishga keltiriladi: $\frac{dy}{dx}=-\frac{x^2+2xy}{x\cdot y}.$ Haqiqatan tenglikning o'ng tomoni bir jinsli funksiya:

$$\boldsymbol{\frac{(xt)^2+2\cdot x\cdot t\cdot y\cdot t}{xt\cdot yt}=\frac{x^2+2xy}{x\cdot y}}.$$

y=u$\cdot$$\boldsymbol{x}$; $\boldsymbol{y'=u'x+u}$. Bular o'rniga qo'yiladi:

$$u^1x+u=-\frac{x^2+2ux^2}{ux^2}\Rightarrow u^1x+u=-\frac{1+2u}{u}\Rightarrow u^1\cdot x=-\frac{1+2u}{u}-u\Rightarrow u^1x=-\frac{u^2+2u+1}{u}\Rightarrow$$

$$\Rightarrow u^1x=-\frac{(u+1)^2}{u}\Rightarrow\frac{du}{dx}\cdot x=-\frac{(u+1)^2}{u}\Rightarrow du\cdot x\cdot u=-(u+1)^2dx\Rightarrow\frac{u\cdot du}{(u+1)^2}=-\frac{dx}{x}\Rightarrow$$

$$\Rightarrow\int\frac{u\cdot du}{(u+1)^2}=\ln c-\int\frac{dx}{x}\Rightarrow\int\frac{(u+1)-1}{(u+1)^2}du=\ln c-\ln x\Rightarrow\int\frac{(u+1)du}{(u+1)^2}-\int\frac{du}{(u+1)^2}=\ln\frac{c}{x}\Rightarrow$$

$$\Rightarrow\int\frac{du}{u+1}-\int(u+1)^{-2}d(u+1)=\ln\frac{c}{x}\Rightarrow\int\frac{d(u+1)}{u+1}-\frac{(u+1)^{-2+1}}{-2+1}=\ln\frac{c}{x}\Rightarrow\ln|u+1|+\frac{1}{u+1}=\ln\frac{c}{x}\Rightarrow$$

$$\Rightarrow\frac{1}{u+1}=\ln\frac{c}{x}-\ln|u+1|\Rightarrow\frac{1}{u+1}=\ln\frac{c}{(x)u+1}.$$

Bunga $u=\frac{y}{x}$ qo'yilsa $\frac{x}{x+y}=\ln\frac{c}{x+y}$.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi differensial tenglamalarning umumiy yechimini toping:

958. $\frac{1}{2}y^1=y.$ Javobi: $y=c\cdot e^{2x}.$

959. $4y^1=y.$ Javobi: $y=c\cdot e^{\frac{x}{4}}.$

960. $y^1=3y.$ Javobi: $y=c\cdot e^{3x}.$

961. $y^1=\frac{1}{3}\cdot y.$ Javobi: $y=c\cdot e^{\frac{x}{3}}.$

962. $y^1=3x^2-2x+1.$ Javobi: $y=x^3-x^2+x+c.$

963. $\frac{dy}{dx}=y^2.$ Javobi: $y=-\frac{1}{x+c}.$

964. $\frac{dx}{2}-\frac{dy}{\cos x}=0.$ Javobi: $y=\frac{1}{2}\sin x+c.$

965. $y^1=x(y^2+1).$ Javobi: $y=tg(\frac{x^2}{2}+c).$

966. $y^2dy+xdx=0.$ Javobi: $y=\sqrt[3]{3(c-\frac{x^2}{2})}.$

967. $(1-x^2)\frac{dy}{dx}+xy=2x.$ Javobi: $y=2-c\sqrt{1-x^2}.$

Koshi masalasini yeching:

968. $(x-1)dx+ydy=0;\quad y(0)=1.$ Javobi: $x^2-2x+y^2=1.$

969. $\frac{dx}{3}+\frac{dy}{\cos x}=0;\quad y|_{x=0}=-1.$ Javobi: $y=\frac{1}{3}\sin x-1.$

970. $x\cdot y^1-y=x\cdot tg\frac{y}{x};\quad y|_{x=1}=\frac{\pi}{2}.$ Javobi: $y=x\cdot\arcsin x.$

971. $2(x+y)\cdot y^1+(3x+3y-1)=0;\quad y|_{x=0}=2.$ Javobi: $3x+2y-4+2\ln|x+y-1|=0.$

34-MAVZU. BIRINCHI TARTIBLI CHIZIQLI DIFFERENSIAL TENGLAMALAR

CHIZIQLI DIFFERENSIAL TENGLAMA

$$y^1+P(x)y=Q(x)$$

Tenglamani yechishda $y=u\cdot v$ almashtirish olinadi.

Tenglamaning umumiy yechimi $y=e^{-\int pdx}(c+\int Qe^{\int pdx}dx).$

Misol 972. $(x^2-x)y^1+y=x^2(2x-1)$ differensial tenglamaning umumiy yechimini toping.

Yechilishi. Tenglama $x^2-x\neq 0$ ga bo'linadi:

$$y^1+\frac{y}{x^2-x}=\frac{x^2(2x-1)}{x^2-x} \text{ yoki } y^1+\frac{y}{x(x-1)}=\frac{x(2x-1)}{x-1}\Rightarrow\begin{cases}P(x)=\dfrac{1}{x(x-1)};\\ Q(x)=\dfrac{x(2x-1)}{x-1}.\end{cases}$$

$\boldsymbol{y=u\cdot v},\quad \boldsymbol{y'=u'\cdot v+u\cdot v'}$ lar oxirgi tenglamaga qo'yiladi:

$$u^1\cdot v+u\cdot v^1+\frac{1}{x(x-1)}\cdot u\cdot v=\frac{x(2x-1)}{x-1}; \qquad u^1\cdot v+(v^1+\frac{v}{x(x-1)})u=\frac{x(2x-1)}{x-1}.$$

u ning oldidagi ko'paytuvchi nolga tenglanadi: $\begin{cases}\boldsymbol{v'+\dfrac{v}{x(x-1)}=0;}\\ \boldsymbol{u^1\cdot v=\dfrac{x(2x-1)}{x-1}.}\end{cases}$

Birinchi tenglamaning istalgan xususiy yechimi topiladi:

$$\boldsymbol{\frac{dv}{dx}=-\frac{v}{x(x-1)}\Rightarrow\frac{dv}{v}=-\frac{dx}{x(x-1)}\Rightarrow\int\frac{dv}{v}=-\int\frac{x-(x-1)}{x(x-1)}dx\Rightarrow\int\frac{dv}{v}=-\int\frac{dx}{x-1}+\int\frac{dx}{x}\Rightarrow}$$

$$\boldsymbol{\Rightarrow\ \ln v=-\ln|x-1|+\ln|x|\Rightarrow\ln v=\ln\frac{x}{x-1}\Rightarrow v=\frac{x}{x-1}.}$$

Bu xususiy yechim sistemaning ikkinchi tenglamasiga qo'yiladi:

$$u^1\cdot\frac{x}{x-1}=\frac{x(2x-1)}{x-1}\Rightarrow\frac{du}{dx}=2x-1\Rightarrow du=2xdx-dx\Rightarrow\int du=2\int xdx-\int dx+c\Rightarrow u=x^2-x+c.$$

Berilgan tenglamaning umumiy yechimi:

$$y=u\cdot v=(x^2-x+c)\cdot\frac{x}{x-1}\Rightarrow y=\frac{x(x^2-x+c)}{x-1}.$$

Misol 973. $(2x-y^2)y^1=2y$ tenglamaning $y\big|_{x=1}=1$ boshlang'ich shartni qanoatlantiruvchi xususiy yechimini toping.

Yechilishi. Tenglama **x** ga nisbatan chiziqli. Haqiqatan

$$(2x-y^2)\cdot\frac{1}{x^1}=2y\Rightarrow 2x-y^2=2y\cdot x^1\Rightarrow x^1-\frac{x}{y}=-\frac{y}{2}\Rightarrow\begin{cases}P(y)=-\dfrac{1}{y};\\ Q(y)=-\dfrac{y}{2}.\end{cases}$$

Almashtirish: $x=u\cdot v,\qquad x^1=u^1\cdot v+u\cdot v^1.$

$$u^1v+uv^1-\frac{1}{y}\cdot u\cdot v=-\frac{y}{2}; \qquad u^1v+\left(v^1-\frac{v}{y}\right)\cdot u=-\frac{y}{2}. \qquad \begin{cases}v^1-\dfrac{v}{y}-=0;\\ u^1v=-\dfrac{y}{2}.\end{cases}$$

Sistemaning birinchi tenglamasi yechiladi:

$$\frac{dv}{dy}=\frac{v}{y}\Rightarrow\frac{dv}{v}=\frac{dy}{y}\Rightarrow v=y.$$

Bu natija sistemaning ikkinchi tenglamasiga qo'yiladi:

$$\frac{dv}{dy}\cdot y=-\frac{y}{2}\Rightarrow du=-\frac{1}{2}dy\Rightarrow u=-\frac{1}{2}y+c.$$

Berilgan tenglamaning umumiy yechimi:

$$x=u\cdot v=y\cdot(c-\frac{y}{2}).$$

$y\big|_{x=1}=1$ boshlang'ich shartdan $1=1\left(c-\frac{1}{2}\right)\Rightarrow c=\frac{1}{2}.$ Demak, $x=\frac{1}{2}y(3-y).$

BERNULLI TENGLAMASI

$$y^1+P(x)y=y^{\alpha}Q(x).$$

$\alpha-const.\quad \alpha\neq 0,\quad \alpha\neq 1.$

Tenglamani yechish uchun yangi $z=y^{1-\alpha}$ funksiya kiritilib chiziqli tenglama hosil qilinadi:

$z^1+(1-\alpha)zP(x)=(1-\alpha)Q(x).$ Bernulli tenglamasini **z** funksiya kiritmasdan $y=u\cdot v$ almashtirish orqali yechsa ham bo'ladi.

Misol 974. $y^1+\frac{y}{x}=y^2\cdot\frac{\ln x}{x}$ differensial tenglamaning umumiy yechimini toping.

Yechilishi. Berilgan tenglama Bernulli tenglamasi bo'lganligidan:

$\alpha=2,\quad y=u\cdot v,\quad y^1=u^1\cdot v+u\cdot v^1$ o'rniga qo'yish bajariladi:

$$u^1v+u\cdot v^1+\frac{1}{x}\cdot u\cdot v=(u\cdot v)^2\cdot\frac{\ln x}{x};\qquad u^1v+(v^1+\frac{v}{x})u=u^2v^2\cdot\frac{\ln x}{x};\qquad \begin{cases} v^1+\frac{v}{x}=0;\\ u^1\cdot v=u^2v^2\cdot\frac{\ln x}{x}.\end{cases}$$

Birinchi tenglamadan:

$\frac{dv}{dx}=-\frac{v}{x}\Rightarrow\frac{dv}{v}=-\frac{dx}{x}\Rightarrow \ln v=-\ln x\Rightarrow \ln v=\ln x^{-1}\Rightarrow v=\frac{1}{x}.$ Bu sistemaning ikkinchi tenglamasiga qo'yiladi:

$$\frac{du}{dx}\cdot\frac{1}{x}=u^2\cdot\frac{1}{x^2}\cdot\frac{\ln x}{x}\Rightarrow\frac{du}{u^2}=\frac{\ln x}{x^2}dx\Rightarrow-\frac{1}{u}=\int\frac{\ln x}{x^2dx}=\left|\begin{matrix} S=\ln x\Rightarrow ds=\frac{dx}{x};\\ dt=\frac{dx}{x^2}\Rightarrow t=-\frac{1}{x}\end{matrix}\right|=-\frac{\ln x}{x}+\int\frac{dx}{x^2}=$$

$$=-\frac{\ln x}{x}-\frac{1}{x}-c\Rightarrow-\frac{1}{u}=-\frac{\ln x}{x}-\frac{1}{x}-c\Rightarrow u=\frac{x}{cx+1+\ln x}.$$

Berilgan Bernulli tenglamasining umumiy yechimi:

$$y=u\cdot v=\frac{1}{x}\cdot\frac{x}{cx+1+\ln x}\Rightarrow y=\frac{1}{cx+1+\ln x}.$$

TO'LIQ DIFFERENSIALLI DIFFERENSIAL TENGLAMA

$M(x;y)dx+N(x;y)dy=0$ tenglamaning chap qismi $u(x;y)$ funksiyaning to'liq differensiali, ya'ni $du=M(x;y)dx+N(x;y)dy$ dan iborat bo'lsa, bunday differensial tenglamani to'liq differensialli tenglama deyiladi.

Berilgan tenglama to'liq differensial tenglama bo'lishi uchun $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ shart bajarilishi kerak.

To'liq differensialli tenglamaning ta'rifidan $du=0\Rightarrow u=c.$ Bundan $u(x;y)=c$ ekanligi kelib chiqadi.

$\boldsymbol{u(x;y)}$ ni topish uchun **y** o'zgarmas deb olinadi, u holda $dy=0$ ekanligidan $du = M(x;y)dx$ bo'ladi. Bu tenglik **x** bo'yicha integrallanadi: $u=\int M(x;y)dx+\varphi(y).$

Oxirgi tenglik **y** bo'yicha differensiallanib, natija $N(x;y)$ ga tenglanadi, chunki $\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{y}} = \boldsymbol{N(x;y)}.$

$$\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{y}} = \int \frac{\partial \boldsymbol{M}}{\partial \boldsymbol{y}} \boldsymbol{dx} + \varphi^1\boldsymbol{(y)} = \boldsymbol{N(x;y)} \quad \text{yoki} \quad \varphi^1\boldsymbol{(y)} = \boldsymbol{N(x;y)} - \int \frac{\partial \boldsymbol{M}}{\partial \boldsymbol{y}} \boldsymbol{dx}.$$

Bu ifoda **y** bo'yicha integrallansa $\varphi\boldsymbol{(y)} = \int \left[\boldsymbol{N(x;y)} - \int \frac{\partial \boldsymbol{M}}{\partial \boldsymbol{y}} \boldsymbol{dx} \right] \boldsymbol{dy} + \boldsymbol{c}.$

Demak, $\boldsymbol{u(x;y)} = \int \boldsymbol{M(x;y)dx} + \int \left[\boldsymbol{N(x;y)} - \int \frac{\partial \boldsymbol{M}}{\partial \boldsymbol{y}} \boldsymbol{dx} \right] \boldsymbol{dy} + \boldsymbol{c}.$

Bu ifoda ixtiyoriy o'zgarmasga tenglanib, differensial tenglamaning umumiy yechimi hosil qilinadi.

Misol 975. $\boldsymbol{(}3x^2+6\boldsymbol{x}\boldsymbol{y}^2\boldsymbol{)dx} + \boldsymbol{(}6\boldsymbol{x}^2\boldsymbol{y}+4\boldsymbol{y}^3\boldsymbol{)dy}=0$ tenglamaning umumiy yechimini toping.

Yechilishi. $\boldsymbol{M(x;y)} = 3x^2 + 6\boldsymbol{xy}^2\boldsymbol{;}$ $\qquad \boldsymbol{N(x;y)} = 6\boldsymbol{x}^2\boldsymbol{y} + 4\boldsymbol{y}^3\boldsymbol{.}$

$\frac{\partial \boldsymbol{M}}{\partial \boldsymbol{y}} = 12\boldsymbol{xy;}$ $\quad \frac{\partial \boldsymbol{N}}{\partial \boldsymbol{x}} = 12\boldsymbol{xy,}$ $\quad$ ya'ni $\quad \frac{\partial \boldsymbol{M}}{\partial \boldsymbol{y}} = \frac{\partial \boldsymbol{N}}{\partial \boldsymbol{x}}$

$\frac{\partial u}{\partial x} = M(x;y) \Rightarrow \frac{\partial u}{\partial x} = 3x^2 + 6xy^2.$

Bu tenglik **x** bo'yicha integrallanadi: $u = x^3 + 3x^2y^2 + \varphi(y).$ Bundan $\frac{\partial u}{\partial y} = 6x^2y + \varphi^1(y) \Rightarrow \varphi^1(y) = \frac{\partial u}{\partial y} - 6x^2y.$ $\quad \frac{\partial u}{\partial y} = N(x;y)$ ekanligi e'tiborga olinsa $\varphi^1(y) = 6x^2y + 4y^3 - 6x^2y = 4y^3.$ Bundan $\varphi(y) = y^4 + c.$

Demak, $u = x^3 + 3x^2y^2 + y^4 = c$ yoki $x^3 + 3x^2y^2 + y^4 = c.$

INTEGRALLOVCHI KO'PAYTUVCHI

$M(x;y)dx + N(x;y)dy = 0$ tenglamaning chap tomoni biror funksiyaning to'liq differensiali emas, ya'ni $\frac{\partial \boldsymbol{M}}{\partial \boldsymbol{y}} \neq \frac{\partial \boldsymbol{N}}{\partial \boldsymbol{x}}$ Bunday holda shunday $\mu\boldsymbol{(x;y)}$ funksiya tanlanib, berilgan differensial tenglama bu funksiyaga ko'paytirilsa, tenglamaning chap tomoni biror funksiyaning to'liq differensiali bo'lib qoladi. $\mu\boldsymbol{(x;y)}$ funksiyani integrallovchi ko'paytuvchi deyiladi.

$\frac{\boldsymbol{d\,ln}\mu}{\boldsymbol{dy}} = \frac{\frac{\partial \boldsymbol{N}}{\partial \boldsymbol{x}} - \frac{\partial \boldsymbol{M}}{\partial \boldsymbol{y}}}{\boldsymbol{M}}$ oddiy differensial tenglama integrallovchi ko'paytuvchining faqat **y** ga bog'liqligini, $\frac{\boldsymbol{d\,ln\,M}}{\boldsymbol{dx}} = \frac{\frac{\partial \boldsymbol{N}}{\partial \boldsymbol{x}} - \frac{\partial \boldsymbol{M}}{\partial \boldsymbol{y}}}{\boldsymbol{N}}$ esa faqat **x** ga bog'liqligini aniqlaydi.

Misol 976. $(y+xy^2)dx - xdy = 0$ differensial tenglamaning umumiy yechimini toping.

Yechilishi. $M = y + xy^2, \quad N = -x, \quad \frac{\partial M}{\partial y} = 1 + 2xy, \quad \frac{\partial N}{\partial x} = -1. \ \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$

Bu tenglamaning chap tomoni biror funksiyaning to'liq differensiali emasligini bildiradi. Endi tenglamaning faqat **y** ga bog'liq bo'lgan integrallovchi ko'paytuvchisi bormi degan masala qo'yiladi:

$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{-1-1-2xy}{y+xy^2} = \frac{-2(1+xy)}{y(1+xy)} = -\frac{2}{y}.$ Olingan natija **y** ga bog'liq integrallovchi ko'paytuvchining borligini bildiradi. Bu ko'paytuvchi topiladi:

$$\frac{d\, ln\mu}{dy} = -\frac{2}{y} \Rightarrow ln\mu = -2\, ln\, y \Rightarrow \mu = y^{-2} \Rightarrow \mu = \frac{1}{y^2}.$$

Berilgan tenglama hadlari $\frac{1}{y^2}$ ga ko'paytiriladi: $(\frac{1}{y} + x)dx - \frac{x}{y^2}dy = 0.$

Bu to'liq differensialli tenglama:

$$M = \frac{1}{y} + x;\ N = -\frac{x}{y^2}, \quad \frac{\partial M}{\partial y} = -\frac{1}{y^2}, \quad \frac{\partial N}{\partial x} = -\frac{1}{y^2} \Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = -\frac{1}{y^2}.$$

Hosil qilingan to'liq differensialli tenglama bundan oldingi mavzu bo'yicha yechiladi:

$$\frac{x}{y} + \frac{x^2}{2} + c = 0 \quad \text{yoki} \quad y = -\frac{2x}{x^2 + 2c}.$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi differensial tenglamalarning umumiy yechimlarini toping:

977. $y^1 - \frac{1-2x}{x^2} \cdot y = 1.$ Javobi: $y = cx^2 e^{\frac{1}{x}} + x^2.$

978. $(1+x^2)y^1 - 2xy = (1+x^2)^2$ Javobi: $y = (x+c)(1+x^2).$

979. $yy^1 = 2y - x.$ Javobi: $y - x = c \cdot e^{\frac{x}{y-x}}.$

980. $x^2 + y^2 - 2xyy^1 = 0.$ Javobi: $x^2 - y^2 = c \cdot x.$

981. $y^1 - \frac{3y}{x} = x.$ Javobi: $y = cx^3 - x^2.$

982. $\left(4 - \frac{y^2}{x^2}\right)dx + \frac{2y}{x}dy = 0.$ Javobi: $4x^2 + y^2 = c \cdot x..$

983. $3x^2e^y dx + (x^3e^y - 1)dy = 0.$ Javobi: $x^3e^y - y = c.$

Quyidagi differensial tenglamalarning integrallovchi ko'paytuvchilarini toping va tenglamalarni yeching:

984. $(x^2 - y)dx + xdy = 0.$ Javobi: $\mu = \frac{1}{x^2}; \quad x + \frac{y}{x} = c.$

985. $y^2dx + (yx-1)dy = 0.$ Javobi: $\mu = \frac{1}{y}; \quad xy - ln\, y = 0$

986. $(x^2 - 3y^2)dx + 2xydy = 0.$ Javobi: $\mu = \frac{1}{x^4}; y^2 = cx^3 + x^{2.}$

987. $2xtgydx + (x^2 - 2\cdot \sin y)dy - 0.$ Javobi: $ln\mu = ln\cos y; \quad x^2 \sin y + \frac{1}{2}\cos 2y = c.$

Quyidagi differensial tenglamalarning chap tomonlari to'liq differensialdan iborat ekanligini tekshiring va tenglamalarni yeching:

988. $(3x^2+2y)dx+(2x-3)dy=0.$ Javobi: $x^3+2xy-3y=c.$

989. $(3x^2y+4xy^2)dx+(x^3-4x^2y+12y^3)dy=0.$ Javobi: $x^3y-2x^2y^2+3y^4=c.$

Koshi masalasini yeching:

990. $(y^2+2y+x^2)y^1+2x=0;$ $y|_{x=1}=0.$ Javobi: $x^2-y\ln\dfrac{4e}{y}.$

991. $x+y\cdot e^x+(y+e^x)y^1=0;$ $y|_{x=0}=4.$ Javobi: $x^2+y^2+2ye^x=24.$

992. $y^1=2y-x+e^x;$ $y|_{x=0}=-1.$ Javobi: $y=\frac{1}{2}x-e^x+\frac{1}{4}(1-e^{2x}).$

35-MAVZU. YUQORI TARTIBLI DIFFERENSIAL TENGLAMALAR

$y^{(n)}=f(x)$ ko'rinishdagi tenglama ketma-ket n marta integrallash orqali yechiladi va umumiy yechimda n ta o'zgarmas c soni qatnashadi.

Misol 993. $y''=x\cdot e^{-x}$ differensial tenglamaning $y\big|_{x=0}=1,$ *va* $y'\big|_{x=0}=0$ boshlang'ich shartlarni qanoatlantiruvchi xususiy yechimini toping.

Yechilishi. $y^1=\dfrac{dy}{dx}$ *va* $y^{11}=\dfrac{d^2y}{dx^2}=\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right).$ $\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)=xe^{-x}\Rightarrow$

$$d\left(\frac{dy}{dx}\right)=xe^{-x}dx\Rightarrow\int d\left(\frac{dy}{dx}\right)=\int xe^{-x}dx\Rightarrow\frac{dy}{dx}=\int xe^{-x}dx=\begin{vmatrix}u=x\Rightarrow du=dx;\\ dv=e^{-x}dx\Rightarrow v=-e^{-x}\end{vmatrix}=$$

$$=-x\cdot e^{-x}-\int(-e^{-x})dx=-xe^{-x}-e^{-x}+c_1\Rightarrow\frac{dy}{dx}=c_1-xe^{-x}-e^{-x}\Rightarrow$$

$$dy=(c_1-xe^{-x}-e^{-x})dx=c_1dx-xe^{-x}dx-e^{-x}dx\Rightarrow$$

$$\Rightarrow\int dy=\int c_1dx-\int xe^{-x}dx-\int e^{-x}dx\Rightarrow$$

$$\Rightarrow y=c_1x-(-xe^{-x}-e^{-x})+e^{-x}+c_2=xe^{-x}+2e^{-x}+c_1x+c_2\Rightarrow$$

$$\Rightarrow y=xe^{-x}+2e^{-x}+c_1x+c_2;\quad y^1=c_1-xe^{-x}-e^{-x}.$$

Aniqlangan y va y' larga boshlang'ich shartlar qo'yilib c_1 va c_2 lar topiladi:

$$\begin{cases}1=2+c_2\\ 0=c_1-1\end{cases}\Rightarrow\begin{cases}c_2=-1;\\ c_1=1.\end{cases}$$

U holda xususiy yechim $y=xe^{-x}+2e^{-x}+x-1.$

Misol 994. $y^{(IV)}=\dfrac{8}{(x-3)^5}$ differensial tenglamaning umumiy yechimini toping.

Yechilishi.

$$y''' = \int y^{IV} dx = \int \frac{8dx}{(x-3)^5} = 8\int \frac{d(x-3)}{(x-3)^5} = 8\cdot \int (x-3)^{-5} d(x-3) = 8\cdot \frac{(x-3)^{-5+1}}{-5+1} + c_1 =$$

$$= -\frac{2}{(x-3)^4} + c_1 \Rightarrow y''' = -\frac{2}{(x-3)^4} + c_1;$$

$$y'' = \int y''' dx = -2\int \frac{dx}{\left(x-3\right)^4} + c_1\int dx = -2\int \frac{d\left(x-3\right)}{\left(x-3\right)^4} + c_1x + c_2 = -2\int \left(x-3\right)^{-4} d\left(x-3\right) + c_1x + c_2 =$$

$$= -2\cdot \frac{\left(x-3\right)^{-4+1}}{-4+1} + c_1x + c_2 = \frac{2}{3\left(x-3\right)^3} + c_1x + c_2 \Rightarrow y'' = \frac{2}{3\left(x-3\right)^3} + c_1x + c_2.$$

$$y' = \int y'' dx = \int \frac{2dx}{3(x-3)^3} + \int c_1 x dx + \int c_2 dx = \frac{2}{3}\int (x-3)^{-3} d(x-3) + c_1\int x dx + c_2 x + c_3 =$$

$$= \frac{2}{3}\cdot \frac{(x-3)^{-3+1}}{-3+1} + c_1\cdot \frac{x^2}{2} + c_2x + c_3 = -\frac{1}{3(x-3)^2} + \frac{1}{2}c_1x^2 + c_2x + c_3;$$

$$\frac{dy}{dx} = -\frac{1}{3(x-3)^2} + \frac{1}{2}c_1x^2 + c_2x + c_3.$$

$$dy = -\frac{dx}{3(x-3)^2} + \frac{1}{2}c_1x^2dx + c_2xdx + c_3dx;\ \int dy = -\frac{1}{3}\int \frac{d(x-3)}{(x-3)^2} + \frac{1}{2}c_1\int x^2dx + c_2\int xdx + c_3\int dx + c_4;$$

$$y = -\frac{1}{3}\int (x-3)^{-2} d(x-3) + \frac{1}{2}c_1\cdot \frac{x^3}{3} + c_2\cdot \frac{x^2}{2} + c_3x + c_4 = -\frac{1}{3}\cdot \frac{(x-3)^{-2+1}}{-2+1} + \frac{1}{6}c_1x^3 +$$

$$+\frac{1}{2}c_2x^2 + c_3x + c_4 = \frac{1}{3\left(x-3\right)} + \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x + c_4;$$

$$y = \frac{1}{3\left(x-3\right)} + \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x^2 + c_3x + c_4.$$

Differensial tenlamada *y* qatnashmasa, *y*^{1} = *z* almashtirish olish orqali tenglamaning tartibi bittaga pasaytiriladi.

Misol 995.

$xy'' = y' \ln \frac{y'}{x}$ differensial tenglamaning

$$y\Big|_{x=1} = e, \quad va \quad y'\Big|_{x=1} = e^2$$

boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.

Yechilishi. $y' = z \Rightarrow xz' = z\ln\frac{z}{x}.$

Bu tenglamani yechish uchun $z = u\cdot x \Rightarrow z' = u + u'\cdot x$ almashtirish olinadi.

U holda

$$x\left(u + u'\cdot x\right) = u\cdot x\cdot \ln\frac{u\cdot x}{x};\ ux + u'x^2 + u\cdot x\ln u;\ u'\cdot x^2 = u\cdot x\left(\ln u - 1\right);\ u'\cdot x = u\left(\ln u - 1\right);$$

$$\frac{du}{dx}\cdot x = u\left(\ln u - 1\right);\quad \frac{du}{u\cdot\left(\ln u - 1\right)} = \frac{dx}{x};$$

$$\int \frac{du}{u\left(\ln u - 1\right)} = \int \frac{dx}{x};\ \int \frac{d\left(\ln u - 1\right)}{\ln u - 1} = \ln x + \ln c_1;\quad \ln\left|\ln u - 1\right| = \ln x\cdot c_1;$$

$$\ln u - 1 = c_1\cdot x;\quad \ln u = c_1x + 1;\quad u = e^{c_1x+1};$$

$z = u \cdot x \Rightarrow u = \dfrac{z}{x}$ bo'lgani uchun $z = x \cdot e^{c_1 x+1}$. $\quad z = y'$ bo'lganligidan

$$\frac{dy}{dx} = xe^{c_1x+1} \Rightarrow dy = xe^{c_1x+1}dx \Rightarrow \ y = \int xe^{c_1x+1}dx = \frac{1}{c_1}\int x \cdot d\left(e^{c_1x+1}\right) =$$

$$= \begin{vmatrix} u = x \Rightarrow du = dx; \\ dv = d\left(e^{c_1x+1}\right) \Rightarrow \\ \Rightarrow v = e^{c_1x+1} \end{vmatrix} = \frac{1}{c_1}\left[x \cdot e^{c_1x+1} - \int e^{c_1x+1}dx\right] =$$

$$= \frac{1}{c_1}\left[xe^{c_1x+1} - \frac{1}{c_1}e^{c_1x+1}\right] = \frac{1}{c_1} \cdot e^{c_1x+1}\left(x - \frac{1}{c_1}\right) = \frac{c_1x-1}{c_1^2} \cdot e^{c_1x+1} + c_2; \qquad y = \frac{c_1x-1}{c_1^2} \cdot e^{c_1x+1} + c_2.$$

Bu umumiy yechimga va yuqorida topilgan $y' = x \cdot e^{c_1x+1}$ ga boshlang'ich shart ma'lumotlarini qo'yib $c_1$va c_2 lar topiladi.

$$e^2 = 1 \cdot e^{C_1 1+1} \Rightarrow e^2 = e^{C_1+1} \Rightarrow 2 = C_1 + 1 \Rightarrow C_1 = 1;$$

$$e = \frac{1 \cdot 1 - 1}{1^2} \cdot e^{1 \cdot 1+1} + C_2 \Rightarrow C_2 = e.$$

U holda xususiy yechim $y = (x-1)e^{x+1} + e.$

Agar differensial tenglamada x qatnashmasa, y' hosila $P(y)$ funksiya bilan almashtirilib, tenglamaning tartibi bittaga pasaytiriladi.

Misol 996. $y^{III} - \dfrac{(y^{II})^2}{y^I} = 6(y^I)^2 \cdot y$ tenglamaning $y(2) = 0, \quad y^I(2) = 1,$

$y^{II}(2) = 0$ boshlang'ich shartlarni qanoatlantiruvchi xususiy yechimi topilsin.

Yechilishi. $P(y) = y^I; \quad y^{II} = [P(y)]^I = P^I(y) \cdot y^I = P(y) \cdot P^I(y);$

$$y^{III} = [P(y) \cdot P^I(y)]^I = [P(y)]^I \cdot P^I(y) + P(y) \cdot [P^I(y)]^I = P^I(y) \cdot y^I \cdot P^I(y) + P(y) \cdot P^{II}(y) \cdot y^I =$$

$$= P^{I^2}(y) \cdot P + P^{II}(y) \cdot P^2(y).$$

Yoki $y^I = P, \ y^{II} = P \cdot P^I; \qquad y^{III} = P^2P^{II} + P \cdot P^{I^2}$

Bular berilgan tenglamaga qo'yiladi:

$$P^2P^{II} + P \cdot P^{I^2} - \frac{(P \cdot P^I)^2}{P} = 6P^2 \cdot y; \ \ P^2P^{II} + PP^{I^2} - P \cdot P^{I^2} = 6P^2 \cdot y;$$

$$P^{II} = 6y \Rightarrow \frac{d^2P}{dy^2} = 6y \Rightarrow \frac{dP}{dy} = 6\int ydy = 6 \cdot \frac{y^2}{2} + c_1 = 3y^2 + c_1 \Rightarrow dP = (3y^2 + c_1)dy = 3y^2dy + c_1dy \Rightarrow$$

$$\Rightarrow P = 3\int y^2dy + c_1\int dy = y^3 + c_1y + c_2 \Rightarrow P = y^I = y^3 + c_1y + c_2.$$

Boshlang'ich shartlardan quyidagilar hosil qilinadi:

$$y^I(x) = P(y) \Rightarrow y^I(2) = P(0) = 1; \ \ y^{II}(x) = P(y) \cdot \frac{dP(y)}{dy} \Rightarrow y^{II}(2) = P(0) \cdot \frac{dP(0)}{dy} = 0.$$

Bu ma'lumotlar $y^I = y^3 + c_1y + c_2$ va $y^{II} = 3y^2 + c_1$ larga qo'yilib C_1 va C_2 lar topiladi:

$$\begin{cases} 1 = 0^3 + c_1 \cdot 0 + c_2 \\ 0 = 3 \cdot 0^2 + c_1 \end{cases} \Rightarrow \begin{cases} c_1 = 0; \\ c_2 = 1. \end{cases}$$

U holda

$$y^I=y^3+1\Rightarrow\frac{dy}{dx}=y^3+1\Rightarrow\frac{dy}{y^3+1}=dx\Rightarrow\int\frac{dy}{y^3+1}=\int dx\Rightarrow\frac{1}{\sqrt{3}}arctg\frac{2y-1}{\sqrt{3}}+$$
$$+\frac{1}{3}\ln\frac{y+1}{\sqrt{y^2-y+1}}x+c_3.$$

$y(2)=0$ shartdan

$$\frac{1}{\sqrt{3}}arctg\frac{2\cdot0-1}{\sqrt{3}}+\frac{1}{3}\ln\frac{0+1}{\sqrt{0^2+0+1}}=2+c_3\Rightarrow-\frac{1}{\sqrt{3}}arctg\frac{1}{\sqrt{3}}+\frac{1}{3}\ln1=2+c_3\Rightarrow$$
$$\Rightarrow-\frac{1}{\sqrt{3}}\cdot\frac{\pi}{6}+0=2+c_3\Rightarrow c_3=-2-\frac{\pi}{6\sqrt{3}}.$$

Demak, $x=\frac{1}{\sqrt{3}}\cdot arctg\frac{2y-1}{\sqrt{3}}+\frac{1}{3}\cdot\ln\frac{y+1}{\sqrt{y^2-y+1}}+2+\frac{\pi}{6\sqrt{3}}.$

Misol 997. $y^{II}=\frac{1+y^{I^2}}{y}$ differensial tenglamani yeching.

Yechilishi.

Tenglamada x qatnashmaganligi uchun $y^I=P(y)$ almashtirish olinadi. U holda $y^{II}=P^I\cdot P$. Bular berilgan tenglamaga qo’yiladi

$$\boldsymbol{P^I P=\frac{1+P^2}{y}\Rightarrow P\cdot\frac{dP}{dy}=\frac{1+P^2}{y}\Rightarrow\frac{dy}{y}=\frac{PdP}{1+P^2}\Rightarrow\int\frac{dy}{y}=\int\frac{PdP}{1+P^2}\Rightarrow \ln y=\frac{1}{2}\int\frac{d(1+P^2)}{1+P^2}\Rightarrow}$$
$$\boldsymbol{\Rightarrow\ln y=\frac{1}{2}\ln(1+P^2)\Rightarrow\frac{1}{2}\ln(1+P^2)=\ln y+\ln c_1\Rightarrow\ln\sqrt{1+P^2}=\ln c_1\cdot y\Rightarrow\sqrt{1+P^2}=c_1y\Rightarrow}$$
$$\boldsymbol{\Rightarrow1+P^2=c_1^2y^2\Rightarrow P^2=c_1^2y^2-1\Rightarrow P=\pm\sqrt{c_1^2y^2-1}\Rightarrow y^I=\pm\sqrt{c_1^2y^2-1}\Rightarrow}$$
$$\boldsymbol{\Rightarrow\frac{dy}{dx}=\pm\sqrt{c_1^2y^2-1}\Rightarrow dy=\pm\sqrt{c_1^2y^2-1}\cdot dx\Rightarrow\frac{dy}{\sqrt{c_1^2y^2-1}}=\pm dx\Rightarrow\frac{1}{c_1}\ln\left|c_1y+\sqrt{c_1^2y^2-1}\right|=\pm(x+c_2).}$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi differensial tenglamalarni ko’rsatilgan boshlang’ich shartlarda tartibini pasaytirish orqali yeching:

998. $y^{III}=\frac{6}{x^3};$ $x=1, y=2, y^I=1, y^{II}=1.$ Javobi: $y=3\ln x+2x^2-6x+6.$

999. $y^{II}=4\cos2x;$ $x=0, y=0, y^I=0.$ Javobi: $y=1-\cos2x.$

Quyidagi y qatnashmagan differensial tenglamalarni tartibini pasaytirish orqali yeching:

1000. $x^3y^{II}+x^2y^I=1.$ Javobi: $y=\frac{1}{x}+c_1\ln x+c_2.$

1001. $y^{II}+y^Itgx=\sin2x.$ Javobi: $y=c_1\sin x-x-\frac{1}{2}\sin2x+c_2.$

1002. $y^{II}\cdot x\ln x=y^I.$ Javobi: $y=c_1x(\ln x-1)+c_2.$

Quyidagi x qatnashmagan differensial tenglamalarni tartibini pasaytirish orqali yeching:

1003. $yy^{II}+(y^I)^2=0.$ Javobi: $y^2=c_1x+c_2.$

1004. $y^{II}+2y(y^I)^3=0.$ Javobi: $y^3+c_1y+c_2=3x.$

1005. $y^{II}\cdot tgy=2(y^{I})^2.$ Javobi: $ctgy=c_2-c_1x.$

36-MAVZU. O'ZGARMAS KOEFFITSIYENTLI BIR JINSLI CHIZIQLI DIFFERENSIAL TENGLAMALAR

BIR JINSLI DIFFERENSIAL TENGLAMALAR

$y''+py'+qy=0$ differensial tenglamaning $k^2+pk+q=0$ xarakteristik tenglamasi, $y=c_1e^{k_1x}+c_2e^{k_2x}$ esa uning umumiy yechimi.

Misol 1006. $y''+3y'-4y=0$ **tenglamaning umumiy yechimini toping.**

Yechilishi. $p=3,\quad q=-4,\quad k^2+3k-4=0.$

$$k_{1,2}=-\frac{3}{2}\pm\sqrt{\left(\frac{3}{2}\right)^2-(-4)}=-\frac{3}{2}\pm\sqrt{\frac{9}{4}+4}=-\frac{3}{2}\pm\frac{5}{5}\Rightarrow\begin{cases}k_1=-4,\\ k_2=1.\end{cases}\Rightarrow\ y=c_1e^{-4x}+c_2e^{x}.$$

Misol 1007. $y''+6y'=0$ differensial tenglamaning umumiy yechimini toping.

Yechilishi.

$$p=6,\quad q=0,\quad k^2+6k=0\Rightarrow k(k+6)=0\quad\begin{cases}k_1=0,\\ k_2=-6.\end{cases}\Rightarrow y=c_1e^{0\cdot x}+c_2e^{-6x}=c_1+c_2e^{-6x}.$$

Xarakteristik tenglama uchun *D=0* bo'lsa, differensial tenglamaning umumiy yechimi $y=e^{kx}(c_1+c_2x)$ ko'rinishda bo'ladi.

Misol 1008. $y''+4y'+4y=0$ differensial tenglamaning umumiy yechimini toping.

Yechilishi. $p=4,\quad q=4,\quad k^2+4k+4=0\Rightarrow k_{1,2}=-2\pm\sqrt{4-4}=-2\Rightarrow k_1=k_2=-2.$

$y=e^{-2x}(c_1+c_2x).$

Xarakteristik tenglama uchun $D<0$ bo'lsa. Differensial tenglamaning umumiy yechimi $y=e^{\alpha x}(c_1cos\beta x+c_2\,sin\beta x)$ bo'ladi. Bunda $\alpha=-\frac{p}{2},\quad \beta=\sqrt{q-\frac{p^2}{4}}.$

Misol 1009. $y''-4y'+13y=0$ differensial tenglamaning umumiy yechimini toping.

Yechilishi. $p=-4,\quad q=13,\quad k^2-4k+13=0\quad\Rightarrow$

$$\Rightarrow k_{1,2}=2\pm\sqrt{4-13}=2\pm\sqrt{-9}=2\pm3\sqrt{-1}=2\pm3i\Rightarrow\begin{cases}k_1=2-3i,\\ k_2=2+3i.\end{cases}\quad\alpha=-\frac{p}{2}=2,\ \beta=\sqrt{q-\frac{p^2}{4}}=3.$$

U holda $y=e^{2x}(c_1cos3x+c_2\,sin3x).$

Misol 1010. $y^{IV}-13y''+36y=0$ differensial tenglamaning umumiy yechimini toping.

Yechilishi. $k^4-13k^2+36=0\Rightarrow k_{1,2}=\pm3,\ k_{3,4}=\pm2.$ Bulardan quyidagi xususiy yechimlar hosil bo'ladi:

$$y_1=e^{3x},\qquad y_2=e^{-3x},\qquad y_3=e^{2x},\qquad y_4=e^{-2x}.$$

Umumiy yechim: $y=c_1e^{3x}+c_2e^{-3x}+c_3e^{2x}+c_4e^{-2x}.$

Misol 1011. $y''-y'-2y=0$ tenglamaning $y|_{x=0}=0,\ y'|_{x=0}=3$ boshlang'ich shartni qanoatlantiruvchi yechimini toping.

Yechilishi. $p=-1,\quad q=-2,\ k^2-k-2=0\ \Rightarrow\ k_1=2,\ k_2=-1\quad\Rightarrow\ y=c_1e^{2x}+c_2e^{-x}.$

Buning hosilasi: $y'=2c_1e^{2x}-c_2e^{-x}.$

U holda $\begin{cases} 0 = c_1 e^{2\cdot 0} + c_2 e^{-0}, \\ 3 = 2\cdot c_1 e^{2\cdot 0} - c_2 e^{-0}. \end{cases} \Rightarrow \begin{cases} 0 = c_1 + c_2, \\ 3 = 2c_1 - c_2. \end{cases} \Rightarrow c_1 = 1, \ c_2 = -1.$

Demak, $y = e^{2x} - e^{-x}.$

BIR JINSLI BO'LMAGAN DIFFERENSIAL TENGLAMALAR

$$y'' + py' + qy = f(x).$$

Bir jinsli bo'lmagan differensial tenglamaning umumiy yechimi, o'ziga mos bir jinsli differensial tenglamaning umumiy yechimi bilan o'zining birorta xususiy yechimining yig'indisiga teng, ya'ni $y = u + \bar{y}.$

I. α soni xarakteristik tenglamaning ildizi emas. Bunda tenglamaning xususiy yechimi $f(x) = P_n(x)e^{\alpha x}$ yoki $\bar{y} = A_0x^2 + A_1x + A_2$ ko'rinishda izlanadi.

Misol 1012. $y'' + 2y' - 8y = x^2$ differensial tenglamaning umumiy yechimini toping.

Yechilishi. 1) $y'' + 2y' - 8y = 0 \Rightarrow k^2 + 2k - 8 = 0 \Rightarrow k_1 = -2, k_2 = 4 \Rightarrow u = c_1e^{-2x} + c_2e^{4x}.$

2) $f(x) = x^2 \Rightarrow x^2 = P_2(x)e^{\alpha x} \Rightarrow x^2 \cdot e^{0\cdot x} = P_2(x)e^{\alpha\cdot x} \Rightarrow \alpha = 0.$

Demak, α soni xarakteristik tenglamaning yechimi emas. Xususiy yechim $f(x) = P_2(x)e^{\alpha x}$ yoki $\bar{y} = A_0x^2 + A_1x + A_2$ ko'rinishda izlanadi. Buning uchun $\bar{y}$ dan hosilalar olinib berilgan tenglamaga qo'yiladi: $\bar{y}' = 2A_0x + A_1; \ \bar{y}'' = 2A_0.$

U holda

$$y'' + 2y' - 8y = x^2 \Rightarrow 2A_0 + 2(2A_0x + A_1) - 8(A_0x^2 + A_1x + A_2) = x^2 \Rightarrow$$

$$2A_0 + 4A_0x + 2A_1 - 8A_0x^2 - 8A_1x - 8A_2 = x^2 \Rightarrow -8A_0x^2 + (4A_0 - 8A_1)x + 2A_0 + 2A_1 - 8A_2 = x^2 \Rightarrow$$

$$\Rightarrow \begin{cases} -8A_0 = 1, \\ 4A_0 - 8A_1 = 0, \\ 2A_0 + 2A_1 - 8A_2 = 0. \end{cases} \Rightarrow \begin{cases} A_0 = -\dfrac{1}{8}; \\ A_1 = -\dfrac{1}{16}; \\ A_2 = -\dfrac{3}{64}. \end{cases} \quad \text{U holda } \ \bar{y} = -\frac{1}{8}x^2 - \frac{1}{16}x - \frac{3}{64}.$$

Demak, $y = c_1e^{-2x} + c_2e^{4x} - \dfrac{1}{8}x^2 - \dfrac{1}{16}x - \dfrac{3}{64}.$

II. α soni xarakteristik tenglamaning ildizi bo'lgan hol.

Misol 1013. $y'' - 2y' = x + 3$ differensial tenglamaning umumiy yechimini toping.

Yechilishi. 1)

$$y'' - 2y' = 0 \Rightarrow p = -2, \ q = 0, \ k^2 - 2k = 0 \Rightarrow k(k-2) = 0 \ \begin{cases} k_1 = 0 \\ k_2 = 2 \end{cases} \Rightarrow u = c_1 + c_2e^{2x}.$$

2) $f(x) = P_n(x)e^{\alpha x} \Rightarrow x + 3 = P_1(x)e^{\alpha x} \Rightarrow (x+3)e^{0\cdot x} = P_1(x)e^{\alpha x} \Rightarrow \alpha = 0.$

Demak, α soni xarakteristik tenglamaning bir ildiziga teng.

3) Bunda xususiy yechim $\bar{y} = x \cdot Q(x)$ yoki $\bar{y} = x(A_0x + A_1)$ ko'rinishda izlanadi: $\bar{y}' = 2A_0x + A_1; \ \bar{y}'' = 2A_0.$

4) $\bar{y}, \ \bar{y}'$ *va* $\bar{y}''$ lar berilgan tenglamaga qo'yiladi:

$$2A_0 - 2(2A_0x + A_1) = x + 3 \Rightarrow 2A_0 - 4A_0x - 2A_1 = x + 3 \Rightarrow -4A_0x + 2A_0 - 2A_1 = x + 3 \Rightarrow$$

$$\Rightarrow \begin{cases} -4A_0 = 1 \\ 2A_0 - 2A_1 = 3 \end{cases} \Rightarrow \begin{cases} A_0 = -\frac{1}{4} \\ 2\cdot\left(-\frac{1}{4}\right) - 2A_1 = 3 \end{cases} \Rightarrow \begin{cases} A_0 = -\frac{1}{4} \\ A_1 = -\frac{7}{4} \end{cases} \Rightarrow \bar{y} = x\cdot\left(-\frac{1}{4}x - \frac{7}{4}\right) = -\frac{1}{4}x^2 - \frac{7}{4}x.$$

5) Umumiy yechim: $y = u + \bar{y} = c_1 + c_2 e^{2x} - \frac{1}{4}x^2 - \frac{7}{4}x.$

III. α soni xarakteristik tenglamaning ikki karrali ildizi bo'lgan hol.

Bunda xususiy yechim $\bar{y} = x^2 Q(x) e^{\alpha x}$ *yoki* $\bar{y} = A_0 x^2 e^{\alpha x}$ ko'rinishda izlanadi.

Misol 1014. $y'' - 6y' + 9y = 5e^{3x}$ differensial tenglamaning umumiy yechimini toping.

Yechilishi. 1) $y'' - 6y' + 9y = 0 \Rightarrow k^2 - 6k + 9 = 0 \Rightarrow k_1 = k_2 = 3 \Rightarrow u = (c_1 + c_2 x)e^{3x}.$

2) $f(x) = P_n(x)e^{\alpha x} \Rightarrow 5\cdot e^{3x} = P_0(x)e^{\alpha x} \Rightarrow \alpha = 3.$

3) $k_1 = k_2 = \alpha = 3.$ Bundan

$\bar{y} = A_0 x^2 e^{3x};\ \bar{y}' = 2A_0 x e^{3x} + 3A_0 x^2 e^{3x};\ \bar{y}'' = 2A_0 e^{3x} + 6A_0 x e^{3x} + 6A_0 x e^{3x} + 9A_0 x^2 e^{3x} =$

$= 9A_0 x^2 e^{3x} + 12A_0 x e^{3x} + 2A_0 e^{3x}.$

4) $\bar{y},\ \bar{y}'$ *va* $\bar{y}''$ lar berilgan tenglamaga qo'yiladi:

$9A_0 x^2 e^{3x} + 12A_0 x e^{3x} + 2A_0 e^{3x} - 12A_0 x e^{3x} - 18A_0 x^2 e^{3x} + 9A_0 x^2 e^{3x} = 5e^{3x} \Rightarrow$

$\Rightarrow 2A_0 e^{3x} = 5e^{3x} \Rightarrow A_0 = \frac{5}{2} \Rightarrow \bar{y} = \frac{5}{2}x^2 e^{3x}.$

5) Umumiy yechim: $y = u + \bar{y} = (C_1 + C_2 x)e^{3x} + \frac{5}{2}x^2 e^{3x} \Rightarrow y = \left(C_1 + C_2 x + \frac{5}{2}x^2\right)e^{3x}.$

Misol 1015. Ushbu

$$a_1(x)y''' + a_2(x)y'' + a_3(x)y' + a_4(x)y = f(x), \qquad (1)$$

ko'rinishdagi differensial tenglamani

1^0. $a_1(x) = \frac{1}{x},\ a_2(x) = a_3(x) = a_4(x) = 0,\ f(x) = \ln x;$

2^0. $a_1(x) = a_4(x) = 1,\ a_2(x) = a_3(x) = 3,\ f(x) = 0;$

3^0. $a_1(x) = a_3(x) = 1,\ a_2(x) = a_4(x) = 0,\ f(x) = 8e^{2x} + 5e^x \sin x$

bo'lgan hollarda yeching.

Yechilishi. I. 1^0. shart bo'yicha berilganlarni (1) tenglamaga qo'yib,

$$\frac{1}{x}y''' = \ln x \text{ yoki } y''' = x\ln x$$

tenglamani hosil qilamiz.Ma'lumki bu uchinchi tartibli differensial tenglama *kema-ket uch marta integrallash yo'li* bilan yechiladi. Oxirgi tenglamani ketma-ket uch marta integrallaymiz:

1) $y'' = \int x\ln x dx + c_1$ .o'ng tomondagi integralni bo'laklab integrallaymiz:

$$\begin{matrix} u = \ln x, du = \frac{dx}{x} \\ dv = xdx, v = \frac{x^2}{2} \end{matrix} \Rightarrow \int x\ln x dx = \frac{x^2\ln x}{2} - \int\frac{x^2 dx}{2x} = \frac{x^2\ln x}{2} - \frac{x^2}{4} + c_1 = \frac{x^2\ln x}{2} - \frac{x^2}{4} + c_1.$$

Demak,

$$y''=\frac{x^2\ln x}{2}-\frac{x^2}{4}+c_1 .$$

2) Yuqoridagi jarayonni davom ettirib,

$$y'=\frac{x^3\ln x}{6}-\frac{5x^3}{36}+c_1x+c_2 \qquad \text{yoki}$$

3) $$y=\frac{x^4\ln x}{24}-\frac{13}{288}x^4+c_1\frac{x^2}{2}+c_2x+c_3$$

ga ega bo'lamiz. Bu esa yuqoridagi $y'''=x\ln x$ uchinchi tartibli differensial tenglamaning umumiy yechimidir.

II. 2^0. shart bo'yicha

$$y'''+3y''+3y'+y=o$$

tenglamani hosil qilamiz. Bu *chiziqli bir jinsli o'zgarmas koeffisiyentli* uchinchi tartibli differensial tenglama bo'lib, uni yechish uchun *xarakteristik tenglamasi*ni tuzamiz:

$$k^3+3k^2+3k+1=0 \quad \text{yoki} \quad (k+1)^3=0 .$$

Ko'rinib turibdi, hosil qilingan differensial tenglamaning xarakteristik tenglamasi uch karali haqiqiy ildizga ega $(k_1=k_2=k_3=-1)$.

Demak berilgan tenglamaning umumiy yechimi:

$$y=e^{-x}(c_1+c_2x+c_3x^2)$$

ko'rinishda bo'ladi.

III. 3^0. shartdagilarni (1) tenglamaga qo'yib,

$$y'''+4y'=8e^{2x}+5e^x\sin x$$

ko'rinishdagi tenglamaga ega bo'lamiz. Bu *chiziqli bir jinsli bo'lmagan o'zgarmas koeffisiyentli* 3- tartibli differensial tenglamadir. Bu tenglamaga mos chiziqli bir jinsli differensial tenglamaning xarakteristik tenglamasi

$$k^3+4k=0 \quad \text{yoki} \quad k(k^2+4)=0$$

Bo'lib, $k_1=o$ haqiqiy ildiz va $k_{1,2}=\pm 2i$ kompleks ildizlarga ega. Demak berilgan tenglamaga mos keluvchi chiziqli bir jinsli differensial tenglamaning umumiy yechimi

$$u=c_1+c_2\cos 2x+c_3\sin 2x \qquad (2)$$

bo'ladi.

Berilgan chiziqli bir jinsli bo'lmagan o'zgarmas koeffisiyentli 3- tartibli differensial tenglamaning umumiy yechimi

$$y=u+y_1 \qquad (3)$$

ko'rinishda bo'ladi. Bu yerda y_1 berilgan chiziqli bir jinsli bo'lmagan o'zgarmas koeffisiyentli 3- tartibli differensial tenglamaning xususiy yechimlaridan biri. y_1 xususiy yechimni *aniqmas koeffisiyentlar usuli* bilan topamiz. Buning uchun berilgan chiziqli bir jinsli bo'lmagan o'zgarmas koeffisiyentli 3- tartibli differensial tenglamaning o'ng tomonidagi funksiya

$$f(x)=8e^{2x}+5e^x\sin x=8e^{2x}+0e^x\cos x+5e^x\sin x=8e^{2x}+e^x(0\cos x+5\sin x)$$

ko'rinishida bo'lganligi sababli y_1 xususiy yechimni

$$y_1=Ae^{2x}+e^x(B\cos x+C\sin x) \qquad (4)$$

ko'rinishda izlaymiz. A,V va S aniqmas koeffisiyentlarni aniqlash uchun y_1 dan kema-ket uch marta hosila olamiz:

$$y_1^{'} = 2Ae^{2x} + e^x[(B+C)\cos x + (C-B)\sin x];$$

$$y_1^{'} = 4Ae^{2x} + 2e^x(C\cos x - B\sin x);$$

$$y_1^{'} = 8Ae^{2x} + 2e^x[(C-B)\cos x - (B+C)\sin x].$$

Bularni berilgan tenglamaga qo'ysak,

$$16Ae^{2x} + 2e^x[(B+3C)\cos x + (C-3B)\sin x] = 8e^{2x} + 5e^x \sin x$$

bo'ladi.

Oxirgi tenglikdagi o'xshash hadlar oldidagi koeffisiyentlarni tenglashtirib,

$$\begin{cases} 16A = 8 \\ 2(B+3C) = 0 \\ 2(C-3B) = 5 \end{cases}$$

Tenglamalar sistemasini hosil qilamiz. Tenglamalar sistemasini yechib,

$A = \frac{1}{2}, B = -\frac{3}{4}$ va $C = \frac{1}{4}$ larni topamiz.

Demak bularni (4) ga qo'ysak,

$$y_1 = \frac{1}{2}e^{2x} + \frac{1}{4}e^x(\sin x - 3\cos x) \qquad (5)$$

bo'ladi.

(2) va (5) ni (3) ga qo'yib, berilgan chiziqli bir jinsli bo'lmagan o'zgarmas koeffisiyentli 3- tartibli differensial tenglamaning umumiy yechimini hosil qilamiz:

$$y = u + y_1 = c_1 + c_2\cos 2x + c_3 \sin 2x + \frac{1}{2}e^{2x} + \frac{1}{4}e^x(\sin x - 3\cos x).$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi bir jinsli tenglamalarning umumiy yechimlarini toping:

1016. $y'' + 4y' + 3y = 0.$ ***J.:*** $y = c_1e^{-x} + c_2e^{-3x}.$

1017. $y'' - 4y' + 4y = 0.$ ***J.:*** $y = e^{2x}(c_1 + c_2x).$

1018. $y'' + 4y' + 8y = 0.$ ***J.:*** $y = e^{-2x}(c_1 \cos 2x + c_2 \sin 2x).$

Koshi masalasini yeching:

1019. $y'' + 4y' + 29y = 0;\ y|_{x=0} = 0;\ y'|_{x=0} = 15.$ ***J.:*** $y = 3e^{-2x}\sin 5x.$

1020. $y'' - 2y' + y = 0;\ y|_{x=2} = 1;\ y'|_{x=2} = -2.$ ***J.:*** $y = (7-3x)e^{x-2}.$

Quyidagi bir jinsli bo'lmagan tenglamalarning umumiy yechimlarini toping:

1021. $y'' + 2y' - 8y = 3x.$ ***J.:*** $y = c_1e^{-2x} + c_2e^{4x} - \frac{3}{8}x - \frac{3}{32}.$

1022. $y'' - 5y = 5x.$ ***J.:*** $y = c_1e^{-\sqrt{5}x} + c_2e^{\sqrt{5}x} - x.$

1023. $y'' - 3y' - 4y = x^2 + 1.$ ***J.:*** $y = c_1e^{-x} + c_2e^{4x} - \frac{1}{4}x^2 + \frac{3}{8}x - \frac{17}{32}.$

1024. $y'' - y' = x^3.$ ***J.:*** $y = c_1 + c_2e^x - \frac{1}{4}x^4 - x^3 - 3x^2 - 6x.$

1025. $y'' - 2y' + y = \frac{e^x}{2}.$ ***J.:*** $y = (c_1 + c_2x + x\ln|x|)e^x.$

37-MAVZU. KOMBINATORIKA

O'RIN ALMASHTIRISH

n elementdan *n* tadan o'rin almashtirish soni $P_n = 1 \cdot 2 \cdot 3 \cdot ... \cdot (n-1) \cdot n = n!$ ga teng. ! – faktorial. $0! = 1, \quad 1! = 1.$

Misol 1026. $3! = 1 \cdot 2 \cdot 3 = 6; \quad 5!! = 1 \cdot 3 \cdot 5 = 15; \quad 8!! = 2 \cdot 4 \cdot 6 \cdot 8 = 384;$

O'RINLASHTIRISH

n elementdan *m* tadan o'rinlashtirishlar soni quyidagi formula yordamida topiladi: $A_n^m = n(n-1)(n-2) \cdot ... \cdot (n-m+1).$

Misol 1027. $A_8^4 = ?$

Yechilishi: $n - m + 1 = 8 - 4 + 1 = 5; \quad A_8^4 = 8 \cdot 7 \cdot 6 \cdot 5 = 1680.$

Misol 1028. $A_8^3 = 8 \cdot 7 \cdot 6 = 336.$

Misol 1029. $A_8^8 = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 8!.$

GRUPPALASH

n elementdan *m* tadan gruppalashlar soni quyidagi formuladan topiladi:

$$C_n^m = \frac{n!}{m!\,(n-m)!}$$

Misol 1030. $C_5^3 = ?$

Yechilishi:

$$C_5^3 = \frac{5!}{3!\,(5-3)!} = \frac{3! \cdot 4 \cdot 5}{3! \cdot 2!} = \frac{20}{1 \cdot 2} = 10.$$

Misol 1031.

$$C_5^2 = \frac{5!}{2!\,(5-2)!} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \cdot 1 \cdot 2 \cdot 3} = 10.$$

Misol 1032.

$$C_5^4 = \frac{4!}{4!\,(5-4)!} = \frac{4! \cdot 5}{4! \cdot 1!} = 5.$$

Misol 1033.

$$C_5^0 = \frac{5!}{0!\,(5-0)!} = \frac{5!}{5! \cdot 0!} = 1.$$

Misol 1034.

$$C_5^5 = \frac{5!}{5!\,(5-5)!} = \frac{5!}{5! \cdot 0!} = 1.$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi ifodalarning son qiymatlarini hisoblang:

1035. $8!$;

1036. $10!$;

1037. $12!!$;

1038. $13!!$;

1039. $\frac{6!}{4!}$;

1040. $\frac{20!}{22!}$;

1041. $\frac{16!!}{14!!}$;

1042. $\frac{15!!}{17!!}$;

1043. $\frac{49!!}{47!!}$.

1044. Tarelkadagi oltita olmaning o'rinlarini almashtirishlar sonini toping.

1045. 12 ta mehmonni 12 kishilik taom tayyorlangan stolga necha xil usulda o'tkazish mumkin?

1046. 10 ta raqamdan foydalanib, bu raqamlarning birortasi ham takrorlanmaydigan nechta to'rt xonali son hosil qilish mumkin (bunda 0 bilan boshlanadigan A_3^3 o'rinlashtirishlar chiqarib tashlanadi)?

1047. Bir sinf o'quvchilari 8 ta o'quv predmetini o'rganishadi. Agar dars jadvaliga har kuni 4 tadan predmet qo'yilsa, bir kunlik darsni necha usul bilan joylashtirish mumkin?

1048. 12 ta shaxmatchining har biri 2 partiyadan o'ynashsa, turnirda jami necha partiya o'yin bo'ladi?

1049. Fazoda 8 ta nuqtaning hech bir to'rttasi bir tekislikda yotmasa, ular orqali nechta turli tekislik o'tkazish mumkin(tekislikning 3 nuqta bilan aniqlanishi e'tiborga olinsin)?

1050. Ketma-ket kelgan har uchtasi bir to'g'ri chiziqda yotmaydigan 7 ta nuqtadan o'tuvchi to'g'ri chiziq kesmalari sonini toping (bunda ikki nuqtadan bitta to'g'ri chiziq o'tishi e'tiborga olinsin).

1051. $C_n^m = C_n^{n-m}$ tenglikning to'g'riligini isbotlang.

1052. Shaxmat turnirida ishtirok etayotgan har bir shaxmatchi ikki partiyadan shaxmat o'ynaydi. Agar turnirda jami 462 partiya o'ynalgan bo'lsa, turnir ishtirokchilari sonini toping.

38-MAVZU. ELEMENTAR HODISA VA EHTIMOLNING TA'RIFLARI

ELEMENTAR HODISA

Tajribaning har qanday natijasi elementar hodisa deyiladi.

Misol 1053. Tajriba simmetrik, bir jinsli tangani ikki marta tashlashdan iborat bo'lsin. Bunda ro'y berishi mumkin bo'lgan elementar hodisalar to'plamini yozing.

Yechilishi. $P = \{ГГ; ГР; РГ; РР\}$. Elementar hodisalar soni $n = 4$ ta elementar hodisadan faqat bittasi ro'y beradi.

Misol 1054. Tajriba yoqlari birdan oltigacha nomerlangan bir jinsli o'yin soqqasini ikki marta tashlashdan iborat bo'lsin. Elementar hodisalar to'plamini yozing.

Yechilishi.

$$P = \begin{Bmatrix} 1;1 & 2;1 & 3;1 & 4;1 & 5;1 & 6;1 \\ 1;2 & 2;2 & 3;2 & 4;2 & 5;2 & 6;2 \\ 1;3 & 2;3 & 3;3 & 4;3 & 5;3 & 6;3 \\ 1;4 & 2;4 & 3;4 & 4;4 & 5;4 & 6;4 \\ 1;5 & 2;5 & 3;5 & 4;5 & 5;5 & 6;5 \\ 1;6 & 2;6 & 3;6 & 4;6 & 5;6 & 6;6 \end{Bmatrix} \quad n = 36 \text{ elementar hodisalar soni}$$

Elementar hodisalar sonini quyidagicha topish qulay: tanganing tomoni ikkita bo'lgani uchun 2 ni, o'yin soqqasining yoqlari 6 ta bo'lgani uchun 6 ni darajaga ko'tarish kerak. Masalan, tanga bir marta tashlansa, elementar hodisalar soni $2^1 = 2$, ikki marta tashlansa $2^2 = 4$, uch marta tashlansa $2^3 = 8$ va hokazo bo'ladi. Shuningdek, o'yin soqqasi bir marta tashlansa $6^1 = 6$, ikki marta tashlansa $6 = 36$, uch marta tashlansa $6^3 = 216$ va hokazo bo'ladi.

EHTIMOLNING KLASSIK TA'RIFI

$$P(A) = \frac{m}{n}.$$

n – sinovning mumkin bo'lgan elementar hodisalari soni; *m* – *A* hodisaning ro'y berishiga qulaylik tug'diruvchi elementar hodisalar soni; *P* – ehtimol.

Misol 1055. Ikkita o'yin soqqasi bir marta tashlangan. Soqqalarning tushgan yoqlaridagi ochkolar yig'indisi juft son, shu bilan birga soqqalardan hech bo'lmaganda bittasining yog'ida olti ochko chiqish ehtimolini toping.

Yechilishi. Ma'lumki $n = 36$.

$$P = \begin{Bmatrix} 2;\,6 \\ 4;\,6 \\ 6;\,2 \\ 6;\,4 \\ 6;\,6 \end{Bmatrix} => m = 5.$$ U holda $P(A) = \frac{5}{36}$ bo'ladi.

Misol 1056. Qutida 15 ta shar bo'lib, ulardan 5 tasi qizil, qolganlari boshqa rangda. Tavakkaliga olingan 1 ta sharning qizil bo'lish ehtimolini toping.

Yechilishi. $n = 15, m = 5.$

$$P(A) = \frac{5}{15} = \frac{1}{3}$$

EHTIMOLNING STATISTIK TA'RIFI.

$$\mathcal{W}(A) = \frac{m}{n}.$$

m – *A* hodisa ro'y bergan sinovlar soni; *n* – o'tkazilgan jami sinovlar soni; $\mathcal{W}$ – nisbiy chastota.

Misol 1057. Nishonga 20 ta o'q uzilgan, shundan 18 ta o'qning nishonga tekkanligi qayd qilingan. Nishonga tegishlar nisbiy chastotasini toping.

Yechilishi. $n = 20, m = 18.$

$$\mathcal{W}(A) = \frac{18}{20} = \frac{9}{10} = 0.9$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

1058. Tajriba simmetrik, bir jinsli tangani uch marta tashlashdan iborat bo'lsin. Ro'y berishi mumkin bo'lgan barcha elementar hodisalar to'plamini yozing.

1059. *B* hodisa bitta tangani uch marta tashlashda hech bo'lmaganda ikki marta gerb tushishidan iborat bo'lsin. *B* hodisani tashkil qiluvchi elementar hodisalar to'plamini yozing.

1060. *C* hodisa tangani uch marta tashlashda hech bo'lmaganda bir marta gerb tushishidan iborat bo'lsin. *C* hodisani tashkil qiluvchi elementar hodisalar to'plamini yozing.

Tajriba ikkita o'yin soqqasini bir marta tashlashdan iborat. Quyidagi hodisalarni tashkil etuvchi elementar hodisalar to'plamini yozing:

1061. Ochkolar yig'indisi juft son.
1062. Ochkolar yig'indisi toq son.
1063. Ochkolar yig'indisi tub son.
1064. Ochkolar ko'paytmasi tub son.

1065. Ochkolar o'zaro tub son.

Tajriba ikkita o'yin soqqasini bir marta tashlashdan iborat. Quyidagi hodisalarning ro'y berish ehtimolini toping:

1066. Ochkolar yig'indisi yettiga teng.
1067. Ochkolar yig'indisi sakkizga, ayirmasi esa to'rtga teng.
1068. Ochkolar yig'indisi beshga, ko'paytmasi esa to'rtga teng.
1069. Qutida 15 ta detal bo'lib ulardan 10 tasi bo'yalgan. Yig'uvchi tavakkaliga 3 ta detal oldi. Olingan detallarning bo'yalgan bo'lish ehtimolini toping.

$$P(A)=\frac{C_{10}^{3}}{C_{15}^{3}}.$$

1070. Konvertdagi 100 ta fotokartochka orasida bitta izlanayotgan fotokartochka bor. Konvertdan tavakkaliga 10 ta fotokartochka olindi. Bular orasida kerakli fotokartochkaning ham bo'lish ehtimolini toping.

$$P(B)=\frac{C_{99}^{9}}{C_{100}^{10}}.$$

1071. Qutida 100 ta detal bo'lib, ulardan 10 tasi yaroqsiz. Tavakkaliga 4 ta detal olingan. Olingan detallar orasida: yaroqsiz detallar bo'lmasligi; yaroqli detallar bo'lmasligi ehtimolini toping.

$$a)\ P(A)=\frac{C_{90}^{4}}{C_{100}^{4}};\quad b)\ P(B)=\frac{C_{10}^{4}}{C_{100}^{4}}.$$

1072. 100 ta mahsulotdan iborat partiyada 10 ta mahsulot yaroqsiz. Olingan 4 ta mahsulotdan 3 tasining sifatli bo'lish ehtimolini toping.

$$P(A)=\frac{C_{90}^{3}\cdot C_{10}^{1}}{C_{100}^{4}}.$$

1073. 36 kartali tsastadan tavakkaliga bitta karta tortib olindi. Chillik karta chiqish ehtimolini toping. $n=36, m=9$.
1074. Texnik nazorat hodimi tasodifan ajratib olingan 100 ta kitobdan iborat partiyada 2 ta yaroqsiz kitob borligini aniqladi. Yaroqsiz kitoblar chiqishining nisbiy chastotasini toping.
1075. Tajriba uchun 1000 dona urug' ajratib ekildi. Ulardan 125 tasi unib chiqmadi: urug'ning unib chiqish nisbiy chastotasini; birinchi olingan urug'ning unib chiqmaslik ehtimolini toping.
1076. 10000 dona tarvuzni transport bilan tashishda 36 tasi buzildi. Buzilmagan tarvuzlar sonining nisbiy chastotasini toping.
1077. 100 detalli partiyadan texnik nazorat bo'limi 5 ta nostandart detal topdi. Nostandart detallar chiqishining nisbiy chastotasini toping.

1078. Miltiqdan o’q uzishda nishonga tekkan o’qlar sonining nisbiy chastotasi 0,85 ga teng. Jami 120 ta o’q uzilgan bo’lsa, nishonga tekkan o’qlar sonini toping.

1079. Tanga 10 marta tashlanadi. 3 marta gerbil tomon tushish ehtimolini toping.

$P(A) = \frac{C_{10}^{3}}{2^{10}}.$

39-MAVZU. EHTIMOLNING ASOSIY TEOREMALARI

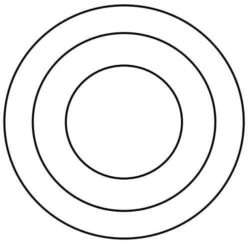
BIRGALIKDA BO’LMAGAN HODISALAR EHTIMOLLARINI QO’SHISH TEOREMASI

$$P(A + B) = P(A) + P(B).$$

Misol 1080. Bir-biri bilan kesishmaydigan 3 ta aylanadan iborat *D* sohaga qarata o’q otildi. O’qning I doiraga tegish ehtimoli $P(A) = \frac{5}{100}$, II doiraga tegish ehtimoli $P(B) = \frac{10}{100}$, III doiraga tegish ehtimoli $P(C) = \frac{17}{100}$. O’qning *D* sohaga tegish ehtimolini toping.

Yechish.

$P(A + B + C) = P(A) + P(B) + P(C) = \frac{5}{100} + \frac{10}{100} + \frac{17}{100} = \frac{32}{100} = 0.32$



Misol 1081. Qutida 50 ta shar bor, ulardan 20 tasi qizil, 12 tasi ko’k, 18 tasi bo’yalmagan. Tavakkaliga bitta shar olindi, uning rangli bo’lish ehtimolini toping.

Yechilishi. $n = 50$

Qizil shar chiqish hodisa(*A*) ehtimoli: $P(A) = \frac{20}{50} = \frac{1}{5} = 0.4$;

Ko’k shar chiqish hodisa(*B*) ehtimoli: $P(B) = \frac{12}{50} = \frac{6}{25} = 0.24$;

A va *B* hodisalar birgalikda emas. Rangli shar chiqish hodisa(*A*+*B*) ehtimoli:

$P(A + B) = P(A) + P(B) = 0.4 + 0.24 = 0.64$

BIRGALIKDA BO’LGAN HODISALAR EHTIMOLLARINI QO’SHISH TEOREMASI.

$$P(A + B) = P(A) + P(B) \vee P(A \cdot B)$$

Misol 1082. Otishmada birinchi to’pdan nishonga tegish ehtimoli $\frac{8}{10}$ ga, ikkinchi to’pdan nishonga tegish ehtimoli esa $\frac{7}{10}$ ga teng. Ikki to’pdan bir vaqtda o’q uzilganda nishonga tegish ehtimolini toping.

Yechilishi.

$$P(A) = \frac{8}{10}; \quad P(B) = \frac{7}{10}$$

$$P(A + B) = P(A) + P(B) - P(A \cdot B) = \frac{8}{10} + \frac{7}{10} - \frac{8}{10} \cdot \frac{7}{10} = 0.94$$

Misol 1083. Agar A hodisa B hodisani ergashtirsa, u holda $P(B) \geq P(A)$ bo'lishini isbotlang.

Isboti. B hodisani birgalikda bo'lmagan A va AB hodisalarning yig'indisi ko'rinishida tasvirlash mumkin: $B = A + AB$

Teoremaga asosan: $P(B) = P(A + \overline{A}B) = P(A) + P(\overline{A}B)$.

Bunda $P(\overline{A}B) \geq 0$ bo'lgani uchun $P(B) \geq P(A)$ bo'ladi.

ERKLI HODISALAR EHTIMOLLARINI KO'PAYTIRISH TEOREMASI.

$$P(AB) = P(A) \cdot P(B)$$

Misol 1084. Ikkita tankdan bitta nishonga qarata o'q uzildi. Birinchi tankdan otilgan o'qning nishonga tegish ehtimoli $\frac{9}{10}$, ikkinchidan esa $\frac{5}{6}$ ga teng. Ikkala tankdan bir vaqtda bittadan o'q uzilgan. Nishonga ikkita o'q tegish ehtimolini aniqlang.

Yechilishi:

$$P(A) = \frac{9}{10}; \quad P(B) = \frac{5}{6}$$

$$P(AB) = P(A) \cdot P(B) = \frac{9}{10} \cdot \frac{5}{6} = \frac{3}{4}.$$

Misol 1085. Ikkita quti bor: birinchi qutida 1 ta oq; 3 ta qora va 4 ta qizil shar bor. Ikkinchi qutida 3 ta oq, 2 ta qora va 3 ta qizil shar bor. Har bir qutidan bittadan olingan sharlarning ranglari bir xil bo'lish ehtimolini toping.

Yechilishi. Birinchi qutidan oq shar olinishi B_1, qora shar olinishi C_1, qizil shar olinishini D_1 hodisa deb belgilaymiz. Bularga mos ravishda ikkinchi quti uchun B_2, C_2, D_2 bo'ladi. U holda

$$A = B_1B_2 + C_1C_2 + D_1D_2 \;=>\; P(A) = P(B_1B_2 + C_1C_2 + D_1D_2) =$$

$$= P(B_1B_2) + P(C_1C_2) + P(D_1D_2) = P(B_1)P(B_2) + P(C_1)P(C_2) + P(D_1)P(D) =$$

$$= \frac{1}{8} \cdot \frac{3}{8} + \frac{3}{8} \cdot \frac{2}{8} + \frac{4}{8} \cdot \frac{3}{8} = \frac{3}{64} + \frac{3}{32} + \frac{3}{16} = \frac{3 + 6 + 12}{64} = \frac{21}{64} = 0.328125$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR.

1086. Qutida 10 ta detal bo'lib, ulardan 4 tasi bo'yalgan. Yig'uvchi tavakkaliga 3 ta detal oldi. Olingan detalning hech bo'lmaganda bittasi bo'yalgan bo'lish ehtimolini toping.

$P = 1 - \frac{C_4^3}{C_{10}^3}$

1087. Ikki mergan nishonga qarata o'q uzmoqda. Bitta o'q uzishda nishonga tekkizish ehtimoli birinchi mergan uchun 0.8 ga teng, ikkinchi mergan uchun esa 0.7 ga teng. Bir yo'la o'q uzishda merganlardan faqat bittasining nishonga tekkizish ehtimolini toping ($P = 0.38$).

1088. Tajriba uchun uch navdagi shaftoli urug'i ekildi. Ulardan 150 donasi *A* navdan, 250 donasi *B* navdan, 100 donasi *C* navdan. Tavakkaliga olingan bir dona urug'ning *B* navdan bo'lmaslik ehtimolini aniqlang.

1089. Ekish uchun 15 ta gilos ko'chati keltirilgan bo'lib, ulardan 5 tasi sariq mevali gilos. Tajriba maydoniga tavakkaliga ekilgan 3 tup ko'chatning hech bo'lmaganda bittasining sariq gilos bo'lish ehtimolini toping.

1090. Talaba o'ziga kerakli formulani uchta adabiyotdan izlamoqda. Formulaning birinchi, ikkinchi, uchinchi adabiyotda bo'lish ehtimoli mos ravishda 0,6; 0,7; 0,8 ga teng. Formulaning: 1) Faqat bitta adabiyotda; 2) Faqat ikkita adabiyotda; 3) Uchala adabiyotda bo'lish ehtimolini toping.

1091. 200 ta qo'ydan 10 tasi nasldor emas. Tavakkaliga 30 ta qo'y ajratildi. Ularning nasldor bo'lish ehtimolini toping.

1092. Urug'chilik punktiga uchta xo'jalikdan urug' keltiriladi. Birinchi xo'jalikdan 15, ikkinchi xo'jalikdan 10, uchinchi xo'jalikdan 5 mashina urug' keltirildi. Keltirilgan ikki mashina urug'ning har ikkalasi birinchi xo'jalikdan keltirilgan bo'lish ehtimolini toping.

1093. Guruhda 12 o'g'il va 8 qiz bola o'qiydi. Navbatchilik uchun tasodifiy ravishda ikki talaba olindi. Ulardan biri o'g'il, ikkinchisining qiz bola bo'lish ehtimolini toping.

40-MAVZU. KAMIDA BITTA HODISANING RO'Y BERISH EHTIMOLI

N TA HODISANING RO'Y BERISH EHTIMOLLARI HAR XIL BO'LGANDA ULARDAN KAMIDA BITTASINING RO'Y BERISH EHTIMOLI

$A_1, A_2, \dots, A_n$ hodisalar birgalikda erkli, hamda $P(A_1) = P_1, P(A_2) = P_2, \dots, P(A_n) = P_n$ bo'lsin. U holda bu hodisalardan kamida bittasining ro'y berishidan iborat *A* hodisaning ro'y berish ehtimolini topish uchun $\overline{A_1}$, $\overline{A_2}$, …, $\overline{A_n}$ qarama-qarshi hodisalar ehtimollari ko'paytmasini birdan ayirish kerak.

$P(A) = 1 - q_1 \cdot q_2 \cdot q_3 \cdot \dots \cdot q_n.$

Misol 1094. Elektr zanjiriga erkli ishlaydigan uchta element ketma-ket ulangan. Birinchi, ikkinchi va uchunchi elementlarning buzilish ehtimollari mos ravishda $P_1 = 0.1, P_2 = 0.15,$ $P_3 = 0.2$ ga teng. Zanjirda tok bo'lmaslik ehtimolini toping.

Yechilishi. Elementlar ketma-ket ulanganligi uchun ulardan kamida bittasi buzilsa zanjirda tok bo'lmaydi. Izlanayotgan ehtimol:

$P(A) = 1 - (1 - P_1) \cdot (1 - P_2) \cdot (1 - P_3) = 1 - (1 - 0.1) \cdot (1 - 0.15) \cdot (1 - 0.2) =$
$= 1 - 0.9 \cdot 0.85 \cdot 0.8 = 1 - 0.612 = 0.388$

n ta hodisaning ro'y berish ehtimollari bir xil bo'lganda ulardan kamida bittasining ro'y berish ehtimoli

$A_1, A_2, \dots, A_n$ hodisalar birgalikda erkli, hamda
$P(A_1) = P, P(A_2) = P, \dots, P(A_n) = P$ bo'lsin.
U holda bu hodisalardan kamida bittasining ro'y berishidan iborat *A* hodisaning ro'y berish ehtimoli quyidagicha bo'ladi:

$$P(A) = 1 - q^n.$$

Misol 1095. Ikki sportchidan har birining mashqni muvaffaqiyatli bajarish ehtimoli 0.5 ga teng. Mashqni sportchilar navbat bilan bajaradilar, bunda har bir sportchi o'z kuchini ikki marta sinab ko'radi. Mashqni birinchi bo'lib muvaffaqiyatli bajargan sportchi mukofot oladi. Ularning mukofotni olishlari ehtimolini toping.

Yechilishi. Mukofotni olish uchun to'rtta sinovdan kamida bittasini muvaffaqiyatli bajarish kifoya. Mashqning muvaffaqiyatli bo'lish ehtimoli $P = 0.5$, muvaffaqiyatsiz bo'lish ehtimoli $q = 1 - P = 0.5$.

Izlanayotgan ehtimol:

$$P(A) = 1 - q^4 = 1 - \left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16} = 0.9375.$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR.

1096. Qurilma o'zaro erkli ishlaydigan ikkita elementni o'z ichiga oladi. Elementlarning buzilish ehtimollari mos ravishda 0.05 va 0.08 ga teng. Qurilmaning buzilishi uchun kamida bitta elementning buzilishi yetarli bo'lsa, qurilmaning ishlamay qolish ehtimolini toping ($P = 0.126$).

1097. Ko'prik yakson bo'lishi uchun bitta aviatsiya bombasining kelib tushishi kifoya. Agar ko'prikka tushish ehtimollari mos ravishda 0.3, 0.4, 0.6 va 0.7 bo'lgan 4 ta bomba tashlansa, ko'prikning yakson bo'lish ehtimolini toping($P = 0.95$).

1098. Uchta talaba bir-birian erkli ravishda biror hayvon qonining tarkibini tahlil qilmoqda. Birinchi talabaning asbob ko'rsatishini o'qishdagi xatoga yo'yish ehtimoli 0.1 ga teng. Ikkinchi va uchinchi talabalar uchun bu ehtimol mos ravishda 0.15 va 0.2. Bir martadan tahlil qilishda talabalarning kamida bittasining xatoga yo'l qo'yish ehtimolini toping ($P = 0.388$).

1099. Qurilma o'zaro erkli, ishonchlilik darajasi bir xil $P = 0.99$ ga teng bo'lgan ikki elementni o'z ichiga olgan. Qurilmaning ishdan chiqishi uchun kamida bitta elementning buzilishi kifoya bo'lsa, qurilmaning ishlamay qolish ehtimolini toping.

1100. Ikki mergandan har birining o'qni nishonga tekkizish ehtimoli 0.9 ga teng. Merganlar navbat bilan ikkitadan o'q uzadilar. Birinchi bo'lib nishonga o'q tekkizgan mergan g'olib bo'ladi. Merganlarning g'olib bo'lish ehtimolini toping.

1101. Merganning uchta o'q uzishda kamida bitta o'qni nishonga tekkizish ehtimoli $P = 0.875$ ga teng. Uning bitta o'q uzishda nishonga tekkizish ehtimolini toping.

1102. To'rtta o'q uzishda kamida bitta o'qning nishonga tegish ehtimoli $P = 0.9984$ ga teng. Bitta o'q uzganda nishonga tekkizish ehtimolini toping.

41-MAVZU. SHARTLI EHTIMOL

TO'LA EHTIMOL FORMULASI

To'la gruppa tashkil etadigan, birgalikda bo'lmagan $B_1, B_2, \dots, B_n$ gipotezalarning biri ro'y bergandagina ro'y berishi mumkin bo'lgan *A* hodisaning ehtimoli gipotezalardan har birining ehtimolini *A* hodisaning tegishli shartli ehtimoliga ko'paytmalari yig'indisiga teng:

$$P(A) = P(B_1) \cdot P(A/B_1) + \cdots + P(B_n) \cdot P(A/B_n) \text{ yoki}$$

$$P(A) = P(B_1) \cdot P_{B_1}(A) + \cdots + P(B_n) \cdot P_{B_n}(A).$$

Misol 1103. Uchta quti bor. Birinchi qutida 5 ta oq va 3 ta qora shar, ikkinchisida 4 ta oq va 4 ta qora shar, uchinchisida esa 8 ta oq shar bor. Qutilardan biri tavakkaliga tanlanib, undan tavakkaliga bitta shar olinsin. Ushbu sharning qora rangli bo'lish ehtimoli topilsin.

Yechilishi. Shar yo birinchi, yo ikkinchi, yoki uchinchi qutidan olinishi mumkin; ushbu gipotezani mos ravishda B_1, B_2, B_3 bilan belgilaymiz. Qutilardan har birini tanlash teng imkoniyatli bo'lgani uchun quyidagicha bo'ladi:

$$P(B_1) = P(B_1) = P(B_3) = \frac{1}{3}$$

A hodisaning ro'y berishi albatta B_1, B_2, B_3 gipotezalarga bog'liq. Shuning uchun *A* hodisaning ehtimolini B_1, B_2, B_3 gipotezalarga nisbatan topamiz:

$$P(A/B_1) = \frac{3}{8}; \quad P(A/B_2) = \frac{4}{8}; \quad P(A/B_3) = \frac{0}{8}$$

Bundan to'la ehtimol formulasiga asosan quyidagi kelib chiqadi:

$$P(A) = P(A/B_1) \cdot P(B_1) + P(A/B_2) \cdot P(B_2) + P(A/B_3) \cdot P(B_3) =$$

$$= \frac{3}{8} \cdot \frac{1}{3} + \frac{4}{8} \cdot \frac{1}{3} + \frac{0}{8} \cdot \frac{1}{3} = \frac{1}{8} + \frac{1}{6} = \frac{3+4}{24} = \frac{7}{24} = 0.291(6)$$

Misol 1104. *X* talaba to'liq tayyorlanmagan holda yakuniy nazorat topshirishga keldi. *Y* talaba 25 ta biletdan 15 tasini o'rgandi. Qaysi holda talaba uchun "yaxshi" bilet olish ehtimoli yuqori bo'ladi? Birinchi bo'lib bilet olgandami yoki ikkinchi bo'lib bilet olgandami?

Yechilishi. *X* talaba birinchi bo'lib bilet olsa, quyidagicha bo'ladi:

$$P(A) = \frac{15}{25} = \frac{3}{5} = 0.6$$

Endi X talaba ikkinchi bo'lib bilet olgan holni qaraymiz. Bu holda X talabaning fikricha ikki gipoteza (yaxshi, yomon bilet chiqish hodisasi) bo'ladi.

B_1 – yaxshi bilet chiqish hodisasi;

B_2 – yomon bilet chiqish hodisasi;

Ravshan-ki,

$$P(B_1) = \frac{15}{25} = \frac{3}{5} = 0.6;$$

$$P(A/B_1) = \frac{15-1}{25-1} = \frac{14}{24} = 0.58(3);$$

$$P(B_2) = \frac{25-15}{25} = \frac{10}{25} = 0.4;$$

$$P(A/B_2) = \frac{15}{25-1} = \frac{15}{24} = 0.625$$

bo'ladi. U holda

$$P(A) = P(B_1)\cdot P(A/B_1) + P(B_2)\cdot P(A/B_2) =$$

$$= \frac{3}{5}\cdot\frac{14}{24} + \frac{2}{5}\cdot\frac{15}{24} = \frac{1}{20}(42+30) = \frac{72}{120} = 0.6$$

kelib chiqadi.

Bulardan ko'rinadi-ki, "yaxshi" bilet olish ehtimoli talabaning birinchi yoki ikkinchi bo'lib bilet olishiga bog'liq emas.

BEYES FORMULASI

Faraz qilamiz, A hodisa hodisalarning to'la guruhini tashkil etadigan, bir-biri bilan birgalikda bo'lmagan $B_1, B_2, \dots, B_n$ gipotezalarning ro'y berish shartidagina ro'y berishi mumkin bo'lsin. Maqsad A hodisa ro'y berdi degan shartda, gipotezalarning ehtimolini topishdan iborat. Bu Beyes formulasi yordamida topiladi:

$$P(B_i/A) = \frac{P(A/B_i)\cdot P(B_i)}{P(A/B_i)\cdot P(B_i) + \dots + P(A/B_n)\cdot P(B_n)}$$

yoki

$$P_A(B_i) = \frac{P(B_i)\cdot P_{B_i}(A)}{P(B_i)\cdot P_{B_i}(A) + \dots + P(B_n)\cdot P_{B_n}(A)}$$

Misol 1106. Quyon tekshirilganda unda ikkita B_1 va B_2 kasalliklardan biri bo'lishi mumkin deb gumon qilindi. Bu holda ularning ehtimollari mos ravishda $P(B_1) = 0.6$ va $P(B_2) = 0.4$ dan iborat. Tashxisni aniqlash uchun quyonni tahlil qilish rejalashtirildi. Tahlil natijasi salbiy yoki ijobiy bo'ladi. B_1 kasallik bo'lgan holda tahlil natijasi ijobiy bo'lish ehtimoli 0.9 ga, salbiy bo'lish ehtimoli 0.1 ga teng. B_2 kasallik uchun esa tahlilning ijobiy va salbiy bo'lishi teng ehtimolli. Tahlil ikki marta o'tkazildi va ikki marta ham natija salbiy bo'lib chiqdi (A hodisa). O'tkazilgan tahlildan so'ng har bir kasallikning ehtimolini topish talab qilindi.

Yechilishi B_1 kasallik bo'lganda A hodisa $P(A/B_1) = 0.1 \cdot 0.1 = 0.01$ ehtimol bilan, B_2 kasallik bo'lganda esa $P(A/B_2) = 0.5 \cdot 0.5 = 0.25$ ehtimol bilan yuz beradi. Demak, Beyes formulasiga asosan:

$$P(B_1/A) = \frac{P(A/B_1) \cdot P(B_1)}{P(A/B_1) \cdot P(B_1) + P(A/B_2) \cdot P(B_2)} =$$

$$= \frac{0.01 \cdot 0.6}{0.01 \cdot 0.6 + 0.25 \cdot 0.4} = \frac{0.006}{0.006 + 0.1} = \frac{0.006}{0.106} = 0.0566.$$

$$P(B_2/A) = \frac{P(A/B_2) \cdot P(B_2)}{P(A/B_1) \cdot P(B_1) + P(A/B_2) \cdot P(B_2)} =$$

$$= \frac{0.25 \cdot 0.4}{0.01 \cdot 0.6 + 0.25 \cdot 0.4} = \frac{0.1}{0.006 + 0.1} = \frac{0.1}{0.106} = 0.9434.$$

Natijadan ko'rinib turibdiki, tahlil natijalari B_2 kasallik deb jiddiy taxmin qilishga asos bo'ladi.

Misol 1107. Fabrikada pillaning 30% ini I mashina, 25% ini II mashina, qolgan qismini III mashina qayta ishlaydi. I mashina qayta ishlagan barcha pilladan 1% i, II mashina uchun 1.5% i, III mashina uchun esa 2% i yaroqsiz bo'ladi. Tavakkaliga tanlangan mahsulot birligi yaroqsiz bo'lib chiqdi(A hodisa). Uning I mashinada qayta ishlangan bo'lish ehtimolini toping.

Yechilishi. Bunda quyidagi gipotezalar bo'ladi:

B_1 – I mashinada qayta ishlangan;

B_2 – II mashinada qayta ishlangan;

B_3 – III mashinada qayta ishlangan;

U holda

$P(B_1) = 0.3; \quad P(B_2) = 0.25; \quad P(B_3) = 0.45.$

$P(A/B_1) = 0.01; \quad P(A/B_2) = 0.015; \quad P(A/B_3) = 0.02$

$$P(A) = P(A/B_1) \cdot P(B_1) + P(A/B_2) \cdot P(B_2) + P(A/B_3) \cdot P(B_3) =$$

$$= 0.01 \cdot 0.3 + 0.015 \cdot 0.25 + 0.02 \cdot 0.45 = 0.015$$

$$P(B_1/A) = \frac{0.01 \cdot 0.3}{0.015} = 0.2$$

Shunday qilib, barcha qayta ishlangan mahsulotning o'rtacha 20% ini I mashina ishlab chiqargan ekan.

Bu misolda B_1 gipotezaning tajribadan keyingi ehtimoli, uning tajribadan oldingi ehtimolidan kichik bo'lib chiqdi. Buni tushuntirishga urinib ko'ring.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

1108. Omborga 360 ta detal keltirildi. Bulardan 300 tasi I korxonada tayyorlangan bo'lib, ularning 250 tasi yaroqli. 40 tasi II korxonada tayyorlangan bo'lib, ularning 30 tasi yaroqli, 20 tasi III korxonada tayyorlangan bo'lib, ularning 10 tasi yaroqli. Tavakkaliga olingan mahsulotning yaroqli bo'lish ehtimolini toping.

1109. Yuqoridagi masalada tavakkaliga olingan detal yaroqli ekanligi ma'lum bo'lsa, uning I korxonada tayyorlangan bo'lish ehtimolini toping.

1110. Sportchilar guruhida 20 chang'ichi, 6 velosipedchi va 4 yuguruvchi bor. Saralash normasini bajarish ehtimoli chang'ichi uchun 0.9 ga, velosipedchi uchun 0.8 ga, yuguruvchi uchun 0.75 ga teng. Tavakkaliga ajratilgan sportchilarning normani bajara olish ehtimolini toping.

1111. Birinchi qutidagi 20 ta detaldan 15 tasi, ikkinchi qutidagi 30 ta detaldan 24 tasi, uchinchi qutidagi 10 tadetaldan 6 tasi standart. Tavakkal tanlangan qutidan tavakkaliga olingan detalning standart bo'lish ehtimolini toping.

1112. Ichida 2 ta shar bo'lgan xaltaga yana bitta oq shar solinib, shundan keyin undan tavakkaliga bitta shar olindi. Sharlarning rangi bo'yicha dastlabki tarkibi haqidagi mumkin bo'lgan gipotezalar teng imkoniyatli bo'lsa, olingan sharning oq rangda bo'lish ehtimolini toping.

1113. Benzokolonka yonidan o'tadigan yuk mashinalar sonining yengil mashinalar soniga nisbati 3:2 kabi. Yuk mashinasining benzin olish ehtimoli 0.1 ga, yengil mashinaning benzin olish ehtimoli esa 0.2 ga teng. Benzokolonka yoniga benzin olish uchun mashina kelib to'xtadi. Uning yuk mashina bo'lish ehtimolini toping.

1114. Piramidada 10 ta miltiq bo'lib, ularning 4 tasi optik nishon bilan ta'minlangan. Merganning optik nishonli miltiqdan o'q uzganda nishonga tekkizish ehtimoli 0.95 ga teng; optik nishon o'rnatilmagan miltiq uchun bu ehtimol 0.8 ga teng. Mergan tavakkaliga olingan miltiqdan nishonga o'q tekkizdi. Qaysi birining ehtimoli aniqroq: mergan optik nishon miltiqdan o'q uzganmi yoki optik nishonsiz miltiqdan o'q uzganmi?

1115. Har birida 20 tadan detal bo'lgan uch partiya detal bor. Birinchi, ikkinchi va uchinchi partiyalardagi standart detallar soni mosh ravishda 20, 15, 10 ga teng. Tavakkaliga tanlangan partiyadan tavakkaliga olingan bitta detal standart bo'lib chiqdi. Bu detalni joyiga qaytarib qo'yib, ikkinchi marta tavakkaliga bitta detal olingan edi, u ham standart detal bo'lib chiqdi. Detallarning uchinchi partiyadan olinganlik ehtimolini toping.

1116. Uchta to'pdan bir yo'la o'q otildi va ulardan ikkitasi nishonga tegdi. Agar birinchi, ikkinchi va uchinchi to'pning nishonga tekkizish ehtimoli mos ravishda 0.4, 0.3, 0.5 bo'lsa, birinchi to'pning nishonga tekkizgan bo'lish ehtimolini toping.

1117. Uch mergan bir yo'la o'q uzishdi, bundan ikki o'q nishonga tegdi. Agar birinchi, ikkinchi, uchinchi merganlarning nishonga tekkizish ehtimoli mos ravishda 0.6, 0.5, 0.4 ga teng bo'lsa, uchinchi merganning nishonga tekkizganlik ehtimolini toping.

1118. Hisoblash qurilmasining bir-biridan erkli ishlaydigan uchta elementinidan ikkitasi ishlamay qo'ydi. Agar birinchi, ikkinchi va uchinchi elementlarning ishlamay qo'yish ehtimoli mos ravishda 0.2, 0.4 va 0.3 ga teng bo'lsa, birinchi va ikkinchi elementlarning ishlamay qo'yish ehtimolini toping.

42-MAVZU. TAJRIBALARNING TAKRORLANISHI

BERNULLI FORMULASI

Har birida hodisaning ro'y berish ehtimoli $p \quad (0 < p < 1)$ ga teng bo'lgan *n* ta erkli sinovda hodisaning rosa *k* marta ro'y berish ehtimoli $P_n(k) = C_n^k \cdot p^k \cdot q^{n-k}$ ga teng. Bunda $q = 1 - p$.

Misol 1119. Tanga 10 marta tashlanadi. Gerb tomonining rosa 3 marta tushish ehtimolini toping.

Yechilishi. Gerb yoki raqam tushishi teng imkoniyatli bo'lgani uchun

$$p = q = \frac{1}{2}.$$

U holda

$$P_{10}(3) = C_{10}^3 \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^{10-3} = \frac{10!}{3!\,(10-3)!} \cdot \frac{1}{2^3} \cdot \frac{1}{2^7} =$$

$$= \frac{7! \cdot 8 \cdot 9 \cdot 10}{3! \cdot 7!} \cdot \frac{1}{2^{10}} = \frac{8 \cdot 9 \cdot 10}{1 \cdot 2 \cdot 3} \cdot \frac{1}{2^{10}} = \frac{1}{128}.$$

Misol 1120. Chigitning unuvchanligi 80% bo'lsa, ekilgan 4 ta chigitdan:

a) rosa uchtasining unib chiqish;

b) hech bo'lmaganda ikkitasining unib chiqish;

c) ko'pi bilan uchtasining unib chiqish ehtimolini toping.

Yechilishi. $n = 4;\ p = 0.8;\ \ q = 1 - p = 0.2;\ \ k = 3.$

a)

$$P_4(3) = C_4^3 \cdot 0.8^3 \cdot 0.2^{4-3} = \frac{4!}{3!\,1!} \cdot 0.512 \cdot 0.2 = 4 \cdot 0.1024 = 0.4096;$$

b) P_4(yoki 2, yoki, 3, yoki 4)= $P_4(2) + P_4(3) + P_4(4)$.

$$P_4(2) = C_4^2 \cdot 0.8^2 \cdot 0.2^{4-2} = \frac{4!}{2!\,2!} \cdot 0.64 \cdot 0.64 = 6 \cdot 0.256 = 0.1536;$$

$P_4(3) = 0.4096;\ \ P_4(4) = C_4^4 \cdot 0.8^4 \cdot 0.2^{4-4} = 0.4096$ bo'ladi.

U holda P_4(yoki 2, yoki, 3, yoki 4)= $0.1536 + 0.4096 + 0.4096 = 0.9728$.

c) P_4(yoki 1, yoki, 2, yoki 3)= $P_4(1) + P_4(2) + P_4(3)$.

$$P_4(1) = C_4^1 \cdot 0.8^1 \cdot 0.2^{4-1} = \frac{4!}{1!\,3!} \cdot 0.8 \cdot 0.008 = 0.0256;$$

$$P_4(2) = 0.1536;\ \ P_4(3) = 0.4096;$$

U holda P_4(yoki 1, yoki, 2, yoki 3)= $0.0256 + 0.1536 + 0.4096 = 0.5888$ bo'ladi.

PUASSON FORMULASI

Tajribalar soni *n* katta bo'lganda

$$P_n(k) = \frac{\lambda^k}{k!} \cdot e^{-\lambda}, \ \ \lambda = n \cdot p.$$

Puasson formulasidan foydalaniladi.

Misol 1121. Darslik 200 000 nusxada bosib chiqarilgan. Darslikning yaroqsiz bo'lish ehtimoli 0.00005 ga teng. Butun tirajda rosa 5 ta yaroqsiz kitob bo'lish ehtimolini toping.

Yechilishi.

$$n = 200000; \ \ k = 5; \ \ p = 0.00005; \ \ \lambda = 200000 \cdot 0.00005 = 10$$

$$P_{200000}(5) = \frac{10^5}{5!} \cdot e^{-10} = \frac{100000}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \cdot \frac{1}{e^{10}} = \frac{100000}{120} \cdot 0.000045 = 0.0375.$$

Misol 4. Hamirga mayiz aralashtirib, uni teng bo'laklarga bo'linib, mayizli bo'g'irsoq yopildi. N barcha bo'laklar, n esa barcha mayizlar soni bo'lsin. Tasodifan tanlangan bo'g'irsoqda rosa k ta mayiz bo'lish ehtimolini toping.

Yechilishi. Har qaysi mayizning hamirga tashlanishi bitta tajriba desak, n ta tajriba o'tkazilgan bo'ladi. Biz tanlagan bo'g'irsoqqa mayizning tushishi A hodisa bo'lsin. Hamirga mayiz yaxshilab aralashtirilgani uchun mayizning har bir bo'g'irsoqqa tushishi teng ehtimolli, ya'ni $p = \frac{1}{N}$ bo'ladi. U holda bitta bo'g'irsoqdagi mayizning o'rtacha soni $\lambda = n \cdot p, \ \lambda = \frac{n}{N}$ ga teng bo'ladi. Mayizning aniq qiymati 8-10 ta atrofida bo'ladi. U holda $P_n(k)$ ehtimol quyidagi ko'rinishda bo'ladi:

$$P_n(k) = \frac{\lambda^k}{k!} \cdot e^{-\lambda}.$$

Masalan, $\lambda = 8$ uchun quyidagilarni topamiz:

$P(0) = 0.000$	$P(5) = 0.092$	$P(10) = 0.099$
$P(1) = 0.003$	$P(6) = 0.122$	$P(11) = 0.072$
$P(2) = 0.011$	$P(7) = 0.139$	$P(12) = 0.048$
$P(3) = 0.029$	$P(8) = 0.139$	$P(13) = 0.030$
$P(4) = 0.057$	$P(9) = 0.124$	$P(14) = 0.017$

va $k > 14$ bo'lsa, $P_n(k) < 0.001$ bo'ladi. Bu bo'g'irsoqlarning 0.3 foizi bittadan mayizga, 1.1 foizi ikkitadan mayizga va hokazo ekanligini ko'rsatadi.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

1122. Ikki teng kuchli raqib shaxmat o'ynashmoqda. Qaysi ehtimol kattaroq:

a) raqiblardan birining ikki partiyadan bittasini yutish ehtimolimi yoki to'rt partiyadan ikkitasini yutish ehtimolimi?

b) to'rt partiyadan kamida ikkitasini yutish ehtimolimi yoki besh partiyadan kamida uchtasini yutish ehtimolimi? Durang natijalar e'tiborga olinmaydi.

1123. Ikki eng kuchli kurashchi kuch sinashmoqda. Qaysi birining ehtimoli kattaroq:

a) ikki partiyadan bir partiyani yutishnimi yoki to'rt partiyadan ikkitasini yutishnimi?

b) to'rt partiyadan kamida ikkitasini yutishnimi yoki besh partiyadan kamida uchtasini yutishnimi? Durang natijalar e'tiborga olinmaydi.

1124. Tanga 5 marta tashlanadi. Gerbli tomon:

a) ikki martadan kam tushish;

b) kamida ikki marta tushish ehtimolini toping.

1125. Agar bitta sinovda *A* hodisaning ro'y berish ehtimoli 0.4 ga teng bo'lsa, to'rtta erkli sinovda *A* hodisaning kamida uch marta ro'y berish ehtimolini toping.

1126. *A* hodisa kamida to'rt marta ro'y berganda *B* hodisa ro'y beradi. Agar har birida *A* hodisaning ro'y berish ehtimoli 0.8 ga teng bo'lgan 5 ta erkli sinov o'tkaziladi. *B* hodisaning ro'y berish ehtimolini toping.

1127. Oilada 5 farzand bor. Bu bolalar orasida:

a) ikki o'g'il bola;

b) ko'pi bilan ikki o'g'il bola;

c) ikkitadan ortiq o'g'il bolalar;

d) kamida ikkita va ko'pi bilan uchta o'g'il bola bo'lish ehtimolini toping. O'g'il bolalar tug'ilish ehtimoli 0.51 ga teng deb olinsin.

1128. 500 betli kitobda 50 ta xato mavjud. Tasodifan tanlangan betda *k* ta xato bo'lish ehtimolini toping.

43-MAVZU. LAPLASNING LOKAL VA INTEGRAL TEOREMALARI

LAPLASNING LOKAL TEOREMASI.

Har birida hodisaning ro'y berish ehtimoli $p \ (0 < p < 1)$ ga teng bo'lgan *n* ta erkli sinovda hodisaning *k* marta ro'y berish ehtimoli

$$P_n(k) \approx \frac{1}{\sqrt{npq}} \cdot \varphi(x)$$

ga teng.

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{x}}, \quad x = \frac{k - np}{\sqrt{npq}}$$

$\varphi(-x) = \varphi(x)$ bo'lib, uning qiymati tegishli jadvaldan topiladi.

Misol 1129. Biror korxonada yaroqsiz mahsulot ishlab chiqarish ehtimoli 0.05 ga teng bo'lsin. 500 ta buyum tekshiriladi. Bular orasida rosa 25 ta yaroqsiz buyum bo'lish ehtimolini toping.

Yechilishi. Masala shartiga ko'ra $n = 500; \ k = 25; \ p = 0.05; \ q = 0.95.$

Masalani yechishga Bernulli formulasini qo'llasak bo'ladi:

$$P_{500}(25) = C_{500}^{25} \cdot (0.05)^{25} \cdot (0.95)^{500-25} = \frac{500!}{25!\,475!} \cdot (0.05)^{25} \cdot (0.95)^{475}.$$

Puasson formulasini qo'llasak ham bo'ladi:

$$P_{500}(25) = \frac{25^{25}}{25!} \cdot e^{-25}.$$

Bularni hisoblash katta texnik qiyinchilik tug'diradi. Ushbu masala Laplasning lokal teoremasi yordamida osongina hal qilinadi:

$$x = \frac{25 - 500 \cdot 0.05}{\sqrt{500 \cdot 0.05 \cdot 0.95}} = \frac{25 - 25}{\sqrt{23.75}} = 0; \quad \varphi(0) = 0.3989;$$

$$P_{500}(25) = \frac{1}{\sqrt{npq}} \cdot \varphi(x) = \frac{0.3989}{4.8734} = 0.0818.$$

Misol 1130. Har bir sovliqning sog'lom qo'zi berish ehtimoli 0.9 ga teng. 150 ta sovliqdan rosa 145 ta sog'lom qo'zi olish ehtimolini toping.

Yechilishi. $n = 150; \ k = 145; \ p = 0.8; \ q = 0.1;$

$$x = \frac{145 - 150 \cdot 0.9}{\sqrt{150 \cdot 0.9 \cdot 0.1}} = \frac{145 - 135}{\sqrt{13.5}} = 2.7; \quad \varphi(2.7) = 0.0104;$$

$$P_{150}(145) = \frac{1}{\sqrt{npq}} \cdot \varphi(x) = \frac{0.0104}{3.67} = 0.028.$$

LAPLASNING INTEGRAL TEOREMASI.

Har birida hodisaning ro'y berish ehtimoli $p \quad (0 < p < 1)$ ga teng bo'lgan n ta sinovda hodisaning k_1 marta va ko'pi bilan k_2 marta ro'y berish ehtimoli

$$P_n(k_1; k_2) = \Phi(x'') - \Phi(x')$$

ga teng.

Bunda

$$\Phi(x) = \frac{1}{2\pi}\int_0^x e^{-\frac{x^2}{2}} \partial x - Laplas\ funksiyasi$$

$$x' = \frac{k_1 - np}{\sqrt{npq}}; \quad x'' = \frac{k_2 - np}{\sqrt{npq}}.$$

$\Phi(-x) = -\Phi(x)$ bo'lib, uning qiymati $0 \le x \le 5$ bo'lganda tegishli jadvaldan olinadi. $x > 5$ bo'lganda $\Phi(x) = 0.5$ deb qabul qilingan.

Misol 1131. Hodisaning 100 ta erkli sinovda ro'y berish ehtimoli o'zgarmas bo'lib, 0.8 ga teng. Hodisaning:

a) kamida 75 marta va ko'pi bilan 90 marta;

b) kamida 75 marta;

c) ko'pi bilan 74 marta ro'y berish ehtimolini toping.

Yechilishi.

a) Masalaning shartiga ko'ra, $n = 100; \ p = 0.8; \ q = 0.2; \ k_1 = 75; \ k_2 = 90$

$$x' = \frac{k_1 - np}{\sqrt{npq}} = \frac{75 - 100 \cdot 0.8}{\sqrt{100 \cdot 0.8 \cdot 0.2}} = \frac{75 - 80}{\sqrt{16}} = -\frac{5}{4} = -1.25;$$

$$x'' = \frac{k_2 - np}{\sqrt{npq}} = \frac{90 - 100 \cdot 0.8}{\sqrt{100 \cdot 0.8 \cdot 0.2}} = \frac{90 - 80}{\sqrt{16}} = \frac{10}{4} = 2.5;$$

$\Phi(-1.25) = -\Phi(1.25) = -0.3944; \ \ \Phi(2.5) = 0.4938.$

$P_{100}(75 \le k \le 90) = 0.4938 + 0.3944 = 0.8882.$

b) Masalaning shartiga ko'ra, $n = 100; \ p = 0.8; \ q = 0.2; \ k_1 = 75; \ k_2 = 100.$

$$x' = \frac{75 - 100 \cdot 0.8}{\sqrt{100 \cdot 0.8 \cdot 0.2}} = \frac{75 - 80}{\sqrt{16}} = -\frac{5}{4} = -1.25;$$

$$x'' = \frac{100 - 100 \cdot 0.8}{\sqrt{100 \cdot 0.8 \cdot 0.2}} = \frac{100 - 80}{\sqrt{16}} = \frac{20}{4} = 5;$$

$\Phi(-1.25) = -\Phi(1.25) = -0.3944; \ \ \Phi(5) = 0.5.$

$P_{100}(75 \le k \le 90) = 0.5 + 0.3944 = 0.8944.$

c) "kamida 75 marta ro'y beradi" va "ko'pi bilan 74 marta ro'y beradi" hodisalari qarama-qarshi hodisalar. Shuning uchun bu hodisalar ehtimollarining yig'indisi birga teng. Demak, izlanayotgan ehtimol:

$$P_{100}(0: 74) = 1 - P_{100}(75: 100) = 1 - 0.8944 = 0.1056.$$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

1132. Korxonada ishlab chiqarilgan detalning yaroqsiz bo'lish ehtimoli 0.005 ga teng. 10000 ta detaldan rosa 40 tasining yaroqsiz bo'lish ehtimolini toping.

1133. Tavakkaliga ajratib olingan qo'yning kasal bo'lish ehtimoli 0.2 ga teng. Tasodifan ajratib olingan 400 ta qo'ydan 80 tasining kasal bo'lish ehtimolini toping.

1134. O'g'il bola tug'ilish ehtimoli 0.51 ga teng. Tug'ilgan 100 chaqaloqning 50 tasi o'g'il bola bo'lish ehtimolini toping.

1135. Har bir otilgan o'qning nishonga tegish ehtimoli 0.001 ga teng. Agar 5000 ta o'q otilgan bo'lsa, kamida 2 ta o'qning nishonga tegish ehtimolini toping.

1136. Hodisaning 2100 ta erkli sinovning har birida ro'y berish ehtimoli 0.7 ga teng. Hodisaning:

a) kamida 1470 marta;

b) kamida 1470 marta va ko'pi bilan 1500 marta;

c) ko'pi bilan 1469 marta ro'y berish ehtimolini toping.

1137. Tavakkaliga olingan pillaning yaroqsiz bo'lish ehtimoli 0.2 ga teng. Tavakkaliga olingan 400 ta pilladan:

a) kamida 70 da, ko'pi bilan 130 ta;

b) kamida 70 ta;

c) ko'pi bilan 69 ta yaroqsiz bo'lish ehtimolini toping.

1138. *n* ta tajribaning har birida ijobiy natija olinish ehtimoli 0.9 ga teng. Kamida 180 ta tajribada ijobiy natija olinishini 0.98 ehtimol bilan kutish mumkin bo'lishi uchun nechta tajriba o'tkazish lozim?

Jadval 1. $\varphi(x)$ funksiyaning qiymatlar jadvali.

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0,3989	0,3989	0,3989	0,3988	0,3986	0,3984	0,3982	0,398	0,3977	0,3973
0,1	0,397	0,3965	0,3961	0,3956	0,3951	0,3945	0,3939	0,3932	0,3925	0,3918
0,2	0,391	0,3902	0,3894	0,3885	0,3876	0,3867	0,3857	0,3847	0,3836	0,3825
0,3	0,3814	0,3802	0,379	0,3778	0,3765	0,3752	0,3739	0,3726	0,3712	0,3698
0,4	0,3683	0,3668	0,3652	0,3637	0,3621	0,3605	0,3589	0,3572	0,3555	0,3538
0,5	0,3521	0,3503	0,3485	0,3467	0,3448	0,3429	0,341	0,3391	0,3372	0,3352
0,6	0,3332	0,3312	0,3292	0,3271	0,3251	0,323	0,3209	0,3187	0,3166	0,3144
0,7	0,3123	0,3101	0,3079	0,3056	0,3034	0,3011	0,2989	0,2966	0,2943	0,292
0,8	0,2897	0,2874	0,285	0,2827	0,2803	0,278	0,2756	0,2732	0,2709	0,2685
0,9	0,2661	0,2637	0,2613	0,2589	0,2565	0,2541	0,2516	0,2492	0,2468	0,2444
1	0,242	0,2396	0,2371	0,2347	0,2323	0,2299	0,2275	0,2251	0,2227	0,2203
1,1	0,2179	0,2155	0,2131	0,2107	0,2083	0,2059	0,2036	0,2012	0,1989	0,1965
1,2	0,1942	0,1919	0,1895	0,1872	0,1849	0,1826	0,1804	0,1781	0,1758	0,1736
1,3	0,1714	0,1691	0,1669	0,1647	0,1626	0,1604	0,1582	0,1561	0,1539	0,1518
1,4	0,1497	0,1476	0,1456	0,1435	0,1415	0,1394	0,1374	0,1354	0,1334	0,1315
1,5	0,1295	0,1276	0,1257	0,1238	0,1219	0,12	0,1182	0,1163	0,1145	0,1127

1,6	0,1109	0,1092	0,1074	0,1057	0,104	0,1023	0,1006	0,0989	0,0973	0,0957
1,7	0,094	0,0925	0,0909	0,0893	0,0878	0,0863	0,0848	0,0833	0,0818	0,0804
1,8	0,079	0,0775	0,0761	0,0748	0,0734	0,0721	0,0707	0,0694	0,0681	0,0669
1,9	0,0656	0,0644	0,0632	0,062	0,0608	0,0596	0,0584	0,0573	0,0562	0,0551
2	0,054	0,0529	0,0519	0,0508	0,0498	0,0488	0,0478	0,0468	0,0459	0,0449
2,1	0,044	0,0431	0,0422	0,0413	0,0404	0,0395	0,0387	0,0379	0,0371	0,0363
2,2	0,0353	0,0347	0,0339	0,0332	0,0325	0,0317	0,031	0,0303	0,0297	0,029
2,3	0,0283	0,0277	0,027	0,0264	0,0258	0,0252	0,0246	0,0241	0,0235	0,0229
2,4	0,0224	0,0219	0,0213	0,0208	0,0203	0,0198	0,0194	0,0189	0,0184	0,018
2,5	0,0175	0,0171	0,0167	0,0163	0,0158	0,0154	0,0151	0,0147	0,0143	0,0139
2,6	0,0136	0,0132	0,0129	0,0126	0,0122	0,0119	0,0116	0,0113	0,011	0,0107
2,7	0,0104	0,0101	0,0099	0,0096	0,0093	0,0091	0,0088	0,0086	0,0084	0,0081
2,8	0,0079	0,0077	0,0075	0,0073	0,0071	0,0069	0,0067	0,0065	0,0063	0,0061
2,9	0,006	0,0058	0,0056	0,0055	0,0053	0,0051	0,005	0,0048	0,0047	0,0046
3	0,0044	0,0043	0,0042	0,004	0,0039	0,0038	0,0037	0,0036	0,0035	0,0034
3,1	0,0033	0,0032	0,0031	0,003	0,0029	0,0028	0,0027	0,0026	0,0025	0,0025
3,2	0,0024	0,0023	0,0022	0,0022	0,0021	0,002	0,002	0,0019	0,0018	0,0018
3,3	0,0017	0,0017	0,0016	0,0016	0,0015	0,0015	0,0014	0,0014	0,0013	0,0013
3,4	0,0012	0,0012	0,0012	0,0011	0,0011	0,001	0,001	0,001	0,0009	0,0009
3,5	0,0009	0,0008	0,0008	0,0008	0,0008	0,0007	0,0007	0,0007	0,0007	0,0006
3,6	0,0006	0,0006	0,0006	0,0005	0,0005	0,0005	0,0005	0,0005	0,0005	0,0004
3,7	0,0004	0,0004	0,0004	0,0004	0,0004	0,0004	0,0003	0,0003	0,0003	0,0003
3,8	0,0003	0,0003	0,0003	0,0003	0,0003	0,0002	0,0002	0,0002	0,0002	0,0002
3,9	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0001

Jadval 2. $\Phi(x)$ funksiyaning qiymatlar jadvali.

x	**Φ(x)**	**x**	**Φ(x)**	**x**	**Φ(x)**	**x**	**Φ(x)**	**x**	**Φ(x)**	**x**	**Φ(x)**
0	0	**0,5**	0,1915	**1**	0,3413	**1,5**	0,4332	**2**	0,4772	**3**	0,49865
0,01	0,004	**0,51**	0,195	**1,01**	0,3438	**1,51**	0,4345	**2,02**	0,4783	**3,2**	0,49931
0,02	0,008	**0,52**	0,1985	**1,02**	0,3461	**1,52**	0,4357	**2,04**	0,4793	**3,4**	0,49966
0,03	0,012	**0,53**	0,2019	**1,03**	0,3485	**1,53**	0,437	**2,06**	0,4803	**3,6**	0,49984
0,04	0,016	**0,54**	0,2054	**1,04**	0,3508	**1,54**	0,4382	**2,08**	0,4812	**3,8**	0,49993
0,05	0,0199	**0,55**	0,2088	**1,05**	0,3531	**1,55**	0,4394	**2,1**	0,4821	**4**	0,49997
0,06	0,0239	**0,56**	0,2123	**1,06**	0,3554	**1,56**	0,4406	**2,12**	0,483	**4,5**	0,5
0,07	0,0279	**0,57**	0,2157	**1,07**	0,3577	**1,57**	0,4418	**2,14**	0,4838	**5**	0,5
0,08	0,0319	**0,58**	0,219	**1,08**	0,3599	**1,58**	0,4429	**2,16**	0,4846		
0,09	0,0359	**0,59**	0,2224	**1,09**	0,3621	**1,59**	0,4441	**2,18**	0,4854		
0,1	0,0398	**0,6**	0,2257	**1,1**	0,3643	**1,6**	0,4452	**2,2**	0,4861		
0,11	0,0438	**0,61**	0,2291	**1,11**	0,3665	**1,61**	0,4463	**2,22**	0,4868		
0,12	0,0478	**0,62**	0,2324	**1,12**	0,3686	**1,62**	0,4474	**2,24**	0,4875		
0,13	0,0517	**0,63**	0,2357	**1,13**	0,3708	**1,63**	0,4484	**2,26**	0,4881		
0,14	0,0557	**0,64**	0,2389	**1,14**	0,3729	**1,64**	0,4495	**2,28**	0,4887		
0,15	0,0596	**0,65**	0,2422	**1,15**	0,3749	**1,65**	0,4505	**2,3**	0,4893		
0,16	0,0636	**0,66**	0,2454	**1,16**	0,377	**1,66**	0,4515	**2,32**	0,4898		
0,17	0,0675	**0,67**	0,2486	**1,17**	0,379	**1,67**	0,4525	**2,34**	0,4904		

0,18	0,0714	**0,68**	0,2517	**1,18**	0,381	**1,68**	0,4535	**2,36**	0,4909		
0,19	0,0753	**0,69**	0,2549	**1,19**	0,383	**1,69**	0,4545	**2,38**	0,4913		
0,2	0,0793	**0,7**	0,258	**1,2**	0,3849	**1,7**	0,4554	**2,4**	0,4918		
0,21	0,0832	**0,71**	0,2611	**1,21**	0,3869	**1,71**	0,4564	**2,42**	0,4922		
0,22	0,0871	**0,72**	0,2642	**1,22**	0,3883	**1,72**	0,4573	**2,44**	0,4927		
0,23	0,091	**0,73**	0,2673	**1,23**	0,3907	**1,73**	0,4582	**2,46**	0,4931		
0,24	0,0948	**0,74**	0,2703	**1,24**	0,3925	**1,74**	0,4591	**2,48**	0,4934		
0,25	0,0987	**0,75**	0,2734	**1,25**	0,3944	**1,75**	0,4599	**2,5**	0,4938		
0,26	0,1026	**0,76**	0,2764	**1,26**	0,3962	**1,76**	0,4608	**2,52**	0,4941		
0,27	0,1064	**0,77**	0,2794	**1,27**	0,398	**1,77**	0,4616	**2,54**	0,4945		
0,28	0,1103	**0,78**	0,2823	**1,28**	0,3997	**1,78**	0,4625	**2,56**	0,4948		
0,29	0,1141	**0,79**	0,2852	**1,29**	0,4015	**1,79**	0,4633	**2,58**	0,4951		
0,3	0,1179	**0,8**	0,2881	**1,3**	0,4032	**1,8**	0,4641	**2,6**	0,4953		
0,31	0,1217	**0,81**	0,291	**1,31**	0,4049	**1,81**	0,4649	**2,62**	0,4956		
0,32	0,1255	**0,82**	0,2939	**1,32**	0,4066	**1,82**	0,4656	**2,64**	0,4959		
0,33	0,1293	**0,83**	0,2967	**1,33**	0,4082	**1,83**	0,4664	**2,66**	0,4961		
0,34	0,1331	**0,84**	0,2995	**1,34**	0,4099	**1,84**	0,4671	**2,68**	0,4963		
0,35	0,1368	**0,85**	0,3023	**1,35**	0,4115	**1,85**	0,4678	**2,7**	0,4965		
0,36	0,1406	**0,86**	0,3051	**1,36**	0,4131	**1,86**	0,4686	**2,72**	0,4967		
0,37	0,1443	**0,87**	0,3078	**1,37**	0,4147	**1,87**	0,4693	**2,74**	0,4969		
0,38	0,148	**0,88**	0,3106	**1,38**	0,4162	**1,88**	0,4699	**2,76**	0,4971		
0,39	0,1517	**0,89**	0,3133	**1,39**	0,4177	**1,89**	0,4706	**2,78**	0,4973		
0,4	0,1554	**0,9**	0,3159	**1,4**	0,4192	**1,9**	0,4713	**2,8**	0,4974		
0,41	0,1591	**0,91**	0,3186	**1,41**	0,4207	**1,91**	0,4719	**2,82**	0,4976		
0,42	0,1628	**0,92**	0,3212	**1,42**	0,4222	**1,92**	0,4726	**2,84**	0,4977		
0,43	0,1664	**0,93**	0,3238	**1,43**	0,4236	**1,93**	0,4732	**2,86**	0,4979		
0,44	0,17	**0,94**	0,3264	**1,44**	0,4251	**1,94**	0,4738	**2,88**	0,498		
0,45	0,1736	**0,95**	0,3289	**1,45**	0,4265	**1,95**	0,4744	**2,9**	0,4981		
0,46	0,1772	**0,96**	0,3315	**1,46**	0,4279	**1,96**	0,475	**2,92**	0,4982		
0,47	0,1808	**0,97**	0,334	**1,47**	0,4292	**1,97**	0,4756	**2,94**	0,4984		
0,48	0,1844	**0,98**	0,3365	**1,48**	0,4306	**1,98**	0,4761	**2,96**	0,4985		
0,49	0,1879	**0,99**	0,3389	**1,49**	0,4319	**1,99**	0,4767	**2,98**	0,4986		

44-MAVZU. TASODIFIY MIQDORLAR

DISKRET TASODIFIY MIQDOR EHTIMOLLARINING TAQSIMOT QONUNI.

X diskret tasodifiy miqdorning taqsimot qonuni deb, uning mumkin bo’lgan $x_1, x_2, \dots, x_n$ qiymatlari bilan ularga mos $P_1, P_2, \dots, P_n$ ehtimollar ro’yxatiga aytiladi:

X	x_1	x_2	...	x_n
P	P_1	P_2	...	P_n

Misol 1139. *X* diskret tasodifiy miqdor ushbu taqsimot qonuni bilan berilgan:

X	1	2	5	6
P	0.5	0.1	0.3	0.1

Taqsimot ko’pburchagini yasang.

$A(1; 0.5)$,

$B(2; 0.1)$,

$C(5; 0.3)$,

$D(6; 0.1)$

$P = P_1 + P_2 + P_3 + P_4 = 1$

BINOMINAL TAQSIMOT QONUNI

Bunda X diskret tasodifiy miqdorning $x_1, x_2, \dots, x_n$ qiymatlariga mos $P_1, P_2, \dots, P_n$ ehtimollar Bernulli formulasi bo'yicha topiladi. $P_n(k) = C_n^k \cdot p^k \cdot q^{n-k}$

Misol 1140. Qurilma bir-biridan erkli ishlaydigan uchta elementdan iborat. Har bir elementning bitta tajribada ishdan chiqish ehtimoli 0.1 ga teng. Bitta tajribada ishdan chiqqan elementlar sonining taqsimot qonunini tuzing.

Yechilishi. $p = 0.1;\ \ q = 0.9;\ \ x_1 = 0;\ \ x_2 = 1;\ \ x_3 = 2;\ \ x_4 = 3.$

U holda Bernulli formulasiga asosan

$$P_3(0) = C_3^0 \cdot p^0 \cdot q^3 = q^3 = 0.9^3 = 0.729;$$

$$P_3(1) = C_3^1 \cdot p^1 \cdot q^2 = \frac{3!}{2!\,1!} \cdot 0.1 \cdot 0.9^2 = 0.3 \cdot 0.81 = 0.243;$$

$$P_3(2) = C_3^2 \cdot p^2 \cdot q^1 = \frac{3!}{2!\,1!} \cdot 0.1^2 \cdot 0.9 = 0.03 \cdot 0.9 = 0.027;$$

$$P_3(3) = C_3^3 \cdot p^3 \cdot q^0 = \frac{3!}{3!\,(3-3)!} \cdot 0.1^3 \cdot 1 = 0.001;$$

bo'ladi. Tekshirish $0.729 + 0.243 + 0.027 + 0.001 = 1$

Bulardan X diskret tasodifiy miqdorning taqsimot qonuni quyidagi ko'rinishni oladi:

X	0	1	2	3
P	0.729	0.243	0.027	0.001

PUASSONNING TAQSIMOT QONUNI

Bunda X diskret tasodifiy miqdorning $x_1, x_2, \dots, x_n$ qiymatlariga mos $P_1, P_2, \dots, P_n$ ehtimollar Puasson formulasi yordamida topiladi:

$$P_n(k) = \frac{\lambda^k}{k!} \cdot e^{-\lambda},\ \ \lambda = n \cdot p.$$

Misol 1141. Lampochka zavodida 10 000 ta lampochka ishlab chiqarilgan. Har qaysi lampochkaning yaroqsiz bo'lish ehtimoli 0.0001 ga teng. Bu lampochkalar ichidan tavakkaliga 4 ta lampochka olingan. Yaroqsiz lampochkalar sonining taqsimot qonunini tuzing.

Yechilishi. $n = 10000;\ \ p = 0.0001;\ \ \lambda = n \cdot p = 1;\ \ k = 0,1,2,3,4.$

U holda

$$P_4(0) = \frac{\lambda^0}{0!} \cdot e^{-\lambda} = e^{-1};$$

$$P_4(1) = \frac{1^1}{1!} \cdot e^{-1} = e^{-1};$$

$$P_4(2) = \frac{1^2}{2!} \cdot e^{-1} = \frac{1}{2} \cdot e^{-1};$$

$$P_4(3) = \frac{1^3}{3!} \cdot e^{-1} = \frac{1}{6} \cdot e^{-1};$$

$$P_4(4) = \frac{1^4}{4!} \cdot e^{-1} = \frac{1}{24} \cdot e^{-1};$$

Bulardan foydalanib tasodifiy miqdorning taqsimot qonunini tuzsak bo'ladi:

X	0	1	2	3	4
P	e^{-1}	e^{-1}	$\frac{1}{2} \cdot e^{-1}$	$\frac{1}{6} \cdot e^{-1}$	$\frac{1}{24} \cdot e^{-1}$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

1142. X diskret tasodifiy miqdor quyidagi taqsimot qonuni bilan berilgan:

a)

X	2	4	5	6
P	0.3	0.1	0.2	0.4

;

b)

X	10	15	20
P	0.1	0.7	0.2

Taqsimot ko'pburchagini yasang.

1143. Qurilma bir-biridan erkli ishlaydigan uchta elementdan iborat. Har bir elementning bitta tajribada ishdan chiqish ehtimoli 0.1 ga teng. Bitta tajribada ishdan chiqqan elementlar sonining binominal taqsimot qonunini tuzing.

1144. Partiyada 10% nostandart detal bor. Tavakkaliga 4 ta detal olingan. Olingan detallarning taqsimot qonunini yozing va hosil qilingan taqsimotning ko'pburchagini yasang.

1145. X diskret tasodifiy miqdor – tangani ikki marta tashlashda gerb tomon tushish sonining binominal taqsimot qonunini tuzing.

1146. Ikkita o'yin soqqasi bir vaqtda 2 marta tashlanadi. X tasodifiy miqdor – ikkita o'yin soqqasida juft ochkolar tushish sonining binominal taqsimot qonuni yozing.

1147. Qutidagi 6 ta detal orasida 4 ta standart detal bor. Tavakkaliga 3 ta detal olingan. X diskret tasodifiy miqdor – olingan detallar orasidagi standart detallar sonining taqsimot qonunini tuzing.

1148. Ekilgan har bir ko'chatning ko'karish ehtimoli 0.9 ga teng. Ekilgan 3 ta ko'chatdan ko'karganlari sonining taqsimot qonunini tuzing.

1149. Otilgan o'qning nishonga, tegish ehtimoli 0.6 ga teng. Otilgan 4 ta o'qdan nishonga tegishlar sonining taqsimot qonunini toping.

1150. Har bir tup g'o'zaning vilt kasaliga chalinish ehtimoli 0.001 bo'lsa, tavakkaliga olingan 2000 tup g'o'zadan vilt kasaliga chalinganlari sonining taqsimot qonunini tuzing. (Puasson taqsimotidan foydalaning).

45-MAVZU. DISKRET TASODIFIY MIQDORLARNING SONLI XARAKTERISTIKALARI

DISKRET TASODIFIY MIQDORNING MATEMATIK KUTILISHI

Diskret tasodifiy miqdorning matematik kutilishi deb, uning mumkin bo'lgan barcha qiymatlari bilan ularga mos ehtimollar ko'paytmasining yig'indisiga aytiladi: $M(x) = x_1p_1 + x_2p_2 + \cdots + x_np_n$.

Matematik kutilishning xossalari:

1^0. O'zgarmas sonning matematik kutilishi shu sonning o'ziga teng: $M(C) = C$.

2^0. Tasodifiy miqdorlar yig'indisining matematik kutilishi qo'shiluvchilar matematik kutilishlarining yig'indisiga teng:
$M(x_1 + x_2 + \cdots + x_n) = M(x_1) + M(x_2) + \cdots + M(x_n)$.

3^0. O'zaro erkli tasodifiy miqdorlar ko'paytmasining matematik kutilishi ko'paytuvchilar matematik kutilishlarining ko'paytmasiga teng:
$M(x_1 \cdot x_2 \cdot \ldots \cdot x_n) = M(x_1) \cdot M(x_2) \cdot \ldots \cdot M(x_n)$.

4^0. Binomial taqsimotning matematik kutilishi sinovlar soni bilan bitta sinovda hodisaning ro'y berish ehtimoli ko'paytmasiga teng: $M(x) = n \cdot p$.

Misol 1151. Quyidagi taqsimot qonuni bilan berilgan diskret tasodifiy miqdorning matematik kutilishini toping.

X	2	4	5	6
P	0.3	0.1	0.2	0.4

Yechilishi. Ta’rifga asosan

$$M(x) = 2\cdot 0.3 + 4\cdot 0.4 + 5\cdot 0.2 + 6\cdot 0.4 = 0.6 + 1.6 + 1 + 2.4 + 5.6.$$

Misol 1152. X diskret tasodifiy miqdorning mumkin bo’lgan qiymatlarining ro’yxati berilgan: $x_1 = 1;\ x_2 = 2;\ x_3 = 3$. Shuningdek, bu miqdorning va uning kvadratining matematik kutilishlari ma’lum: $M(x) = 2.3;\ M(x^2) = 5.9$.

X ning mumkin bo’lgan qiymatlariga mos ehtimollarni toping.

Yechilishi:

X	1	2	3
P	p_1	p_2	p_3

$p_1 + p_2 + p_3 = 1;$
$M(x) = 1\cdot p_1 + 2\cdot p_2 + 3\cdot p_3;$
$M(x^2) = 1^2\cdot p_1 + 2^2\cdot p_2 + 3^2\cdot p_3;$

Bundan

$$\begin{cases} p_1 + p_2 + p_3 = 1 \\ p_1 + 2p_2 + 3p_3 = 2.3 \\ p_1 + 4p_2 + 9p_3 = 5.9 \end{cases} <=> \begin{cases} p_1 + p_2 + p_3 = 1 \\ p_2 + 2p_3 = 1.3 \\ 3p_2 + 8p_3 = 4.9 \end{cases} \le>$$

$$\le> \begin{cases} p_1 + p_2 + p_3 = 1 \\ 3p_2 + 6p_3 = 3.9 \\ 3p_2 + 8p_3 = 4.9 \end{cases} \le> \begin{cases} p_1 + p_2 + p_3 = 1 \\ p_2 + 2p_3 = 1.3 \\ 2p_3 = 1 \end{cases} \le>$$

$$<=> \begin{cases} p_1 = 0.2 \\ p_2 = 0.3\,. \\ p_3 = 0.5 \end{cases}$$

DISKRET TASODIFIY MIQDORNING DISPERSIYASI VA O’RTACHA KVADRATIK CHETLANISHI

X tasodifiy miqdorning dispersiyasi deb, chetlanish kvadratining matematik kutilishiga aytiladi: $D(x) = M[x - M(x)]^2$.

Dispersiyani $D(x) = M(x^2) - [M(x)]^2$ formula orqali hisoblash qulay.

Dispersiyani xossalari:

1^0. O’zgarmas sonning dispersiyasi nolga teng: $D(C) = 0$.

2^0. O’zgarmas son dispersiya belgisidan kvadratga ko’tarilib chiqadi:
$D(Cx) = C^2D(x)$.

3^0. Erkli tasodifiy miqdorlar yig’indisining dispersiyasi qo’shiluvchilar dispersiyalarining yig’indisiga teng:
$D(x_1 + x_2 + \cdots + x_n) = D(x_1) + D(x_2) + \cdots + D(x_n)$.

Binomial taqsimotning dispersiyasi sinovlar soni bilan hodisaning bitta sinovda ro’y berish va ro’y bermaslik ehtimollari ko’paytmasiga teng:
$M(x) = n\cdot p\cdot q$.

Tasodifiy miqdorning o’rtacha kvadratik chetlanishi deb, dispersiyaning olingan kvadrat ildizga aytiladi:
$\delta(x) = \sqrt{D(x)}$.

Misol 1153. X diskret tasodifiy miqdor

X	10	12	20	25	30
P	0.1	0.2	0.1	0.2	0.4

taqsimot qonuni bilan berilgan. Tasodifiy miqdorlar matematik kutilishini, dispersiyasini va o'rtacha kvadratik chetlanishini toping.

Yechilishi:

a) $M(x) = 10 \cdot 0.1 + 12 \cdot 0.2 + 20 \cdot 0.1 + 25 \cdot 0.2 + 30 \cdot 0.4 =$

$= 1 + 2.4 + 2 + 5 + 12 = 22.4.$

b) $M(x^2) = 10^2 \cdot 0.1 + 12^2 \cdot 0.2 + 20^2 \cdot 0.1 + 25^2 \cdot 0.2 + 30^2 \cdot 0.4 =$

$= 100 \cdot 0.1 + 144 \cdot 0.2 + 400 \cdot 0.1 + 625 \cdot 0.2 + 900 \cdot 0.4 =$

$= 10 + 28.8 + 40 + 125 + 360 = 563.8$

$D(x) = M(x^2) - [M(x)]^2 = 563.8 - 22.4^2 = 563.8 - 501.76 = 62.04$

c) $\delta(x) = \sqrt{D(x)} = \sqrt{62.04} = 7.9.$

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

***X* tasodifiy miqdor taqsimot qonuni bilan berilgan. Uning matematik kutilishi, dispersiyasi va o'rtacha kvadratik chetlanishini toping.**

1154.

X	8	12	18	24	30
P	0.3	0.1	0.3	0.2	0.1

1155.

X	21	25	32	40	50
P	0.1	0.2	0.3	0.2	0.2

1156.

X	10.2	12.4	16.5	18.1	20
P	0.2	0.2	0.4	0.1	0.1

1157.

X	12	16	21	26	30
P	0.2	0.1	0.4	0.1	0.1

1158.

X	12	13.6	15	18	18.5
P	0.1	0.2	0.4	0.2	0.1

1159.

X	11.5	13.5	15	17.5	18
P	0.1	0.5	0.2	0.1	0.1

1160.

X	4.6	0.2	6.8	7.2	8.4
P	0.3	0.3	0.1	0.2	0.1

1161.

X	1.4	2.2	3.5	4.1	5.2
P	0.3	0.2	0.3	0.1	0.1

1162. Ikki tasodifiy miqdorning taqsimot qonunlari berilgan:

a)

X	4	6	10	15	20
P	0.15	0.16	0.2	0.4	0.1

b)

Y	12	10	8	25
P	0.2	0.1	0.3	0.4

$X + Y$ yig'indining taqsimot qonunini tuzing. X, Y va $X + Y$ miqdorlarning dispersiyasini toping.

1163. X va Y tasodifiy miqdorlarning taqsimot qonunlari berilgan:

a)

X	10	20	30
P	0.2	0.3	0.5

b)

Y	6	8
P	0.4	0.6

$X-Y$ yig'indining taqsimot qonunini tuzing. $D(X)$, $D(Y)$, $D(X-Y)$ larni toping.

1164. X va Y ning matematik kutilishi ma'lum bo'lsa, Z tasodifiy miqdorning matematik kutilishini toping.

1165. X diskret tasodifiy miqdor uchta mumkin bo'lgan qiymatni qabul qiladi: $x_1 = 4$ ni $p_1 = 0.5$ ehtimol bilan, $x_2 = 6$ ni $p_2 = 0.3$ ehtimol bilan, x_3 ni p_3 ehtimol bilan. $M(X) = 8$ ni bilgan holda, x_3 va p_3 ni toping.

1166. X va Y tasodifiy miqdorlar erkli. Agar $D(X) = 5$, $D(Y) = 6$ bo'lsa, $Z = 3X + 2Y$ tasodifiy miqdorning dispersiyasini toping.

1167. A hodisaning har bir sinovda ro'y berish ehtimoli o'zgarmas hamda $M(X) = 1.2$ ekanligi ma'lum bo'lsa, X diskret tasodifiy miqdor – ikkita erkli sinovda A hodisaning ro'y berish sonining dispersiyasini toping.

1168. X diskret tasodifiy miqdor faqat ikkita mumkin bo'lgan x_1 va x_2 qiymatga ega bo'lib, $x_2 \geq x_1$. X ning x_1 qiymatni qabul qilish ehtimoli 0.6 ga teng. Matematik kutilish va dispersiya ma'lum: $M(X) = 1.4$; $D(X) = 0.24$. X miqdorning taqsimot qonunini tuzing.

46-MAVZU. TANLANMA METOD

TANLANMANING STATISTIK TAQSIMOTI

Tanlanmaning statistik taqsimoti deb, variatsion qatorning variantalari va ularga mos keluvchi n_i chastotalari yoki W_i nisbiy chastotalari ro'yxatiga aytiladi va quyidagicha yoziladi:

x_i: x_1 x_2 ... x_k

n_i: n_1 n_2 ... n_k

$n_1 + n_2 + \cdots + n_k = n$

yoki

x_i: x_1 x_2 ... x_k

W_i: W_1 W_2 ... W_k

$W_1 + W_2 + \cdots + W_k = 1$

Misol 1169. Gruppadagi 20 talabaning bo'yi o'lchanib, quyidagi ma'lumotlar olindi:

Talabaning tartib nomeri	Bo'yi (sm)	Talabaning tartib nomeri	Bo'yi (sm)
1	145	11	163
2	147	12	171
3	151	13	174
4	155	14	145
5	163	15	169
6	147	16	163
7	168	17	166
8	155	18	168
9	179	19	151
10	155	20	155

Bu ma’lumotlarga ko’ra taqsimotni tuzing.

Yechilishi: $n = 20$.

x_i	Bo’yi (sm)	145	147	151	155	163	166	168	169	171	174	179
n_i	Tala-balar soni	2	2	2	4	3	1	2	1	1	1	1
W_i	$\frac{n_i}{n}$	0.1	0.1	0.1	0.2	0.15	0.05	0.1	0.05	0.05	0.05	0.05

$W_1 = \frac{n_1}{n} = \frac{2}{20} = 0.1;\ W_4 = \frac{n_4}{n} = \frac{4}{20} = 0.2;$

$W_5 = \frac{n_5}{n} = \frac{3}{20} = 0.15;\ W_6 = \frac{n_6}{n} = \frac{1}{20} = 0.05.$

Tekshirish: $W_1 + W_2 + \cdots + W_k = 1 \Rightarrow$

$\Rightarrow 0.1 \cdot 4 + 0.2 \cdot 1 + 0.15 \cdot 1 + 0.05 \cdot 5 = 0.4 + 0.2 + 0.15 + 0.25 = 1.$

Misol 1170. Tanlanma

x_i	5	7	12
n_i	2	5	3

chastotalar taqsimoti ko’rinishida berilgan. Nisbiy chastotalar taqsimotini tuzing.

Yechilishi:

Tanlanma hajmi $n = 2 + 5 + 3 = 10;$

U holda

$W_1 = \frac{n_1}{n} = \frac{2}{10} = 0.2;\ W_2 = \frac{n_2}{n} = \frac{5}{10} = 0.5;\ W_3 = \frac{n_3}{n} = \frac{3}{10} = 0.3.$

Demak

x_i	5	7	12
W_i	0.2	0.5	0.3

bo'ladi.

STATISTIK TAQSIMOTNING INTERVALLI VARIATSION QATORI

Tanlanma hajmi *n* katta bo’lganda tanlanmaning statistik taqsimotini tuzishda intervallardan foydalaniladi. Variantalar(ma’lumotlar)ning har biri birorta intervalning ichida joylashishi shart.

Intervallar soni *k* Sterlej formulasi yordamida topiladi:

$k = 1 + 3.322 lgn.$

Intervalning uzunligi

$h = \frac{x_{max} - x_{min}}{k}.$

formula yordamida topiladi.

x_{max} – eng katta varianta;

x_{min} – eng kichik varianta.

Misol 1171. Paxta maydonidan tasodifiy ravishda $n = 50$ tup g'o'za olindi va har biridan terib olingan paxta hosili alohida-alohida tortilganda og'irligi gramm hisobida quyidagicha bo'ldi:

38.0	51.5	48.3	40.8	33.2	40.2	49.2	34.6	32.0	27.5
41.3	43.2	42.0	30.3	48.0	43.0	36.0	39.6	38.2	56.0
47.4	53.8	45.6	33.2	38.2	39.0	35.0	40.5	45.0	44.4
30.0	35.7	43.5	42.1	42.0	37.3	42.8	50.3	44.6	46.3
59.0	49.0	37.8	45.0	36.1	44.3	51.7	44.5	48.5	36.4

Shu tanlanma to'plamning statistik taqsimotini tuzing.

Yechilishi. $n = 50$.

Sterlej formulasi yordamida intervallar sonini topamiz:

$$k = 1 + 3.322lgn = 1 + 3.322lg50 = 1 + 3.322lg10 \cdot 5 = 1 + 3.322 \cdot (lg10 + lg5)$$
$$== 1 + 3.322(1 + 0.6990) + 1 + 3.322 + 2.3227 = 6.644 \approx 7$$

Intervalning uzunligini topamiz:

$$h = \frac{x_{max} - x_{min}}{k} = \frac{59 - 27.5}{6.644} = \frac{31.5}{6.644} = 4.74$$

Uzunligi $h = 4.74$ ga teng bo'lgan intervallar yoziladi (bunda biror son ixtiyoriy tanlanib unga 4.74 qo'shiladi. Natijada yuqoridagi variantalar hosil bo'lgan intervalning ichiga tushadigan bo'ladi):

$[26.50 - 31.24]$, $[31.24 - 35.96]$, $[35.96 - 40.72]$,
$[40.72 - 45.46]$, $[45.46 - 50.20]$, $[50.20 - 54.94]$,
$[54.94 - 59.68]$.

Har bir intervalga tegishli n_i chastotalar soni topiladi:

Birinchi intervalga tushadigan variantalar: $27.5;\ 30.0;\ 30.3;\ n_1 = 3;$

Ikkinchi intervalga tushadigan variantalar:

$$32.0;\ 33.2;\ 33.2;\ 34.6;\ 35.0;\ 35.7;\ n_2 = 6;$$

Uchinchi intervalga tushadigan variantalar:

$$36.0;\ 36.1;\ 36.4;\ 37.3;\ 37.8;\ 38.0;\ 38.2;\ 39.0;\ 39.6;\ 40.2;\ 40.5;\ n_3 = 12;$$

To'rtinchi intervalga tushadigan variantalar:

$$40.8;\ 41.3;\ 42.0;\ 42.0;\ 42.1;\ 42.6;\ 43.0;\ 43.2;$$
$$43.5;\ 44.3;\ 44.4;\ 44.5;\ 44.6;\ 45.0;\ 45.0;\ n_4 = 15;$$

Beshinchi intervalga tushadigan variantalar:

$$45.6;\ 46.0;\ 46.3;\ 47.4;\ 48.0;\ 48.3;\ 48.5;\ 49.2;\ n_5 = 8;$$

Oltinchi intervalga tushadigan variantalar: $50.3;\ 51.5;\ 51.7;\ 53.8;\ n_6 = 4;$

Yettinchi intervalga tushuvchi variantalar: $56.0;\ 59.0;\ n_7 = 2;$

Tekshirish:

$$n = n_1 + n_2 + n_3 + n_4 + n_5 + n_6 + n_7 = 50$$

Nisbiy chastotalar hisoblanadi:

$$W_1 = \frac{n_1}{n} = \frac{3}{50} = 0.06; \quad W_2 = \frac{n_2}{n} = \frac{6}{50} = 0.12; \quad W_3 = \frac{n_3}{n} = \frac{12}{50} = 0.24;$$
$$W_4 = \frac{n_4}{n} = \frac{15}{50} = 0.30; \quad W_5 = \frac{n_5}{n} = \frac{8}{50} = 0.16; \quad W_6 = \frac{n_6}{n} = \frac{4}{50} = 0.08;$$
$$W_7 = \frac{n_7}{n} = \frac{2}{50} = 0.04;$$

Tekshirish:

$$W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 =$$

$= 0.06 + 0.12 + 0.24 + 0.3 + 0.16 + 0.08 + 0.04 = 1$

Yuqoridagi hisoblashlar asosida 50 ta ma’lumotning statistik taqsimoti tuziladi:

Variantalar intervallari	Intervalga tegishli variantalar soni (chastotasi) n_i	Intervalning o’rtasi	Nisbiy chastota W_i
$26.50 - 31.24$	3	28.87	0.06
$31.24 - 35.96$	6	33.61	0.12
$35.96 - 40.72$	12	38.35	0.24
$40.72 - 45.46$	15	43.09	0.30
$45.46 - 50.20$	8	47.83	0.16
$50.20 - 54.94$	4	52.57	0.08
$54.94 - 59.68$	2	57.31	0.04

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

1172. Tajriba uchastkasidagi olmalardan 45 tasi tortib ko’rilganda ularning og’irliklari gramm hisobida quyidagicha bo’ldi:

65	28	38	55	77	100	40	46	52
80	74	70	25	30	58	46	80	69
60	25	33	50	60	74	87	90	98
22	39	60	74	80	55	92	95	38
78	70	49	30	92	100	50	43	95

Ushbu tanlanmaning variatsion qatori va statistik taqsimotini tuzing.

1173. Chastotali statistik taqsimot berilgan:

x_i	6	8	12	16	25
n_i	3	5	8	2	2

Nisbiy chastotali taqsimotni tuzing.

1174. Tanlanma

x_i	2	5	7
n_i	1	3	6

chastotalar taqsimoti ko’rinishida berilgan. Nisbiy chastotalar taqsimotini toping.

1175. Tanlanma

x_i	4	7	8	12
n_i	5	2	3	10

chastotalar taqsimoti ko’rinishida berilgan. Nisbiy chastotalar taqsimotini toping.

47-MAVZU. POLIGON VA GISTOGRAMMA

POLIGON

Hajmi *n* bo'lgan tanlanma statistik taqsimoti bilan berilgan:

x_i:	x_1	x_2	...	x_k
n_i:	n_1	n_2	...	n_k
W_i:	W_1	W_2	...	W_k

x_i – variantalar;

n_i – chastotalar;

W_i – nisbiy chastotalar;

$(x_1;\ n_1), (x_2;\ n_2), \dots, (x_k;\ n_k)$

nuqtalarni tutashtiruvchi siniq chiziqqa chastotali poligoni deyiladi.

$(x_1;\ W_1), (x_2;\ W_2), \dots, (x_k;\ W_k)$

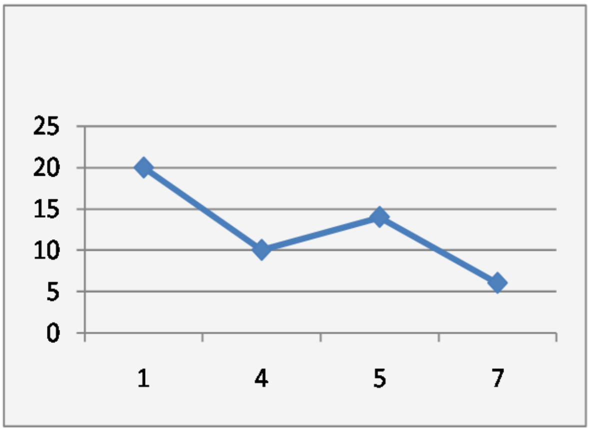
nuqtalarni tutashtiruvchi siniq chiziqqa nisbiy chastotalar poligoni deyiladi.

Siniq chiziqni yasash uchun variantalar abssissa o'qiga, chastotalar yoki nisbiy chastotalar esa ordinate o'qiga qo'yiladi.

Misol 1176. Tanlanmaning quyida berilgan taqsimoti bo'yicha chastotalar poligonini yasang:

x_i	1	4	5	7
n_i	20	10	14	6

$A(1;20)$,
$B(4;10)$,
$C(5;14)$,
$D(7;6)$.



GISTOGRAMMA

Statistik taqsimotning gistogrammasini yasash uchun variantalarni uzunligi *h* gat eng bo'lgan ketma-ket intervallarga bo'linadi va har bir intervalga tushgan variantalarning chastotalari topiladi.

Asoslari *h* uzunlikdagi intervallardan, balandliklari $\frac{n_i}{h}$ nisbatdan iborat bo'lgan pog'onaviy to'g'ri to'rtburchaklarga chastotalar gistogrammasi deyiladi.

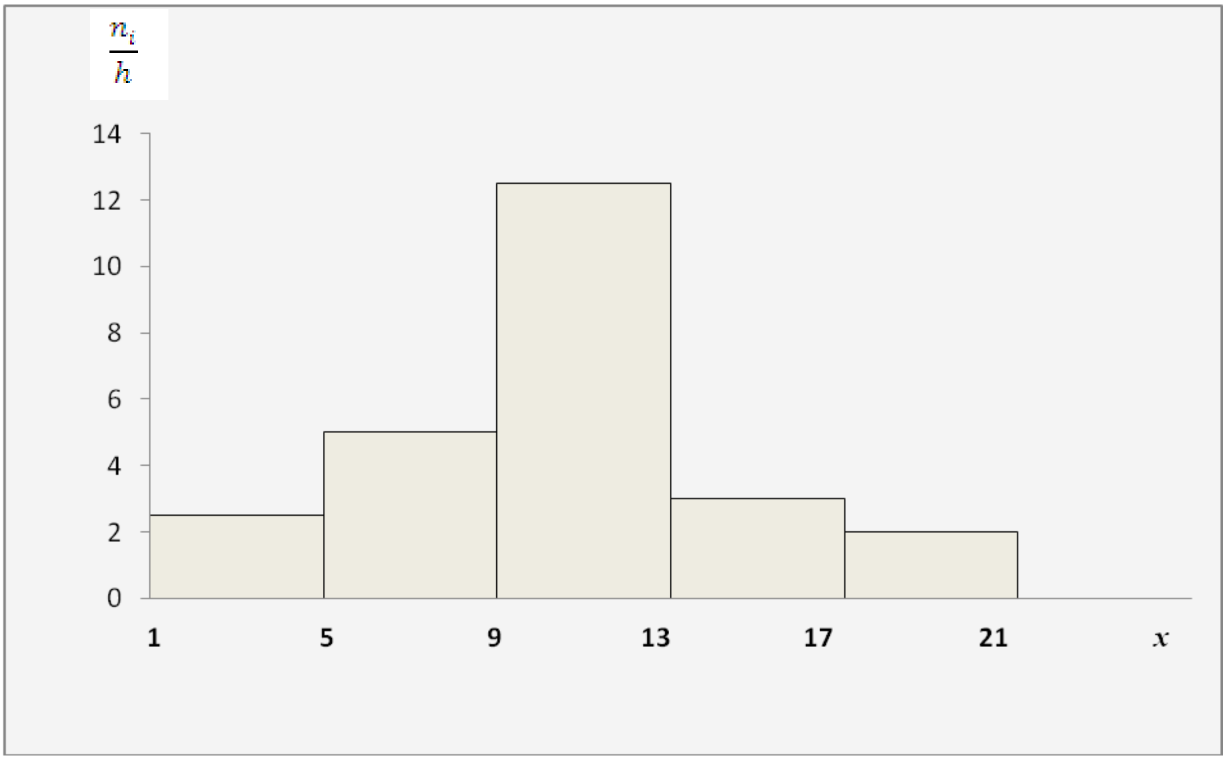
Asoslari *h* uzunlikdagi intervallardan, balandliklari $\frac{W_i}{h}$ nisbatdan iborat bo'lgan pog'onaviy to'g'ri to'rtburchaklarga nisbiy chastotalar gistogrammasi deyiladi.

Chastotalar gistogrammasining yuzi tanlanma hajmi *n* ga, nisbiy chastotalar gistogrammasining yuzi esa birga teng bo'ladi.

Misol 1177. $n = 100$ hajmli tanlanmaning quyida berilgan taqsimoti bo'yicha chastotalar gistogrammasini yasang:

Interval nomeri	Qismiy intervallar	Intervaldagi variantalar chastotalari yig'indisi	Chastota zichligi
1	1 – 5	10	2.5
2	5 – 9	20	5
3	9 – 13	50	12.5
4	13 – 17	12	3
6	17 – 21	8	2

Yechilishi. Abssisaga $h = 4$ intervallarni qo'yib chiqamiz, $\frac{n_i}{h}$ zichlikni ordinataga joylashtiramiz va ulardan abssissaga parallel to'g'ri chiziqlar o'tkazib, pog'onaviy to'rtburchaklarni, ya'ni chastotalar gistogrammasini yasaymiz. Chastotalar gistogrammasining yuzi tanlanma hajmi *n* ga teng bo'ladi.



Misol 1178. Tanlanmaning quyidagi taqsimoti bo'yicha nisbiy chastotalar gistogram-masini yasang.

Interval nomeri	Qismiy intervallar	Qismiy intervaldagi variantalar chastotalarining yig'indisi
i	$h = x_i + x_{i+1}$	n_i
1	0 – 2	20
2	2 – 4	30
3	4 – 6	50

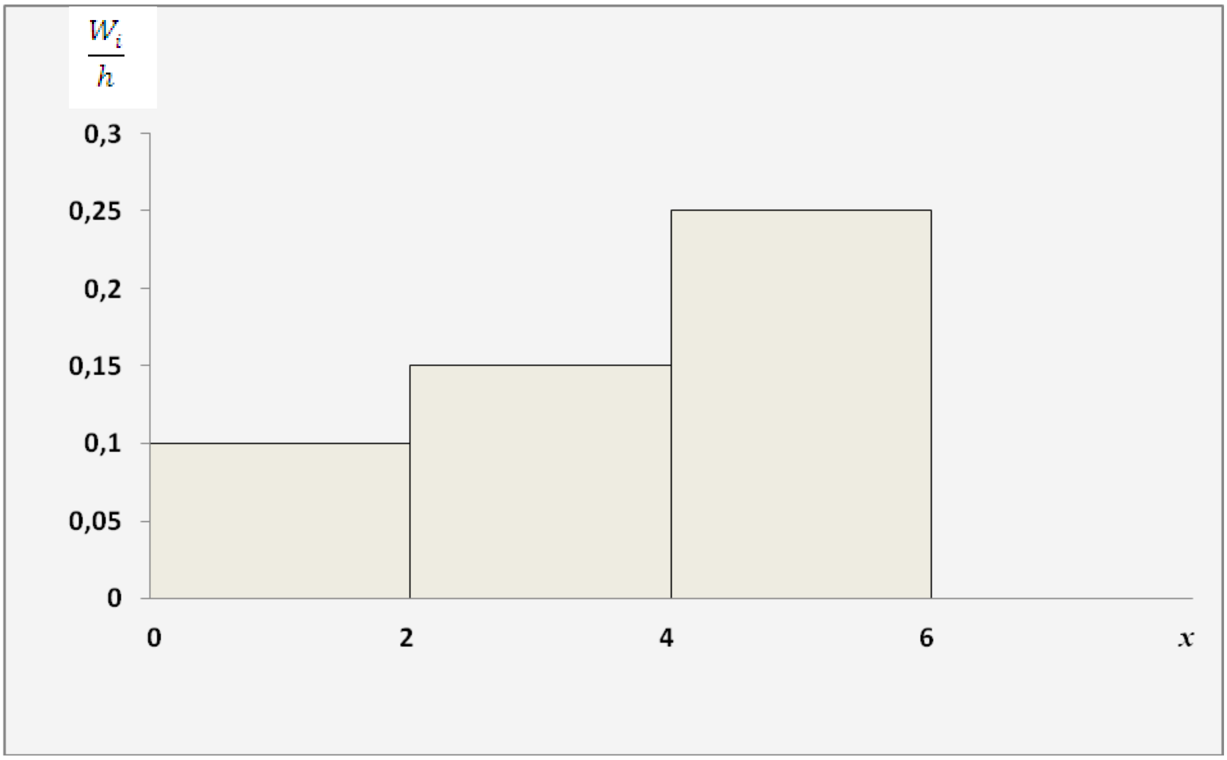
$n = 20 + 30 + 50 = 100$

Yechilishi.

$$W_1 = \frac{20}{100} = 0.2;\ \ W_2 = \frac{30}{100} = 0.3;\ \ W_3 = \frac{50}{100} = 0.5.\ \ h = 2.$$

Nisbiy chastotalar zichligini topamiz:

$$\frac{W_1}{h} = \frac{0.2}{2} = 0.1;\ \ \frac{W_2}{h} = \frac{0.3}{2} = 0.15;\ \ \frac{W_3}{h} = \frac{0.5}{2} = 0.25.$$



Nisbiy chastotalar gistogrammasining yuzi 1 ga teng.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

1179. Tanlanmaning quyida berilgan taqsimoti bo'yicha chastotalar poligonini yasang:

a)

x_i	2	3	5	6
n_i	10	15	5	20

b)

x_i	15	20	25	30	10
n_i	10	15	30	30	25

1180. Tanlanmaning quyidagi berilgan taqsimoti bo'yicha nisbiy chastotalar poligonini yasang:

a)

x_i	2	4	5	7	10
W_i	0.15	0.2	0.1	0.1	0.45

b)

x_i	1	4	5	8	9
W_i	0.15	0.25	0.3	0.2	0.1

c)

x_i	20	40	65	80
W_i	0.1	0.2	0.3	0.4

1181. Tanlanmaning quyida berilgan taqsimoti bo'yicha chastotalar gistogrammasini yasang.

a)

i	$h = x_i - x_{i+1}$	n_i	$\frac{n_i}{h}$
1	2 – 7	5	
2	7 – 12	10	
3	12 – 17	25	
4	17 – 22	6	
5	22 – 27	4	

b)

i	$h = x_i - x_{i+1}$	n_i	$\frac{n_i}{h}$
1	3 – 5	4	
2	5 – 7	6	
3	7 – 9	20	
4	9 – 11	40	
5	11 – 13	20	
6	13 – 15	4	
7	15 – 17	6	

1182. Tanlanmaning quyida berilgan taqsimoti bo'yicha nisbiy chastotalar gistogrammasini yasang:

a)

i	$h = x_i - x_{i+1}$	n_i
1	10 – 15	2
2	15 – 20	4
3	20 – 25	8
4	25 – 30	4
5	30 – 35	2
		$n = 20$

b)

i	$h = x_i - x_{i+1}$	n_i
1	2 – 5	6
2	5 – 8	10
3	8 – 11	4

4	11 – 14	5
		$n = 25$

48-MAVZU: KVADRATIK FORMA

V vektor fazo va *R* haqiqiy sonlar to'plami berilgan bo'lib, *V* vektor fazoning har bir $\vec{x}$ vektoriga, *R* haqiqiy sonlar to'plamidan aniq bitta son mos keltirilgan bo'lsa, *V* vektor fazoda vektor argumentli skalyar funksiya berilgan deyiladi va u $\varphi = \varphi(\vec{x})$ ko'rinishda yoziladi.

Agar *V* vektor fazoning ixtiyoriy ikki $\vec{x}$ va $\vec{y}$ vektorlariga, *R* haqiqiy sonlar to'plamidan aniq bitta haqiqiy soni mos kelsa, u holda *V* vektor fazoda ikki argumentli skalyar funksiya aniqlangan deyiladi va uni $\varphi = \varphi(\vec{x},\vec{y})$ ko'rinishda yoziladi.

Agar shu ikkita vektor argumentli funksiyaning o'ng tomoni ikkinchi darajali ko'phaddan iborat, ya'ni, $\varphi(\vec{x},\vec{y}) = a_{11}x_1y_1 + a_{12}x_1y_2 + a_{13}x_1y_3 + \dots + a_{nn}x_ny_n$ yoki qisqacha

$$\varphi(\vec{x},\vec{y}) = \sum_{i,j=1}^{n} a_{ij}\cdot x_i \cdot y_j$$

bo'lsa, bu o'ng tomondagi ko'phadni bichiziqli forma deyiladi.

Bichiziqli formaning koeffitsiyentlaridan tuzilgan

$$M = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

matrisani, bichiziqli formaning matrisasi deyiladi.

Xususiy holda

$\varphi(\vec{x},\vec{y}) = x_1y_1 + x_2y_2 + \dots + x_ny_n$

Bichiziqli formaning matrisasi

$$M = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

ko'rinishda bo'ladi.

Ta'rif. Agar ixtiyoriy $\vec{x},\vec{y} \in V$ vektorlar uchun $\varphi(\vec{x},\vec{y}) = \varphi(\vec{y},\vec{x})$ munosabat o'rinli bo'lsa, $\varphi(\vec{x},\vec{y})$ bichiziqli formani simmetrik bichiziqli forma deyiladi. $\varphi(\vec{x},\vec{y}) = -\varphi(\vec{y},\vec{x})$ bo'lsa, $\varphi(\vec{x},\vec{y})$ bichiziqli formani antisimmetrik bichiziqli forma deyiladi.

Simmetrik bichiziqli formaning matrisasi ham simmetrik bo'ladi va bu matrisaning rangi deb yuritiladi. Antisimmetrik bichiziqli formaning bosh diagonalidagi elementlari nol bo'ladi.

$\varphi(\vec{x},\vec{y})$ simmetrik bichiziqli formada $\vec{x} = \vec{y}$ deb olishdan hosil qilinadigan $\varphi(\vec{x},\vec{x})$ formaga kvadratik forma deyiladi. Bunday holda $\varphi(\vec{x},\vec{y})$ qutbiy forma deb yuritiladi.

Misol. Ikkita x_1 va x_2 o'zgaruvchili kvadratik formaning umumiy ko'rinishi quyidagicha bo'ladi:

$\varphi(\vec{x},\vec{x}) = a_{11}x_1x_1 + 2a_{12}x_1x_2 + a_{22}x_2x_2$

Shunga o'xshash uchta x_1, x_2 va x_3 o'zgaruvchilarning kvadratik formasi quyidagi ko'rinishda bo'ladi:

$\varphi(\vec{x},\vec{x}) = a_{11}x_1x_1 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{21}x_2x_1 + a_{22}x_2x_2 + a_{23}x_2x_3 +$
$+a_{31}x_3x_1 + a_{32}x_3x_2 + a_{33}x_3x_3 =$

$= a_{11}x_1x_1 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3 + a_{22}x_2x_2 + a_{33}x_3x_3$

Bularni quyidagi ko’rinishda yozsa ham bo’ladi:

$\varphi(\vec{x},\vec{x}) = a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2;$

$\varphi(\vec{x},\vec{x}) = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3;$

$$\varphi(\vec{x},\vec{x}) = a_{11}x_1^2 + a_{22}x_2^2 + \ \dots \ + \ a_{nn}x_n^2.$$

ko’rinishdagi kvadratik formani, uning kanonik ko’rinishi deyiladi.

Kanonik ko’rinishdagi kvadratik formaning matrisasi

$$M = \begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix}$$

ko’rinishda bo’ladi.

1-teorema. Agar kvadratik formada birorta ham o’zgaruvchining kvadrati qatnashma-gan bo’lsa, uni chiziqli almashtirishlar yordamida eng kamida bitta o’zgaruvchining kvadrati qatnashgan kvadratik formaga keltirish mumkin.

Isboti. Teorema shartiga ko’ra $a_{11} = a_{22} = \ \dots \ = a_{nn} = 0.$

U holda kvadratik forma $\varphi(\vec{x},\vec{x}) = 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + \ \dots \ + 2a_{n-1,n}x_{n-1}, x_n$ ko’rinishda bo’ladi.

Bunda eng kamida bitta o’zgaruvchining kvadratini hosil qilish uchun quyidagi chiziqli almashtiruvchi olinadi:

$x_1 = y_1 + y_2$

$x_2 = y_1 - y_2$

$x_3 = y_3$

$$\dots$$

$x_n = y_n$

Bu chiziqli almashtirish aynimagan, sababi $\begin{vmatrix} 1 & 1 & 0 & \dots & 0 \\ 1 & -1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{vmatrix} \neq 0$

Bu holda

$\varphi(\vec{x},\vec{x}) = 2a_{12}y_1^2 - 2a_{12}y_2^2 + 2a_{13}y_1y_3 + 2a_{23}y_2y_3 + \ \dots + 2a_{n-1,n}y_{n-1}y_n$

kelib chiqadi.

Misol. $\varphi(\vec{x},\vec{x}) = 2x_1x_3 - x_2x_3$ kvadratik formani o’zgaruvchilarning kvadratlari qatnashgan holga keltiring.

Yechilishi. Chiziqli almashtirish olinadi:

$$\begin{cases} x_1 = y_1 + y_3 \\ x_2 = y_2 \\ x_3 = y_1 + y_3 \end{cases}$$

Chiziqli almashtirishning aynimaganligi tekshiriladi:

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = -1 - 1 = 2 \neq 0$$

Demak, chiziqli almashtirish aynimagan.

Endi x_1, x_2, x_3 lar almashtirilgan ifodalar $\varphi(\vec{x},\vec{x})$ kvadratik formaga qo’yiladi:

$\varphi(\vec{x},\vec{x}) = 2(y_1 + y_3)(y_1 - y_3) - y_2(y_1 - y_3) = 2(y_1^2 - y_3^2) - y_1y_2 + y_2y_3 \Rightarrow$

$\Rightarrow \varphi(\vec{x},\vec{x}) = 2y_1^2 - 2y_3^2 - y_1y_2 + y_2y_3.$

2-teorema. Bir o'zgaruvchining kvadrati va undan boshqa hadlarda shu o'zgaruvchi qatnashsa, chiziqli almashtirishlar yordamida, bunday kvadratik formani, barcha o'zgaruvchilarning kvadratlari qatnashgan kvadratik formaga keltirish mumkin.

Bu teorema aynimaganligini misolda ko'rsatamiz.

Misol. $\varphi(\vec{x},\vec{x}) = x_1^2 - 4x_1x_2 - 4x_1x_3 + 3x_2^2$.

Yechilishi. x_1^2 va shu o'zgaruvchi qatnashayotgan hadlar bor. Bunda

$$\begin{cases} y_1 = a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ y_2 = x_2 \\ \dots \\ y_n = x_n \end{cases}$$

Almashtirish asosida, quyidagi almashtirishni olamiz:

$y_1 = x_1 - 2x_2 - 2x_3;$
$y_2 = x_2;$
$y_3 = x_3.$

Chiziqli almashtirishning aynimaganligi tekshriladi:

$$\begin{vmatrix} 1 & -2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \neq 0$$

$y_1$almashtirish kvadratga ko'paytiriladi:

$y_1^2 = x_1^2 + 4x_2^2 + 4x_3^2 - 4x_1x_2 - 4x_1x_3 + 8x_2x_3.$

$\varphi(\vec{x},\vec{x}) - y_1^2$ topiladi.

U holda

$$\varphi(\vec{x},\vec{x}) - y_1^2 = x_1^2 - 4x_1x_2 - 4x_1x_3 + 3x_2^2 - \\ -(x_1^2 + 4x_2^2 + 4x_3^2 - 4x_1x_2 - 4x_1x_3 + 8x_2x_3)$$

$$\varphi(\vec{x},\vec{x}) - y_1^2 = -x_2^2 - 4x_3^2 - 8x_2x_3$$

U holda,

$$\varphi(\vec{x},\vec{x}) = y_1^2 - y_2^2 - 4y_3^2 - 8y_2y_3$$

bo'ladi.

3-teorema. Chiziqli almashtirishlar yordamida har qanday kvadratik formani kanonik ko'rinishga keltirish mumkin.

Misol. $\varphi(\vec{x},\vec{x}) = x_1^2 + 2x_1x_2 + 2x_2^2 + 4x_2x_3 + 8x_3^2$ kvadratik formani kanonik ko'ri-nishga keltiring.

Yechilishi. Berilgan kvadratik formani quyidagi ko'rinishda yozib olamiz.

$$\varphi(\vec{x},\vec{x}) = x_1^2 + 2x_1x_2 + x_2^2 + x_2^2 + 4x_2x_3 + (2x_3)^2 + 4x_3^2 = \\ = (x_1 + x_2)^2 + (x_2 + x_3)^2 + 4x_3^2$$

Oxirgi natijani e'tiborga olgan holda quyidagi chiziqli almashtirish olinadi:

$y_1 = x_1 + x_2;$
$y_2 = x_2 + 2x_3;$
$y_3 = x_3.$

Chiziqli almashtirishning aynimaganligi tekshiriladi:

$$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = 1 \neq 0$$

Natijada

$$\varphi(\vec{x},\vec{x}) = y_1^2 + y_2^2 + 4y_3^2$$

kanonik shakli hosil bo'ladi.

MUSTAQIL BAJARISH UCHUN TOPSHIRIQLAR

Quyidagi kvadratik formalarni kanonik ko'rinishga keltiring.

1. $\varphi(\vec{x},\vec{x}) = 27x_1^2 - 10x_1x_2 + 3x_2^2$
2. $\varphi(\vec{x},\vec{x}) = 2x_1^2 + 8x_1x_2 + 8x_2^2$
3. $\varphi(\vec{x},\vec{x}) = 3x_1^2 + 2x_2^2 + x_3^2 + 4x_1x_2 + 4x_2x_3$
4. $\varphi(\vec{x},\vec{x}) = 6x_1^2 + 3x_2^2 + 3x_3^2 + 4x_1x_2 + 4x_1x_3 - 8x_2x_3$.

49-MAVZU: XOSMAS INTEGRALLAR.
Cheksiz chegarali integrallar

Integrallash chegarasi cheksiz bo'lgan hol uchun aniq integral tushunchasini umumlashtiramiz.

Ta'rifga o'tishdan oldin misol qaraymiz.

Misol. $y = \frac{1}{x^2}$ funksiya $[1;+\infty)$ intervalda uzluksiz. Shuning uchun istalgan $[1;b]$ segmentda $\int_1^b \frac{dx}{x^2} = 1 - \frac{1}{b}$ integral mavjud. Bu integral $b \to \infty$ da 1 ga teng limitga ega. Bu limitni $\frac{1}{x^2}$ funksiyadan olingan xosmas integral deyiladi va $\int_1^{+\infty} \frac{dx}{x^2}$ ko'rinishda yoziladi.

Shunday qilib, $\int_1^{+\infty} \frac{dx}{x^2} = \lim_{b\to\infty} \int_1^b \frac{dx}{x^2} = \lim_{b\to\infty}\left(1 - \frac{1}{b}\right) = 1$ bo'ladi.

Ushbu misolni umumlashtirib, $a \le x < +\infty$ intervalda uzluksiz $y = f(x)$ funksiyani qaraymiz.

$[a;b]$ segmentda $\int_a^b f(x)dx$ integral mavjudligi bizga oldingi mavzulardan ma'lum.

Ta'rif. Agar $b \to \infty$ da, $\int_a^b f(x)dx$ integral chekli limitga intilsa, u holda bu limitni $f(x)$ funksiyaning xosmas integrali deyiladi va $\int_a^{+\infty} f(x)dx$ ko'rinishida belgilanadi.

Demak,

$$\int_a^{+\infty} f(x)dx = \lim_{b\to\infty} \int_a^b f(x)dx \text{ bo'ladi.}$$

Bunday holda $\int_a^{+\infty} f(x)dx$ xosmas integral mavjud yoki yaqinlashadi deyiladi. Agar shu limit mavjud bo'lmasa yoki cheksiz bo'lsa, u holda integral mavjud emas yoki uzoqlashmadi deyiladi.

Quyi chegarasi cheksiz bo'lgan xosmas integral ham shunday aniqlanadi:

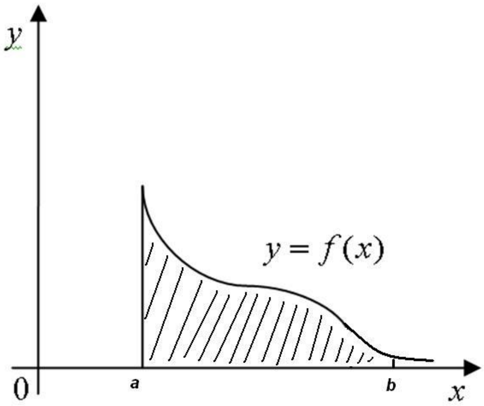
$$\int_{-\infty}^{b} f(x)dx = \lim_{a\to-\infty} \int_a^b f(x)dx.$$

Ikki chegarasi ham cheksiz bo'lgan xosmas integral quyidagicha yoziladi:

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-\infty}^{c} f(x)dx + \int_c^{+\infty} f(x)dx.$$

Bu yerda *c – (Ox)* o'qining ixtiyoriy nuqtasi.

Shunday qilib, $\int_{-\infty}^{+\infty} f(x)dx$ integral $\int_{-\infty}^{c} f(x)dx$ va $\int_{c}^{+\infty} f(x)dx$ integrallarning har biri mavjud bo'lgandagina mavjud bo'ladi.



Keltirilgan ta'riflardan ko'rinib turibdiki, xosmas integral integral yig'indidan emas, balki o'zgaruvchan chegarali aniq integralning limitidan iborat ekan.

Agar $f(x)$ funksiya $[a;+\infty]$ intervalda uluksiz va $\int_{a}^{+\infty} f(x)dx$ musbat integral mavjud bo'lsa, uni chapdan $x=a$ to'g'ri chiziq, yuqoridan $y=f(x)$ funksiya quyidan *(Ox)* abscissa o'qi bilan chegaralangan va o'ng tomondan chegaralanmagan egri chiziqli trapetsiya sifatida talqin qilish mumkin.

Misol 1. $\int_{1}^{+\infty} \frac{dx}{x^{\alpha}}$ integralni qaraymiz.

1) Agar $\alpha \neq 1$ bo'lsa, $\int_{1}^{b} \frac{dx}{x^{\alpha}} = \frac{x^{1-\alpha}}{1-\alpha}\Big|_{1}^{b} = \frac{1}{1-\alpha}\left(b^{1-\alpha}-1\right)$.

2) Agar $\alpha = 1$ bo'lsa, $\int_{1}^{b} \frac{dx}{x} = \ln x\big|_{1}^{b} = \ln b.$

3) Agar $\alpha > 1$ bo'lsa, $\alpha - 1 > 0$ bo'ladi. Shuning uchun $b^{1-\alpha}$ ning limiti: $\lim_{b\to+\infty}\left(b^{1-\alpha}\right) = \lim_{b\to+\infty}\frac{1}{b^{\alpha-1}} = 0$ bo'ladi. Demak, bu holda $\lim_{b\to+\infty}\int_{1}^{b}\frac{dx}{x^{\alpha}} = \lim_{b\to+\infty}\frac{1}{1-\alpha}\left(b^{1-\alpha}-1\right) = \frac{1}{\alpha-1}$.

4) $\alpha < 1$ bo'lsin, u holda $\lim_{b\to+\infty}\int_{1}^{b}\frac{dx}{x^{\alpha}} = \lim_{b\to+\infty}\frac{b^{1-\alpha}-1}{1-\alpha} = +\infty$ bo'ladi.

$\alpha = 1$ bo'lsa, $\lim_{b\to+\infty}\int_{1}^{b}\frac{dx}{x} = \lim_{b\to+\infty}\ln x = +\infty$ bo'ladi.

Demak, $\alpha > 1$ bo'lsa, $\int_{1}^{+\infty}\frac{dx}{x^{\alpha}}$ yaqinlashadi. $\alpha \le 1$ da uzoqlashadi.

Misol 2. $\int_{-\infty}^{+\infty}\frac{dx}{1+x^2}$ integralning yaqinlashishini tekshiring.

Yechilishi. $c=0$ deb olinsa, $\int_{-\infty}^{+\infty}\frac{dx}{1+x^2} = \int_{-\infty}^{0}\frac{dx}{1+x^2} + \int_{0}^{+\infty}\frac{dx}{1+x^2}$ bo'ladi. Bundan

$$\int_{-\infty}^{0}\frac{dx}{1+x^2} = \lim_{a\to-\infty}\int_{a}^{0}\frac{dx}{1+x^2} = \lim_{a\to-\infty} arctg\big|_{a}^{0} = \lim_{a\to-\infty}\left(arctg0 - arctga\right) = 0 - \left(-\frac{\pi}{2}\right) = \frac{\pi}{2}$$

yaqinlashar ekan.

Shunga o'xshash $\int_{0}^{+\infty}\frac{dx}{1+x^2} = \frac{\pi}{2}$ kelib chiqadi.

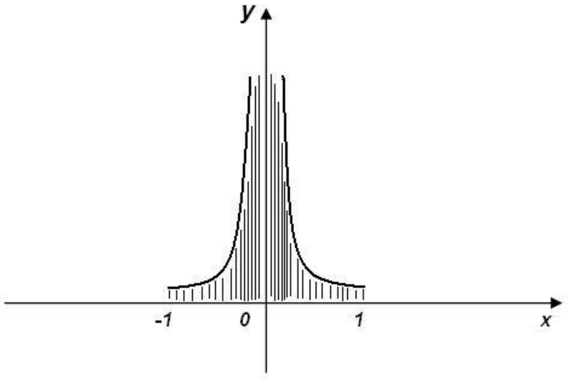
Bulardan $\int\limits_{-\infty}^{+\infty} \frac{dx}{1+x^2} = \frac{\pi}{2} + \frac{\pi}{2} = \pi$ bo'lib, integralning yaqinlashuvchi ekanligi kelib chiqadi.

Misol 3. $\int\limits_{-1}^{1} \frac{dx}{\sqrt[3]{x^2}}$ integralning yaqinlashi-shini tekshiring.

Yechilishi. $\frac{1}{\sqrt[3]{x^2}}$ funksiya $x=0$ da uzilishga ega, ya'ni $\lim\limits_{x\to 0} \frac{1}{\sqrt[3]{x^2}} = \infty$.

$$\int\limits_{-1}^{0} \frac{dx}{\sqrt[3]{x^2}} = 3; \quad \int\limits_{0}^{1} \frac{dx}{\sqrt[3]{x^2}} = 3.$$

Demak, $\int\limits_{-1}^{1} \frac{dx}{\sqrt[3]{x^2}} = \int\limits_{-1}^{0} \frac{dx}{\sqrt[3]{x^2}} + \int\limits_{0}^{1} \frac{dx}{\sqrt[3]{x^2}} = 3+3=6.$



Integral yaqinlashar ekan.

Xosmas integrallarning yaqinlashish alomatlari

Ayrim hollarda xosmas integralni hisoblab o'tirishga hojat yo'q. Uning yaqinlashuvchi yoki uzoqlashuvchi ekanligini bilish yetarli. Bunday hollarda berilgan xosmas integralni yaqinlashishi yoki uzoqlashishi ma'lum bo'lgan boshqa xosmas integral bilan taqqoslash kifoya.

Teorema 1. $[a;+\infty)$ intervalda $f(x)$ va $g(x)$ funksiyalar uzluksiz bo'lsin va $0 \le g(x) \le f(x)$ tengsizlikni qanoatlantirsin. U holda:

1) agar $\int\limits_{a}^{+\infty} f(x)dx$ integral yaqinlashsa, $\int\limits_{a}^{+\infty} g(x)dx$ integral ham yaqinlashadi;

2) agar $\int\limits_{a}^{+\infty} g(x)dx$ integral uzoqlashsa, $\int\limits_{a}^{+\infty} f(x)dx$ integral ham uzoqlashadi.

Misol 1. $\int\limits_{1}^{+\infty} \frac{xdx}{\sqrt{x^5+1}}$ integralning yaqinlashishini tekshiring.

Yechilishi. Integral ostidagi $\frac{x}{\sqrt{x^5+1}}$ funksiyani $\frac{x}{\sqrt{x^5}}$ funksiya bilan taqqoslaymiz.

Ma'lumki $1 \le x < +\infty$ intervalda $\frac{x}{\sqrt{x^5+1}} < \frac{x}{\sqrt{x^5}} = \frac{1}{x^{\frac{3}{2}}}$.

$\int\limits_{1}^{+\infty} \frac{dx}{x^{\frac{3}{2}}}$ integral yaqinlashadi, chunki $\alpha = \frac{3}{2} > 1$.

Demak, berilgan integral ham yaqinlashadi.

Misol 2. $\int\limits_{2}^{+\infty} \frac{\ln(x^2+1)}{x}dx$ integralning yaqinlashishini tekshiring.

Yechilishi. $2 \le x < +\infty$ intervalda $\ln(x^2+1) > 1$ bo'ladi.

Demak, bu intervalda $\frac{\ln\left(x^2+1\right)}{x} > \frac{1}{x}$. Lekin $\int_2^{+\infty}\frac{dx}{x}$ integral uzoqlashadi. U holda berilgan integral ham uzoqlashadi.

Teorema-2. $f(x)$ va $g(x)$ funksiyalar $[a;b]$ segmentda uzluksiz bo'lsin va $0 \le g(x) \le f(x)$ tengsizlikni qanoatlantirsin, $x=b$ nuqtada esa uzilishga ega bo'lsin. U holda:

1) agar $\int_a^b f(x)dx$ integral yaqinlashsa, $\int_a^b g(x)dx$ integral ham yaqinlashadi;

2) agar $\int_a^b g(x)dx$ integral uzoqlashsa, $\int_a^b f(x)dx$ integral ham uzoqlashadi.

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MUNDARIJA

ILOVA

30.03.2013 й. босишга рухсат этилди.
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60х84 1/16, *100* нусха.
"Shabnam omad nur" матбаа бўлимида чоп этилди.
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